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A Coordinated Planning Model with Price-Dependent Demand

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Abstract. This paper presents a coordinated planning model of price-dependent demand for a single-manufacturer and a single-retailer. The demand is assumed to be normally distributed, with its mean being price dependent. The manufacturer and retailer coordinate with each other to jointly and simultaneously determine the retail selling price and the retailer order quantity to maximize the joint expected total profit. This model is then compared to a 'returns' policy model where manufacturer buys back unsold items from the retailers. It is shown that the optimal total profit is higher for coordinated planning model than that for the returns policy model, in which the retail price is set by the retailer. A compensation or profit sharing scheme is then suggested and it is shown that the coordinated model with profit sharing yields a 'win-win' situation. Numerical results are presented to illustrate the profit patterns for both linear and nonlinear demand functions. The coordinated planning model, in addition, has a lower optimal price than for a returns policy model, which would result in higher sales, thus expanding the markets for the whole supply chain.

Keywords: Coordinated Planning, Supply Chain Management, Returns Policy, Price-dependent Demand, Manufacturer-retailers, Revenue Sharing

1. INTRODUCTION

Supply chains of fashion goods are complex because of the enormous variety of products and short life cycles, and multiple retailers. The fashion products must be often manufactured without retailer orders, making efficient management of market channels and inventory levels difficult. In addition, with the globalization of markets and the dynamics of demand caused by rapidly changing customer tastes, manufacturers have been faced with stiffer competitive prices and quality. The manufacturers and retailers should think of strategies to cooperate to bring down prices, improve quality, and cut down costs. An efficient, seamless supply chain, with a high level of coordination between the two parties becomes a necessity.

To implement these strategies, information sharing and joint planning are required. A mutual trust and a faith in working together are needed for successful cooperation. At the beginning of the spectrum are the price discounts for retail and wholesale prices, offered by retailers and wholesalers/manufacturers, respectively independent of each other. Retail price discount works only if there is demand elasticity and the demand increases with discounts. Wholesale price discounts motivate the retailers to order larger quantities. Several researchers have developed analytical models that optimize price discount and order quantity to maximize the revenue and profit (Banerjee, 1986; Weng and Wong, 1993; Parlar and Wang, 1994; Wang and Wu, 2000; Viswanathan and Piplani, 2001). For the case of consumer goods with seasonal demands and short life cycles, it has been concluded that markdown pricing increases sales quantity, especially at the end of season (Pashigian, 1988; Bils, 1989; Pashigian and Bowen, 1991; Warner and Barsky, 1995; MacDonald, 2000).

A strategy that stands at an extended level of cooperation is the coordinated ordering. Retailers and manufacturer jointly carry out the lot size planning. The objective is to maximize the total profit or minimize the total cost. Many researchers (Goyal, 1976; Banerjee, 1986; Goyal, 1995; Zahir and Sarker, 1991; Zahir, 1997) use this concept to improve inventory levels.

Another policy is a Returns Policy. Here, retailers

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can return all the unsold products to the manufacturer at a full or fraction of the wholesale price. With this type of incentive, retailers are willing to order larger quantitiy of products, increasing the level of availability to customers, and hence obtain higher profits. Emmons and Gilbert (1998) consider the returns policy to maximize the retailer profit. The optimal selling price and optimal order quantity are determined with multiplicative demand function of selling price for single-retailer case. Mantrala and Raman (1999) further develop the model subject to normally distributed demand. Demands at the retailers are linearly related, and the demand levels may be different. The optimal repurchase prices and retailer order quantities are determined to maximize individual profit of both parties.

Lau and Lau (1999) developed a model of pricing strategy and returns policy for a monopolistic manufacturer using the channel coordination. To maximize the manufacturer profit, the optimal wholesale and repurchase prices are determined under the normally distributed demand assumption. At these two optimal values, the optimal order quantity is determined to maximize the retailer profit. This paper focuses on the strategy proposed by Mantrala and Raman (1999).

Weng (1995) presents a deterministic model for analyzing the impact of joint decision policies on channel coordination. The results show that quantity discounts alone are not sufficient to guarantee joint profit maximization. Weng (1997) studies some coordinated pricing and ordering policies with non-linear demand and exponential distribution for single distributor. The optimal selling price, optimal order quantity, and optimal wholesale price are determined to maximize joint total profits. Petruzzi and Dada (1999) optimize the order quantity and selling price for maximizing retailer profit with price-dependent and stochastic demand.

Donohue (2000) examines a problem of developing supply contracts for seasonal and fashion products. He develops a decentralized system following a two-stage optimization problem and using a returns policy for stochastic demand. Nagarur *et al*. (2003) develop a coordinated planning model that uses a joint planning policy, in conjunction with a returns policy and a profit sharing scheme. The demand is assumed to have a normal distribution. The optimal order quantity and the effective wholesale price for a single manufacturer and a single retailer are determined that maximize the total profit. The results are compared to a corresponding returns policy, and a mathematical proof is presented to show that it is better than the returns policy. Since a coordinated policy can be implemented only if both the sides realize better profits, a profit sharing scheme is suggested that imbeds a returns policy.

1.1 Problem Description

This paper addresses various types of coordination in supply chains between suppliers and the retailers. The returns or buy back contracts as described above are very attractive, but their benefit has a limited scope. There is no information sharing in the supply chains under such scenario. Hence it may happen that there is no risk sharing either. In addition, the retailers bear little risk of unsold items as all the unsold goods is bought back by the manufacturer. There is not much incentive for the retailer to forecast the demands accurately, or determine safety levels carefully. Also, there is not much incentive either to move or sell the stocked items (Chopra and Meindl, 2003). Manufacturers on the other hand, bear all the risk of unsold or over stocked items. Sometimes they may be fortunate enough to ship the items bought back to other retailers. Otherwise they are stuck with the excess stock. Sometimes, it could be frustrating to the manufacturer to see that the retailers are not putting as much effort as themselves in selling the products. There is another problem with such an arrangement. The manufacturers may not get an accurate estimate of the forecasts from the retailers, since there is not much incentive for the retailers to do so, as described above. Actual sales data may be estimated from the original quantities of supplies, and goods bought back. However these estimations would only give sales aggregated over a long season, at the end of which the goods are taken back. In addition, these estimates may have little practical usefulness as they would be available only at the end of the season.

Some of the extensions to the returns contract are information sharing and coordination in order quantities. Information sharing takes care of all the deficiencies mentioned above. In addition, if order quantities are jointly decided, they would be more efficient. They will be more accurate as the information is shared and the decision is taken at the aggregate level. The total costs will be lower and profits will be higher. But then, the challenge will be how to get the retailers agree to such arrangements. What kind of incentives can be given to the retailers?

Nagarur *et al.* (2003) addresses this problem. It was shown that information sharing and joint ordering improve the performance of the supply chains in terms of total profits. The ordering quantities and overstock quantities are less and the joint profits are higher than in a corresponding returns policy or buy back scenario. As for the profit sharing, it was suggested that since the total joint profits would be higher, retailers can be given at least the increased profits they would obtain under buy back situation (Mantrala and Raman, 1999), and even a little more, if needed. Specific terms regarding profit sharing could be pre-negotiated. The manufacturers too, would improve their profits, compared to either isolation or buy back scenarios. The joint coordination scenario thus creates a ‘win-win’ situation, and would encourage both the parties to join forces.

The synergetic activities of information sharing and planning can be extended even further for mutual benefit. In such a virtual, single party/company scenario, similar activities can be envisaged and planned. Once a trust is established and benefits are realized, it would be less difficult and less problematic for such collaborations.

A logical extension would be a joint effort in fixing the price of the product. Typically, the manufacturer fixes the sale price when selling the product to the retailer, and the retailer in turn, fixes the price the customers have to pay. In either situation, the price may not be completely arbitrarily fixed, as competition and other market forces play a role, however there is some flexibility for the sellers in fixing the price. They take advantage of such flexibility, particularly if there is demand elasticity with respect to price. The profit per unit can be balanced against the higher demands and sales generated by lower prices, to increase the total profits. In a highly coordinated planning model, the manufacturer and retailers can jointly fix the price of product that the retailers sell to the final customers and, or consumers. Know-ledge about markets and market behavior can be utilized in determining a price level. Similar to the coordinated model described previously where there is a profit sharing, at a specific price, the total profits can be shared to the advantage of both the parties. The total profits here would be higher than in the previous situation, as the joint profits are maximized over a price range. The sharing of profits can be at a mutually agreeable level, and the profit share of retailers would be higher than they would if the retailers had determined on the price in isolation, and definitely much more than under buy back contracts. One may notice that in such a virtually integrated enterprise, the price of the product as sold to the retailer becomes irrelevant, as the manufacturer is virtually selling the product to the end customer directly.

Such an extended coordination opens the doors for other joint activities. Manufacturers will have a much better feel of the pulse of the markets, and they can use this information in their strategy formulations, from incorporating innovations into their products to reaping more benefits from reduction of production costs. In addition, the virtual enterprise can jointly plan at its entire chain level in coordinating marketing and promotional activities with production, sourcing, and transportation activities.

The joint determination of price and the analysis of resulting benefits is the main theme of this paper. Most of the cooperation models described above assume the price to be a constant. In the present work, price is taken as a decision variable, in an environment where demand depends on price. The demand is assumed to be random variable, with a known distribution, the mean of which depends on the retail price. The effective wholesale price, optimal selling price, and optimal order quantity are determined to maximize the total system profit. For the price dependent demand, both linear and exponential behavioral functions are analyzed.

The paper is organized as follows: in section 2, we discuss our assumptions and notation, formulate our model, and provide structural results. In section 3, characteristics of solution are presented. In section 4, numerical examples are given. It is shown that the total profits are higher for coordinated planning than in the case of individual planning. For the coordinated planning scenarios, a profit sharing scheme is suggested to obtain a ‘win-win’ situation so that the policy can be acceptable to both parties.

2. MODEL DEVELOPMENT

In this section, the mathematical models of returns policy and the coordinated planning with both linear and assumed non-linear demands are presented. The system considered represents a fashion goods product that has a very short life span, and has only one order from a retailer to the manufacturer. Based on demand forecast, retailers place orders with the manufacturer. The manufacturer may agree to take back any unsold items, at the returns price agreed in advance. The demand is assumed to depend on price. There is a vast literature available on demand elasticity. For example, Zahir and Sarker (1991) examine the benefits of supply chain compensation schemes under linear and constant price elasticity (exponential function) demand functions. In the present paper, it is assumed that the demand has a Gaussian (normal) distribution, with the mean of the demand depending on the price of the product. There is a penalty or holding cost for any overstock. Inventory holding cost is assumed to be negligible, since the operational horizon has a short span. Two types of price dependency are considered, in the first case the mean is linearly dependent, and in the second case, it is non-linearly dependent, the relationship represented by an exponential function. The retailer tries to maximize the individual profit by selecting optimal selling price and order quantity.

In the coordinated planning model, the manufacturer and the retailer take advantage of the demand elasticity to jointly determine the selling price that can yield the maximum profits. The order quantity from retailer to manufacturer is also jointly determined. The analysis of return policy in the present context serves two purposes. It shows the impact of implementing the returns strategy and serves as a base for comparison with coordinated planning strategy. It also serves as a basis for profit sharing under the coordinated planning strategy. For the whole analysis, a single manufacturer and only a single retailer are considered.

Notation:

HC	expected holding cost of overstock at retailer,
LC	expected cost of lost sales,
PC	expected total production cost,
RP	expected repurchase cost,
SP	expected sales revenue,
WP	expected total wholesale cost,

hc holding cost per unit of overstock at retailer,
lc cost of lost sales per unit of under-stock,
pc production cost per unit,
rp repurchase cost per unit,
sp selling price per unit of product at retailer,
sp_i selling price per unit of product at retailer, model i.
wp_s wholesale price per unit of product,
wp effective wholesale price per unit ,
X demand, a random variable,
f(x) probability density function of X,
F(x) cumulative density function of X,
Q order quantity,
Q_i order quantity for model i,
π_i joint expected total profit of model i,
$\pi_{R,i}$ expected retailer profit of model i,
$\pi_{S,i}$ expected manufacturer profit of model i.

The index i = 2 for the returns policy, and = 3 for the coordinated planning.

2.1 Linear demand

The case of the linear demand function is considered first. As already mentioned, the demand is assumed to be normally distributed. The price dependency shows up as the mean of the demand varying with selling price linearly as $\mu = \alpha - \beta * sp$. The values of the parameters α and β are assumed to be known. Later the results are analyzed for different values of these two parameters to check the sensitivity. The optimal selling price and the optimal order quantity are simultaneously determined to maximize the retailer profits for the case of returns policy, and the joint expected total profit for the case of coordinated planning.

2.1.1 Returns policy

Under this policy, a manufacturer offers to buy back from the retailer any unsold items at a price mutually agreed upon at the beginning. The retailer sets the price and the order quantity levels that would maximize his or her profit. The objective function of returns policy, which is the expected retailer profit, is given by

$$\pi_{R,2} = SP + RP - WP - HC - LC \tag{1}$$

Now, for any Q, expected total revenues for retailer from sales,

$$SP = \left\{ sp_2 \int_0^Q x f(x) dx + sp_2 \int_Q^\infty Q f(x) dx \right\} \tag{2}$$

Expected cost of lost sales,

$$LC = \left\{ lc \int_Q^\infty (x - Q) f(x) dx \right\} \tag{3}$$

Expected retailer holding cost of overstock (when $X < Q$) is given by

$$HC = \left\{ hc \int_0^Q (Q - x) f(x) dx \right\} \tag{4}$$

The expected wholesale revenue of manufacturer or the expected purchase cost of retailer is equal to the product of the wholesale price and the retailer order quantity,

$$WP = (wp_s \times Q) \tag{5}$$

and the expected repurchase revenue is given by

$$RP = \left\{ rp \int_0^Q (Q - x) f(x) dx \right\} \tag{6}$$

Since the demand is assumed to have a normal distribution, we will be using the error function for computational purposes. The error function is given as

$$Erf(Q) = \frac{2}{\sqrt{\pi}} \int_0^Q e^{-u^2} du$$

By substituting Eq. (2) to Eq. (6) into Eq. (1) and expressing the equation in terms of error functions, we get

$$\begin{aligned}
\pi_{R,2} = \Bigg(& \frac{\sigma(hc + sp_2 - rp)}{\sqrt{2\pi}} e^{-\frac{(-\alpha+\beta sp_2)^2}{2\sigma^2}} \\
& + \frac{\sigma(rp - sp_2 - lc - hc)}{\sqrt{2\pi}} e^{-\frac{(Q_2-\alpha+\beta sp_2)^2}{2\sigma^2}} \\
& + \frac{\beta sp_2(sp_2 - rp + hc) + \alpha(rp - hc - sp_2) + Q_2(hc - rp)}{2} \\
& \times Erf\left[\frac{-\alpha + \beta sp_2}{\sqrt{2}\sigma}\right] \\
& + \frac{\begin{pmatrix} Q_2(rp - hc - lc - sp_2) + \alpha(hc + lc + sp_2 - rp) \\ + \beta sp_2(rp - hc - lc - sp_2) \end{pmatrix}}{2} \\
& \times Erf\left[\frac{Q_2 - \alpha + \beta sp_2}{\sqrt{2}\sigma}\right] \\
& + \frac{(Q_2 sp_2 + Q_2 lc + \beta\, lcsp_2 - \alpha lc - 2Q_2 wp_s)}{2} \Bigg)
\end{aligned} \tag{7}$$

To obtain the optimal order quantity and the optimal selling price, first derivatives with respect to Q_2 and sp_2 are taken for the above quantity,

$$\partial_{Q2}(\pi_{R,2}) = \frac{(hc - rp)}{2} Erf\left[\frac{-\alpha + \beta sp_2}{\sqrt{2}\sigma}\right]$$

$$+\frac{(rp-hc-lc-sp_2)}{2}\mathrm{Erf}\left[\frac{Q_2-\alpha+\beta sp_2}{\sqrt{2}\sigma}\right]+\frac{(lp+sp_2-2wp_s)}{2} \quad (8)$$

and

$$\partial_{SP2}(\pi_{R,2})=\frac{\sigma}{\sqrt{2\pi}}e^{-\frac{(\alpha+\beta sp_2)^2}{2\sigma^2}}-\frac{\sigma}{\sqrt{2\pi}}e^{-\frac{(Q_2-\alpha+\beta sp_2)^2}{2\sigma^2}}+\frac{-\alpha+\beta(hc-rp+2sp_2)}{2}\mathrm{Erf}\left[\frac{-\alpha+\beta sp_2}{\sqrt{2}\sigma}\right]+\frac{(Q_2+\beta lc)}{2}+\frac{\beta Q_2(hc-rp)}{\sqrt{2\pi}\,\sigma}e^{-\frac{(\alpha+\beta sp_2)^2}{2\sigma^2}}+\frac{\alpha-Q_2+\beta(rp-hc-lc-2sp_2)}{2}\times\mathrm{Erf}\left[\frac{Q_2-\alpha+\beta sp_2}{\sqrt{2}\sigma}\right] \quad (9)$$

The optimal selling price sp^*_2 and order quantity Q^*_2 are obtained by setting Eq. (8) and Eq. (9) equal to zero and solving them simultaneously. Substituting these values into Eq. (7), the maximum retailer profit is obtained as,

$$\pi^*_{R,2}=\Big(\frac{(\beta sp^*_2-\alpha)(sp^*_2-rp+hc)+Q^*_2\ (hc-rp)}{2}\mathrm{Erf}[\tau_2]+\frac{Q^*_2(sp^*_2+lc-2wp_s)+(\beta sp^*_2-\alpha)lc}{2}+\frac{\sigma(rp-sp^*_2-lc-hc)}{2}e^{-\theta_2{}^2}+\frac{\sigma(hc+sp^*_2-rp)}{\sqrt{2\pi}}e^{-\tau_2{}^2}+\frac{(\beta sp^*_2-2Q^*_2wp_s-\alpha)(rp-hc-lc-sp^*_2)}{2}\mathrm{Erf}[\theta_2]\Big) \quad (10)$$

where,

$$\tau_2=\frac{-\alpha+\beta sp^*_2}{\sqrt{2}\sigma},\ \theta_2=\frac{Q^*_2-\alpha+\beta sp^*_2}{\sqrt{2}\sigma}$$

The manufacturer profit in this returns policy is equal to the wholesale revenue less the sum of the production cost and the repurchase cost as given by

$$\pi_{S,2}=\mathrm{WP-PC-RP} \quad (11)$$

Where PC = pc × Q.

Substituting the corresponding demand function in the expressions for WP, PC, and RP, and simplifying in terms of error function,

$$\pi_{S,2}=(wp_s-pc)Q_2+\frac{\sigma rp}{\sqrt{2\pi}}e^{-\frac{(-\alpha+\beta sp_2)^2}{2\sigma^2}}+\frac{Q_2rp-\alpha rp+\beta rpsp_2}{2}\mathrm{Erf}\left[\frac{-\alpha+\beta sp_2}{\sqrt{2}\sigma}\right]-\frac{\sigma rp}{\sqrt{2\pi}}e^{-\frac{(Q_2-\alpha+\beta sp_2)^2}{2\sigma^2}}-\frac{Q_2rp-\alpha rp+\beta rpsp_2}{2}\mathrm{Erf}\left[\frac{Q_2-\alpha+\beta sp_2}{\sqrt{2}\sigma}\right] \quad (12)$$

The manufacturer profit, at the levels of optimal order quantity and the optimal selling price that maximize the retailer profit, is now given by,

$$\pi_{S,2}=(wp_s-pc)Q^*_2+\frac{\sigma rp}{\sqrt{2\pi}}e^{-\tau_2{}^2}-\frac{\sigma rp}{\sqrt{2\pi}}e^{-\theta_2{}^2}+\frac{(Q^*_2-\alpha-\beta sp_2)rp}{2}\mathrm{Erf}[\tau_2]-\frac{Q^*_2rp-\alpha rp+\beta rpsp^*_2}{2}\mathrm{Erf}[\theta_2] \quad (13)$$

2.1.2 Coordinated planning model

Here both the retailer and the manufacturer collaborate with each other and jointly set the order quantity and the selling price to maximize the expected joint total profit. The profits of retailer and manufacturer are given by,

$\pi_{R,3}$ = SP - WP - HC - LC, and
$\pi_{S,3}$ = WP - PC.

The expected joint total profit is given by

$$\pi_3=\pi_{S,3}+\pi_{R,3}=\mathrm{SP-HC-LC-PC} \quad (14)$$

The expressions for SP, LC, HC, and PC are the same as in the previous model. For a demand with normal distribution, the expected joint total profit can be expressed and simplified in terms of error functions as,

$$\pi_3=\frac{Q_3(sp_3+lc-2pc)-(\alpha-\beta sp_3)lc}{2}+\frac{\sigma(sp_3+hc)}{\sqrt{2\pi}}e^{-\frac{(-\alpha+\beta sp_3)^2}{2\sigma^2}}+\frac{Q_3hc+(\beta sp_3-\alpha)(hc+sp_3)}{2}\mathrm{Erf}\left[\frac{-\alpha+\beta sp_3}{\sqrt{2}\sigma}\right]-\frac{\sigma(sp_3+hc+lc)}{\sqrt{2\pi}}e^{-\frac{(Q_3-\alpha+\beta sp_3)^2}{2\sigma^2}}+\frac{(\alpha-\beta sp_3-Q_3)(sp_3+hc+lc)}{2}\mathrm{Erf}\left[\frac{Q_3-\alpha+\beta sp_3}{\sqrt{2}\sigma}\right] \quad (15)$$

Taking the first derivatives with respect to selling price sp_3 and order quantity Q, and solving,

$$\pi_3^* = \frac{Q_3^*(sp_3^* + lc - 2pc) - (\alpha - \beta sp_3^*)lc}{2} + \frac{\sigma(sp_3^* + hc)}{\sqrt{2\pi}} e^{-\tau_3^2} + \frac{Q_3^* hc + (\beta sp_3^* - \alpha)(hc + sp_3^*)}{2} \mathrm{Erf}[\tau_3] - \frac{\sigma(sp_3^* + hc + lc)}{\sqrt{2\pi}} e^{-\theta_3^2} + \frac{(\alpha - \beta sp_3^* - Q_3^*)(sp_3^* + hc + lc)}{2} \mathrm{Erf}[\theta_3] \tag{16}$$

where

$$\tau_3 = \frac{-\alpha + \beta sp_3^*}{\sqrt{2}\sigma}, \quad \theta_3 = \frac{Q_3^* - \alpha + \beta sp_3^*}{\sqrt{2}\sigma}$$

The Hessian matrices at the optimal points are not mathematically tractable; however, numerical testing shows that the objective functions for both returns policy and coordinated planning model are concave functions with respect to order quantity and selling price, thus assuring minimum points.

2.2 Non-linear demand

In this section, the mean of demand, μ is assumed to vary with selling price (sp) as $\mu = \lambda sp^{-\varepsilon}$. The demand is normally distributed, as in the previous model. The values of a scaling parameter (λ) and price elasticity (ε) are assumed to be known and constant. All notation of the previous section is kept the same for this model. The sequence of analysis is the same as in the previous linear case.

2.2.1 Returns policy

Substituting $\mu = \lambda sp^{-\varepsilon}$ into Eq. (1) and Eq. (11), the expected retailer and manufacturer profits are derived as,

$$\pi_{R,2} = \frac{(sp_2 + lc - 2wp_s)Q_2 - \lambda lc sp_2^{-\varepsilon}}{2} + \frac{(rp - hc)Q_2 + (sp_2 - rp + hc)\lambda sp_2^{-\varepsilon}}{2} \mathrm{Erf}\left[\frac{\lambda sp_2^{-\varepsilon}}{\sigma\sqrt{2}}\right] + \frac{\sigma(sp_2 + hc - rp)}{\sqrt{2\pi}} e^{-\frac{(\lambda sp_2^{-\varepsilon})^2}{2\sigma^2}} + \frac{\sigma(rp - sp_2 - hc - lc)}{\sqrt{2\pi}} e^{-\frac{(Q_2 - \lambda sp_2^{-\varepsilon})^2}{2\sigma^2}} + \frac{(rp - sp_2 - hc - lc)Q_2 + (sp_2 - rp + hc + lc)\lambda sp_2^{-\varepsilon}}{2} \times \mathrm{Erf}\left[\frac{Q_2 - \lambda sp_2^{-\varepsilon}}{\sigma\sqrt{2}}\right] \tag{17}$$

$$\pi_{S,2} = (wp_s - pc)Q_2 - \frac{Q_2 rp - \lambda rp sp_2^{-\varepsilon}}{2} \mathrm{Erf}\left[\frac{\lambda sp_2^{-\varepsilon}}{\sigma\sqrt{2}}\right] + \frac{Q_2 rp - \lambda rp sp_2^{-\varepsilon}}{2} \mathrm{Erf}\left[\frac{Q_2 - \lambda sp_2^{-\varepsilon}}{\sigma\sqrt{2}}\right] - \frac{\alpha rp}{\sqrt{2\pi}} e^{-\frac{(\lambda sp_2^{-\varepsilon})^2}{2\sigma^2}} + \frac{\alpha rp}{\sqrt{2\pi}} e^{-\frac{(Q_2 - \lambda sp_2^{-\varepsilon})^2}{2\sigma^2}} \tag{18}$$

The maximum retailer profit at optimal order quantity Q_2^* and selling price sp^*_2 is obtained as,

$$\pi^*_{R,2} = \frac{(sp_2^* + lc - 2wp_s)Q_2^* - \lambda lc(sp_2^*)^{-\varepsilon}}{2} + \frac{(rp - hc)Q_2^* + (sp_2^* - rp + hc)\lambda sp_2^*)^{-\varepsilon}}{2} \mathrm{Erf}[\tau_2] + \frac{\sigma(sp_2^* + hc - rp)}{\sqrt{2\pi}} e^{-\tau_2^2} + \frac{\sigma(rp - sp_2^* - hc - lc)}{\sqrt{2\pi}} e^{-\theta_2^2} + \frac{(rp - sp_2^* - hc - lc)Q_2^* + (sp_2^* - rp + hc + lc)\lambda sp_2^*)^{-\varepsilon}}{2} \mathrm{Erf}[\theta_2] \tag{19}$$

Where

$$\tau_2 = \frac{\lambda(sp_2^*)^{-\varepsilon}}{\sigma\sqrt{2}}, \quad \theta_2 = \frac{Q_2^* - \lambda(sp_2^*)^{-\varepsilon}}{\sigma\sqrt{2}}$$

The manufacturer profit at Q_2^* and sp^*_2 is given by Eq. (20) as below.

$$\pi^*_{S,2} = (wp_s - pc)Q_2^* - \frac{Q_2^* rp - \lambda rp(sp_2^*)^{-\varepsilon}}{2} \mathrm{Erf}\left[\frac{\lambda sp_2^{-\varepsilon}}{\sigma\sqrt{2}}\right] + \frac{Q_2^* rp - \lambda rp(sp_2^*)^{-\varepsilon}}{2} \mathrm{Erf}\left[\frac{Q^*_2 - \lambda sp_2^{-\varepsilon}}{\sigma\sqrt{2}}\right] - \frac{\alpha rp}{\sqrt{2\pi}} e^{-\frac{(\lambda sp_2^{-\varepsilon})^2}{\sigma\sqrt{2}}} + \frac{\alpha rp}{\sqrt{2\pi}} e^{-\frac{(Q_2^* - \lambda sp_2^{-\varepsilon})^2}{\sigma\sqrt{2}}} \tag{20}$$

2.2.2 Coordinated planning model

In this section, the optimal values of order quantity and selling price are provided to maximize the joint expected total profit that is the sum of retailer and manufacturer profits. The expected joint total profit is given by Eq. (14). Substituting the expressions of SP, HC, LC, and PC into it, the joint expected total profit for the non-linear function ($\mu = \lambda sp^{-\varepsilon}$) is derived and simplified in terms of the error functions, as shown below.

$$\pi^*_3 = \frac{(sp_3 - lc - 2pc)Q_3 - \lambda lc sp_3^{-\varepsilon}}{2} + \frac{(sp_3 + hc)\lambda sp_3^{-\varepsilon} - Q_3 hc}{2} \mathrm{Erf}\left[\frac{\lambda sp_3^{-\varepsilon}}{\sigma\sqrt{2}}\right] + \frac{\sigma(sp_3 + hc)}{\sqrt{2\pi}} e^{-\frac{(\lambda sp_3^{-\varepsilon})^2}{2\sigma^2}} + \frac{\sigma(-sp_3 - hc - lc)}{\sqrt{2\pi}} e^{-\frac{(Q_3 - \lambda sp_3^{-\varepsilon})^2}{2\sigma^2}}$$

$$+\frac{(-sp_3-hc-lc)Q_3+(sp_3+hc+lc)\lambda sp_3^{-\varepsilon}}{2}\mathrm{Erf}\left[\frac{Q_3-\lambda sp_3^{-\varepsilon}}{\sigma\sqrt{2}}\right] \quad (21)$$

The maximum expected total joint profit is,

$$\begin{aligned}\pi_3^* = {} & \frac{(sp_3^*-lc-2pc)Q_3^*-\lambda lc(sp_3^*)^{-\varepsilon}}{2} \\ & +\frac{(sp_3^*+hc)\lambda(sp_3^*)^{-\varepsilon}-Q_3^*hc}{2}\mathrm{Erf}\left[\tau_3\right] \\ & +\frac{\sigma(sp_3^*+hc)}{\sqrt{2\pi}}e^{-\tau_3^2}+\frac{\sigma(-sp_3^*-hc-lc)}{\sqrt{2\pi}}e^{-\theta_3^2} \\ & +\frac{(-sp_3^*-hc-lc)Q_3^*+(sp_3^*+hc+lc)\lambda(sp_3^*)^{-\varepsilon}}{2}\mathrm{Erf}\left[\theta_3\right] \quad (22)\end{aligned}$$

where

$$\tau_3=\frac{\lambda(sp_3^*)^{-\varepsilon}}{\sigma\sqrt{2}},\ \theta_3=\frac{Q_3^*-\lambda(sp_3^*)^{-\varepsilon}}{\sigma\sqrt{2}}$$

As before, the numerical testing of the Hessian matrix shows concavity for the total profit function with respect to order quantity and sales price.

3. CHARACTERISTICS OF THE SOLUTIION

In order to understand how the results with price-dependent demand including linear and non-linear functions of the coordinated planning model are better than the returns policy, the results of joint expected total profit, manufacturer profit, retailer profit and the optimal order quantity are first analyzed. The proofs are presented for some analytical results in the case of single retailer. Let π_2^* and π_3^* be joint expected total profits, which are the sum of retailer and manufacturer profits, under the returns policy and coordinated planning respectively. It is to be noted that the maximum total profit under returns policy is the total profit in which only the retailer's profit is maximized.

LEMMA 1: The optimal total profit under coordinated planning is greater or equal to the optimal total profit under the retail policy. That is, $\pi^*_3 > \pi^*_2$.

Proof: The optimal total profits under returns policy is the total profits in which only the retailer profit portion is maximized with respect to order quantity and selling price. The manufacturer profit is simply the profit at the set order quantity and selling price.

From Eq. (1), (11), and (14), we get

$$\pi^*_2 = \pi_{S,2} + \pi^*_{R,2}$$
$$\pi^*_2 = \pi_S(Q^*_2, sp^*_2) + \pi^*_R(Q^*_2, sp^*_2),$$

where Q^*_2 and sp^*_2 maximize the retailer profit.

In the coordinated model, the total profit is maximized with respect to the order quantity and the selling price.

$$\pi^*_3 = (\pi_{S,3} + \pi_{R,3})^*$$

Now, $\pi^*_3 = \pi^*_S(Q^*_3, sp^*_3) + \pi^*_R(Q^*_3, sp^*_3)$, for all lot sizes including Q^*_2 from the returns policy, and all sales prices including sp^*_2.

Hence, $\pi^*_3 > \pi^*_2$.

This result shows that coordinated planning will have equal or even more total profit than for the returns policy alone.

LEMMA 2: at any given rp, $\pi^*_{R,2} > \pi^*_{R,3}$. That is, the maximum retailer profit under returns policy is equal to or more than that under coordinated planning.

Proof: Straightforward. $\pi^*_{R,2}$ is the maximum of retailer profit for all order quantities and sales prices including Q^*_3 and sp^*_3.

LEMMA 3: At any given repurchase price rp, the optimal profit of manufacturer is greater than or equal to the manufacturer profit under returns policy. That is $\pi^*_{S,3} > \pi_{S,2}$

Proof: From Lemma 1, $\pi_3^* > \pi_2^*$

$$\pi^*_{S,3} + \pi^*_{R,3} > \pi_{S,2} + \pi^*_{R,2}$$

However, from Lemma 2, $\pi^*_{R,3} < \pi^*_{R,2}$.

$$\pi^*_{S,3} + \pi^*_{R,3} - \pi^*_{R,3} > \pi_{S,2} + \pi^*_{R,2} - \pi^*_{R,2}.$$

Therefore, $\pi^*_{S,3} > \pi_{S,2}$

That is, the manufacturer profit is greater for Model 3.

4. PROFIT SHARING

If the optimal retailer profit under returns policy is equal to or greater than retailer profit under coordinated planning, a proper profit sharing scheme has to be devised to make the retailer agree to a coordinated planning. There can be numerous ways of splitting additional profits so as to make the coordinated planning viable. One such arrangement is suggested here. The manufacturer agrees to give the retailer a profit that is at least equal to the maximum profit the retailer would get under the returns policy. If it is equal, then the amount of compensation, CP, will be given by the difference of the expected retailer profit under the returns policy and the coordinated planning model,

$$CP = \pi_{R,2} - \pi_{R,3}$$

From Lemmas 1, 2, and 4, one can see that this mode of profit sharing is beneficial to both parties as it creates a "win-win" situation, and hence would be readily acceptable.

For the coordinated model, the effective (implicit discounted) wholesale price is given by

$$wp = wp_s - \frac{CP}{Q_3^*} \tag{23}$$

This is the effective wholesale price the retailer is paying under the above described compensation scheme. This is also the maximum wholesale price the retailer is willing to pay, since the retailer may require even more compensation than CP, as an incentive to choose coordinated planning over returns policy. From the manufacturer's view point, the compensation cannot be more than the difference between manufacturer's profits under coordinated planning and returns policy.

5. NUMERICAL TESTING AND ANALYSIS

Numerical experiments are carried out to test and analyze the models. In Section 5.1 only a linear model is analyzed, and a major part of the initial set of data used is the same as that in Mantrala and Raman (1999). This was mainly done for comparative purposes. In Section 5.2 both linear and non-linear models are tested on a set of synthetic data. All the models are tested for various levels of the parameters to check the behavior and sensitivity. In the following discussions, returns policy, coordinated planning model without profit sharing, and coordinated planning model with profit sharing are sometimes referred to as Model 2, Model 3, and Model 3/PS, respectively.

5.1 Linear demand

The demand is distributed as N~(μ, σ) and μ = α –

Table 1. Results of returns policy with linear demand.

	rp	wp	sp*	Q*	π_R	π_S	π^*
σ = 10, pc = 0.75, α = 150, β = 0.5,	0pc	3	151.7	94.1	10943.20	211.73	11154.93
	1pc	3	151.5	95.2	10958.60	198.44	11157.04
	2pc	3	151.3	96.5	10974.80	183.83	11158.63
	3pc	3	151.2	98.3	10992.00	167.34	11159.34
	4pc	3	151.0	101.6	11010.90	147.27	11158.17
σ = 10, pc = 0.75, α = 150, β = 1,	0pc	3	76.7	90.2	5328.55	202.95	5531.50
	1pc	3	76.5	91.5	5341.83	192.27	5534.11
	2pc	3	76.3	93.1	5355.89	180.23	5536.12
	3pc	3	76.2	94.9	5371.08	165.91	5536.99
	4pc	3	76.0	98.6	5388.00	147.98	5536.98
σ = 10, pc = 1, α = 150, β = 0.5,	0pc	4	152.1	92.8	10849.60	278.40	11128.00
	1pc	4	151.9	94.0	10869.10	261.96	11131.06
	2pc	4	151.7	95.4	10889.80	243.58	11133.38
	3pc	4	151.5	97.5	10911.90	222.65	11134.55
	4pc	4	151.2	101.6	10936.60	195.96	11132.56
σ = 10, pc = 1, α = 150, β = 1,	0pc	4	77.1	88.7	5238.93	266.10	5505.03
	1pc	4	76.9	90.0	5255.48	252.91	5508.38
	2pc	4	76.7	91.6	5273.18	237.94	5511.12
	3pc	4	76.5	93.9	5292.49	220.27	5512.76
	4pc	4	76.2	98.7	5314.40	196.42	5510.82
σ = 20, pc = 1, α = 150, β = 1,	0pc	4	77.0	104.5	5148.98	313.50	5462.48
	1pc	4	76.8	107.0	5182.01	286.84	5468.85
	2pc	4	76.6	110.2	5217.42	256.51	5473.93
	3pc	4	76.4	114.7	5256.15	220.40	5476.55
	4pc	4	76.2	123.4	5300.44	171.69	5472.13
σ = 10, pc = 1, α = 200, β = 1,	0pc	4	102.2	114.9	9508.64	344.70	9583.34
	1pc	4	102.0	116.1	9526.49	330.06	9856.55
	2pc	4	101.7	117.9	9545.42	314.31	9859.73
	3pc	4	101.5	120.1	9565.92	295.34	9861.26
	4pc	4	101.2	124.8	9588.93	270.34	9859.27

$\beta *$ sp. The standard deviation σ value is taken as 10 and 20, α as 150 and 200, and β as 0.5 and 1.0. Other variables are assumed as pc = 0.75 and 1, wp_s = 4pc, rp = {0pc, 1pc, 2pc, 3pc, 4pc}, hc = 0.5, and lc = 0.25. The results are calculated by using Mathematica. Table 1 and Table 2 present the results for the returns policy and the coordinated planning, respectively.

Table 3 presents the profits under the profit sharing scheme for the coordinated planning model. As described earlier, the manufacturer agrees to give the retailer the optimal profit the retailer would achieve under the returns policy. Therefore, in this table, the total profit represents the total profits under the coordinated model shown in Table 2, while the retailer profits are the maximized profits of the retailer shown in Table 1, and the manufacturer profits are the difference between these two columns. The wholesale price (wp) shown in Table 3 is actually the effective wholesale price per unit for the retailer when the wholesale price is discounted by the profit sharing scheme under the coordinated planning model.

These tables show that the coordinated planning model can provide better expected total profit than the returns policy in all the cases, as already proved theoretically. For example, at σ = 10, α = 150, β = 0.5, rp = 4pc, and wp_s = 3, the π^*_3(11159.70) is greater than π^*_2(11158.17). For the retailer profit, it is higher for returns case compared to the coordinated case.

Selling price, sp: The optimal selling price for the coordinated model is less than the corresponding price in the case of the returns policy for all the repurchase prices considered, for the same production costs and the demand function. This shows that coordination can lower the target selling price thereby increasing the demand as well as the intangible customer satisfaction.

Repurchase price, rp: The repurchase price has no effect on the total profit for the coordinated model, as it does not appear in the objective function. However the retailer profit increases and manufacturer profit decreases for any increase in repurchase prices. For example, in Table 3, at σ = 10, α = 150, β = 1, and wp_s = 3, the retailer profit $\pi_{R,3}$ = 5386.07 when rp = 4pc, which is greater than $\pi_{R,3}$ = 5353.44 when rp = 2pc. The value of manufacturer profit $\pi_{S,3}$ = 151.64 at rp = 4pc, which is less than $\pi_{S,3}$ = 184.27 when rp = 2pc. Compared to the corresponding case, retailer profit in the returns case is

Table 2. Results of coordinated planning model without profit sharing, linear demand.

	rp	wp	sp*	Q*	π_R	π_S	π^*
σ = 10, pc = 0.75, α = 150, β = 0.5,	0pc	3	150.3	98.9	10937.20	222.50	11159.70
	1pc	3	150.3	98.9	10955.20	204.50	11159.70
	2pc	3	150.3	98.9	10973.30	186.40	11159.70
	3pc	3	150.3	98.9	10991.40	168.30	11159.70
	4pc	3	150.3	98.9	11009.40	150.30	11159.70
σ = 10, pc = 0.75, α = 150, β = 1,	0pc	3	75.3	96.4	5320.81	216.90	5537.71
	1pc	3	75.3	96.4	5337.13	200.58	5537.71
	2pc	3	75.3	96.4	5353.44	184.27	5537.71
	3pc	3	75.3	96.4	5369.76	167.95	5537.71
	4pc	3	75.3	96.4	5386.07	151.64	5537.71
σ = 10, pc = 1, α = 150, β = 0.5,	0pc	4	150.5	98.0	10841.10	294.00	11135.10
	1pc	4	150.5	98.0	10864.40	270.70	11135.10
	2pc	4	150.5	98.0	10887.70	247.40	11135.10
	3pc	4	150.5	98.0	10911.00	224.10	11135.10
	4pc	4	150.5	98.0	10934.20	200.90	11135.10
σ = 10, pc = 1, α = 150, β = 1,	0pc	4	75.5	95.0	5228.84	285.00	5513.84
	1pc	4	75.5	95.0	5249.42	264.42	5513.84
	2pc	4	75.5	95.0	5269.99	243.85	5513.84
	3pc	4	75.5	95.0	5290.57	223.28	5513.84
	4pc	4	75.5	95.0	5311.14	202.70	5513.84
σ = 20, pc = 1, α = 150, β = 1,	0pc	4	75.4	116.0	5129.48	348.00	5477.48
	1pc	4	75.4	116.0	5171.01	306.47	5477.48
	2pc	4	75.4	116.0	5212.54	264.94	5477.48
	3pc	4	75.4	116.0	5254.07	223.41	5477.48
	4pc	4	75.4	116.0	5295.60	181.88	5477.48
σ = 10, pc = 1, α = 200, β = 1,	0pc	4	100.5	121.2	9498.71	363.60	9862.31
	1pc	4	100.5	121.2	9520.47	341.84	9862.31
	2pc	4	100.5	121.2	9542.22	320.09	9862.31
	3pc	4	100.5	121.2	9563.97	298.34	9862.31
	4pc	4	100.5	121.2	9585.73	276.58	9862.31

Table 3. Results of coordinated planning model with profit sharing, for linear demand.

	Rp	wp	sp^*	Q^*	π_R	π_S	π^*
$\sigma = 10$, $wp_s = 3$, $\alpha = 150$, $\beta = 0.5$, $pc = 0.75$	0pc	2.94	150.3	98.9	10943.20	216.50	11159.70
	1pc	2.97	150.3	98.9	10958.60	201.10	11159.70
	2pc	2.99	150.3	98.9	10974.80	184.90	11159.70
	3pc	2.99	150.3	98.9	10992.00	167.70	11159.70
	4pc	2.99	150.3	98.9	11010.90	148.80	11159.70
$\sigma = 10$, $wp_s = 3$, $\alpha = 150$, $\beta = 1$, $pc = 0.75$	0pc	2.92	75.3	96.4	5328.55	209.16	5537.71
	1pc	2.95	75.3	96.4	5341.83	195.88	5537.71
	2pc	2.98	75.3	96.4	5355.89	181.82	5537.71
	3pc	2.99	75.3	96.4	5371.08	166.63	5537.71
	4pc	2.98	75.3	96.4	5388.03	149.68	5537.71
$\sigma = 10$, $wp_s = 4$, $\alpha = 150$, $\beta = 0.5$, $pc = 1$	0pc	3.91	150.5	98.0	10849.60	285.50	11135.10
	1pc	3.95	150.5	98.0	10869.10	266.00	11135.10
	2pc	3.98	150.5	98.0	10889.80	245.30	11135.10
	3pc	3.99	150.5	98.0	10911.90	223.20	11135.10
	4pc	3.98	150.5	98.0	10936.60	198.50	11135.10
$\sigma = 10$, $wp_s = 4$, $\alpha = 150$, $\beta = 1$, $pc = 1$	0pc	3.89	75.5	95.0	5238.93	274.91	5513.84
	1pc	3.94	75.5	95.0	5255.48	258.36	5513.84
	2pc	3.97	75.5	95.0	5273.18	240.66	5513.84
	3pc	3.98	75.5	95.0	5292.49	221.35	5513.84
	4pc	3.97	75.5	95.0	5314.40	199.44	5513.84
$\sigma = 20$, $wp_s = 4$, $\alpha = 150$, $\beta = 1$, $pc = 1$	0pc	3.83	75.4	116.0	5148.98	328.50	5477.48
	1pc	3.91	75.4	116.0	5182.01	295.47	5477.48
	2pc	3.96	75.4	116.0	5217.42	260.06	5477.48
	3pc	3.98	75.4	116.0	5256.15	221.33	5477.48
	4pc	3.96	75.4	116.0	5300.44	177.04	5477.48
$\sigma = 10$, $wp_s = 4$, $\alpha = 200$, $\beta = 1$, $pc = 1$	0pc	3.92	100.5	121.2	9508.64	353.67	9862.31
	1pc	3.95	100.5	121.2	9526.49	335.82	9862.31
	2pc	3.98	100.5	121.2	9545.42	316.88	9862.31
	3pc	3.98	100.5	121.2	9565.92	296.39	9862.31
	4pc	3.97	100.5	121.2	9588.93	273.38	9862.31

higher than that of the coordinated model, as proven mathematically. Furthermore, in case of the returns policy, with any increased repurchase prices, the optimal order quantity and the retailer profit increase.

Demand variables, σ, α, and β: At any $0 < rp < wp_s$, if the standard deviation of demand σ alone increases, it will result in a decrease of the optimal joint expected total profit in the coordinated planning model and the optimal retailer profit and the total profit. This can be seen in Table 2, where for $\sigma = 10$, $pc = 1$, $\alpha = 150$, and $\beta = 1$, the joint total profit is equal to 5513.84, while it is 5477.48 for the same scenario but with $\sigma = 20$. Likewise, for the same scenario in the returns model, the retailer profit decreases from 5314.40 to 5300.44 when the standard deviation increases from 10 to 20, for the case of repurchase price being equal to four times the wholesale price.

Production costs, pc: When the production costs go up, the total profit and retailer profit decrease for both models. In addition, the optimal selling price goes up and the order quantity goes down.

5.2 Linear and Non-linear demands

Here it is assumed that the mean of the demand $\mu = \lambda sp^{-\varepsilon}$. Cost parameters are assumed to be $wp_s = 4pc$, $lc = 0.25$, $hc = 0.50$, $pc = 1$, and $rp = \{0pc, 1pc, 2pc, 3pc, 4pc\}$, to represent some general scenarios. The parameters are expressed in ratios of other cost parameters to give a wide range of applicability.

Table 4. Parameter values for Price-demand functions.

	Linear function		Non-linear function	
	α	β	λ	ε
Case 1	1901.8	205.97	1280.7	1.348
Case 2	2461.0	290.19	1280.7	1.957
Case 3	2777.6	300.82	1970.2	1.348

Behavior of product demand for several levels of price can be hard to obtain because price may not be market-tested at its full range of values. Hence, for this current study, the price dependent demand function is constructed by collecting some actual data pertaining to a particular family of fashion goods and then synthetically generating some more data mimicking the actual data. For each case, both linear and non-linear functions were fitted. The purpose of fitting linear functions was to generate a yardstick to compare the non-linear scenarios. Selling prices were defined in the range of (0.5, 5.5), in steps of 0.5. Sales quantities were considered in the range of (0, 1500), from observing the assumed demand curve. Microsoft Excel was used to obtain the linear and non-linear functions for the hypothesized functions. The parameter values are presented in Table 4.

The parameter values of the demand functions are substituted in the mathematical models developed in section 4. Optimal solutions are obtained using the Ma-

Table 5. Results for nonlinear demand functions.

	Model 2					Model 3	Model 3/P.S				
rp	0pc	1pc	2pc	3pc	4pc		0pc	1pc	2pc	3pc	4pc
Case 1:											
Q^*	43.37	43.62	44.20	45.86	50.04	216.85	216.85	216.85	216.85	216.85	216.85
sp^*	13.43	13.83	14.29	14.72	15.17	3.79	3.79	3.79	3.79	3.79	3.79
wp_k	4.00	4.00	4.00	4.00	4.00	4.00	2.21	2.21	2.20	2.18	2.14
π_R	313.59	320.99	329.85	340.78	355.23	- 74.10	313.59	320.99	329.85	340.78	355.23
π_S	130.11	122.81	113.12	100.64	80.45	650.55	262.86	255.45	246.6	235.67	221.21
π^*	443.70	443.80	442.97	441.42	435.68	576.45	576.45	576.45	576.45	576.45	576.45
Case 2:											
Q^*	26.35	26.94	28.09	30.30	35.59	306.79	306.79	306.79	306.79	306.79	306.79
sp^*	6.87	7.03	7.20	7.37	7.50	2.06	2.06	2.06	2.06	2.06	2.06
wp_k	4.00	4.00	4.00	4.00	4.00	4.00	1.86	1.86	1.85	1.85	1.83
π_R	52.64	55.91	60.17	66.03	75.02	- 605.15	52.64	55.91	60.17	66.03	75.02
π_S	79.05	77.13	74.43	69.83	59.52	920.37	262.57	259.31	255.05	249.19	240.20
π^*	131.69	133.04	134.60	135.86	134.54	315.22	315.22	315.22	315.22	315.22	315.22
Case 3:											
Q^*	58.28	58.45	59.15	60.54	64.48	300.01	300.01	300.01	300.01	300.01	300.01
sp^*	14.03	14.33	14.63	14.98	15.34	3.93	3.93	3.93	3.93	3.93	3.93
wp_k	4.00	4.00	4.00	4.00	4.00	4.00	2.23	2.22	2.21	2.20	2.17
π_R	481.55	489.18	498.23	509.30	523.87	- 50.79	481.55	489.18	498.23	509.30	523.87
π_S	174.84	167.09	157.61	144.29	123.41	900.03	367.69	360.06	351.02	339.94	325.37
π^*	656.39	656.27	655.83	653.60	647.28	849.24	849.24	849.24	849.24	849.24	849.24

Table 6. Results for linear demand functions, for cases shown in Table 4.

		Model 2				Model		Model 3/P.S			
rp	0pc	1pc	2pc	3pc	4pc	3	0pc	1pc	2pc	3pc	4pc
Case 1:											
Q^*	537.46	536.92	539.03	546.53	550.88	853.73	853.73	853.73	853.73	853.73	853.73
sp^*	6.61	6.62	6.62	6.60	6.61	5.12	5.12	5.12	5.12	5.12	5.12
wp	4.00	4.00	4.00	4.00	4.00	4.00	3.45	3.45	3.46	3.46	3.46
π_R	1382.07	1385.07	1388.95	1394.21	1402.49	910.54	1382.07	1385.07	1388.95	1394.21	1402.49
π_S	1612.38	1607.41	1608.34	1620.42	1607.47	2561.19	2089.66	2086.66	2082.79	2077.52	2069.24
π^*	2994.45	2992.48	2997.28	3014.63	3009.96	3471.73	3471.73	3471.73	3471.73	3471.73	3471.73
Case 2:											
Q^*	658.27	656.90	653.10	653.32	656.69	1094.50	1094.50	1094.50	1094.50	1094.50	1094.50
sp^*	6.20	6.21	6.23	6.24	6.25	4.73	4.73	4.73	4.73	4.73	4.73
wp	4.00	4.00	4.00	4.00	4.00	4.00	3.39	3.39	3.39	3.40	3.40
π_R	1430.19	1433.00	1436.58	1441.34	1448.95	757.99	1430.42	1433.00	1436.58	1441.34	1448.95
π_S	1974.81	1967.64	1951.34	1942.76	1928.81	3283.50	2611.07	2608.49	2604.91	2600.15	2592.54
π^*	3405.00	3400.64	3387.92	3384.10	3377.77	4041.49	4041.49	4041.49	4041.49	4041.49	4041.49
Case 3:											
Q^*	789.39	794.04	790.03	793.23	796.49	1240.81	1240.81	1240.81	1240.81	1240.81	1240.81
Sp^*	6.60	6.59	6.61	6.61	6.62	5.13	5.13	5.13	5.13	5.13	5.13
wp	4.00	4.00	4.00	4.00	4.00	4.00	3.46	3.46	3.46	3.47	3.47
π_R	2031.53	2034.48	2038.45	2043.71	2051.97	1356.73	2031.53	2034.48	2038.45	2043.71	2051.97
π_S	2368.17	2378.68	2361.23	2360.68	2345.06	3722.43	3047.62	3044.68	3040.71	3035.44	3027.19
π^*	4399.70	4413.16	4399.68	4404.39	4397.03	5079.16	5079.16	5079.16	5079.16	5079.16	5079.16

thematica software. The optimal levels of profits, corresponding lot quantities and selling prices for various factor levels are shown in Table 5 and Table 6.

The results show that there is a considerable difference between linear and nonlinear functions for the same cases, i.e., for the same data set. The optimal profits for nonlinear functions range in hundreds only while corresponding values for linear functions are in the range of thousands. Similarly, the magnitude of optimum order quantities for nonlinear functions is in the order of tens, and for the linear function it is in the hundreds. Since both functions were derived for the same data set, these results indicates that one must exercise utmost caution in obtaining and fitting appropriate forms of the function for the data.

It can also be seen that the joint expected total profit of Model 3 is better than that of Model 2 for any case of non-linear and linear demand functions. This has already been proven mathematically. Another interesting factor is that the coordinated planning model contribution compared to the returns policy is greater in the case of non-linear function as opposed to the linear function. For Case 1, where rp = 0pc, the values of π_2^* and π_3^* are equal to 443.70 and 576.45 for non-linear function, respectively. It shows that π^* increases by 29.92% from Model 2 to Model 3. For the same case, with linear function, π_3^* = 3471.75 and π_2^* = 2994.45, which shows that the value π^* is increased by 16.02%.

Moreover, the results indicate that the retailer profits can go into negative values in the case of Model 3, that is the coordinated planning model, for nonlinear demand functions. For Case 2 at rp = 0pc, the value $\pi^*_{R,3}$ is negative and is equal to -605.15. It means that the coordinated planning model cannot be used without a proper profit sharing scheme. After compensation, the retailer profit is $\pi^*_{R,3\text{-P.S}}$ = 52.64.

The effects of repurchase price and demand variables (α, β, and σ) on the returns policy and the coordinated planning model have been described in a previous section for linear demand functions. In the following paragraphs we only describe the influence of the other parameters, λ and ε, for nonlinear cases.

Scaling constant λ: If the value of λ increases, the values of all decision variables also increase. For example, at rp = 4pc and other parameters constant, when the value λ increases from 1280.7 to 1970.2 (case 1 and 3), the values of π_3^*, Q_3^*, and sp_3^* are increased from 576.45 to 849.24, from 216.85 to 300.01, and from 3.79 to 3.93, respectively. Moreover, the values of $\pi^*_{R,3}$, $\pi^*_{S,3}$, and wp_3 also increased from 355.23 to 523.87, from 221.21 to 325.37, and from 2.14 to 2.17, respectively.

Price elasticity ε: Only optimal order quantity is increased if the price elasticity increases. The value ε varies inversely with the optimal selling price, effective wholesales price, retailer profit, manufacturer profit, and joint expected total profit. For example, at rp = 4pc, if the value ε increases from 1.348 to 1.957 (case 1 and 2), the value Q_3^* is increased from 216.85 to 306.79. Other results decrease, for example, the optimal selling price drops from 3.79 to 2.06, the retailer profit decreases from 355.23 to 75.02, and the value of π_3^* decreases from 576.45 to 315.22. These parameters are quite sensitive to any changes in the price elasticity parameter.

6. CONCLUSIONS AND RECOMMENDATIONS

Different levels of cooperation and integration between manufacturers and retailers, that would improve sales, improve profits, and strengthen supply chains, were examined. The case of a single manufacturer and a single retailer was considered for the study. The product was fashion goods with a one season lifetime.

It was seen that a returns policy in which the manufacturer agrees to buy back all the unsold items from the retailer at a price agreed beforehand helps retailers obtain much better profits as the policy encourages retailer to stock more quantity and hence increasing the availability. A coordinated model was developed for a higher level of cooperation. It models both parties planning together to maximize the total profit by fixing up optimal levels of the order quantity. In addition, retailer price is also jointly determined. It is proved that the coordinated model yields higher total profit than even that of the returns policy.

A profit sharing scheme to encourage a retailer to participate in coordinated planning was designed, by which the retailer would get the same level of profit he would under the returns policy. It is mathematically shown, and the numerical results illustrate this point, that this profit sharing scheme gives a 'win-win' situation to both players.

The optimal prices for the coordinated model are less than the corresponding returns policy model. The implications for supply chains is that joint planning and control will increase the demand for the product. Such a joint planning also would help to improve synergetic planning and control of other activities like sales promotion, production, and inventory.

The returns and coordinated models were analyzed for demand under two kinds of price dependency, linear and nonlinear (exponential). The results show that profits and order quantities are highly sensitive to the type of demand function. The benefits of a coordinated model are higher for the assumed nonlinear demands.

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