IEMS Vol. 8, No. 2, pp. 72-79, June 2009.

The Impact of Nonconforming Items on (*s*, *S*) Inventory Model with Customer Order Reservation and Cancellation

Yasuhiko Takemoto†
Faculty of Management and Information Systems
Prefectural University of Hiroshima, Hiroshima, 734-8558, JAPAN
Tel: +81-82-251-9579, Fax: +81-82-251-9405, E-mail: ys-take@pu-hiroshima.ac.jp

Ikuo Arizono
Division of Electrical Engineering and Information Science
Graduate School of Engineering, Osaka Prefecture, University, Sakai, Osaka 599-8531, JAPAN
Tel: +81-72-254-9346, Fax: +81-72-254-9915, E-mail: arizono@eis.osakafu-u.ac.jp

Received Date, February 6, 2008; Accepted Date, February 23, 2009

Abstract. The ultimate goal of inventory management is to decide the timing and the quantity of ordering in response to uncertain demands. Recently, some researchers have focused upon an impact of distortions in the information, *e.g.*, customer order cancellation, on an economical inventory policy. The customer order cancellation is considered a kind of distortions in demands, because a demand that is eventually cancelled is equivalent to a phony demand. Also, there are some additional distortions in the inventory information. For instance, the procurement of suppliers may include some nonconforming items as a result of imperfect production and inspection by the suppliers, and/or damage in transit. The nonconforming item should be considered a kind of distortions in the inventory information, because the nonconforming item is equivalent to a phony stock. In this article, we consider an inventory model under the situation that customers can cancel their orders and the procurement of suppliers may include some nonconforming items. Then, we introduce the customer order reservation into the inventory model for the purpose of avoiding the costly backlogs, because the customer order reservation gives retailers a period to fulfill customer's requests. We formulate a periodic review (*s*, *S*) inventory model and investigate the economical operation under the situation mentioned above. Further, through the sensitivity analysis, we show the impact of these distortions and the effect of the customer order reservation on the inventory policy.

Keywords: Customer Order Cancellation, Customer Order Reservation, Inspection, Nonconforming Items, Periodic Review (*s*, *S*) Inventory Model

1. INTRODUCTION

The inventory management is ultimately concerned with the timing and the quantity of ordering in response to uncertain demands. Then, many researchers have proposed various inventory management techniques such as continuous review and periodic review inventory policies. Some researchers have focused upon an impact of the inventory information distortion when they have researched an economical inventory policy. Lee *et al*. (1997) have suggested that the propagation of the variance of demand along a supply chain, *i.e*., the Bullwhip effect, is triggered by the rational ordering behavior of the chain members. Then, they have identified the source of information distortion which leads to the Bullwhip effect as demand forecast updating, order batching, price fluctuation, and shortage gaming. Cheung and Zhang (1999) and Yuan and Cheung (2003) have considered the customer order cancellation to be a kind of information distortions for demands because a demand that is eventually cancelled is equivalent to a phony demand. Then, they have investigated the impact of the customer order cancellation on the inventory policy.

† : Corresponding Author

Also, there are some additional distortions in the inventory information. For instance, the procurement of suppliers may include some nonconforming items as a result of imperfect production and inspection by the suppliers, and/or damage in transit (Moinzadeh and Lee 1987). The nonconforming item should be considered a kind of distortions in the inventory information, because the nonconforming item is equivalent to a phony stock. The inclusion of nonconforming items may cause the extra holding cost in the inventory management as well as the compensation for the delivery of nonconforming items to customers. Therefore, the nonconforming items need to be removed from the stock by inspection beforehand. For the purpose of removing the nonconforming items from the procurement items completely, it is desirable to execute the total inspection for the procurement items. In contrast, it is impossible to deny that the removal of nonconforming items may lead to the further shortage of items. Then, the shortage may bring many backlogs until the next procurement. Therefore, we should consider the impact of the nonconforming items in the planning of the inventory policy.

An implicit assumption in most of the inventory models in the literature is that the procurement items ordered will not include any nonconforming items. Moinzadeh and Lee (1987) have considered a continuous review (Q, R) inventory model, provided that the procurement of suppliers includes some nonconforming items, where Q and R imply the order quantity and the reorder point, respectively. Then, Wu and Ouyang (2001) have considered a continuous review inventory model with nonconforming items in which the lead time is a decision variable in addition to the order quantity and the reorder point. Khouja (2003) has investigated the impact of nonconforming items in a supply chain.

On the other hand, the customer order reservation is an important practice in today's business world. In many retail situations such as sales of electronic appliance, computers, and automobiles, it is common for customers to order in advance (Cheung and Zhang, 1999). In the customer order reservation, customers give the retailer a period to fulfill customers' requests. Under this situation, while the retailer has the extra cost for holding, he can avoid the costly lost sales or backlogs. In other words, the retailer may increase investment on the inventory while maintaining customers' requests.

In this article, we consider an inventory model under the situation that customers can cancel their orders and the procurement of suppliers may include some nonconforming items. In other words, the customer order cancellation and the nonconforming items included in the procurement are considered in the viewpoint of the information distortions for demand and inventory. Then, we formulate the periodic (s, S) inventory model where the customer order reservation is practiced for the purpose of avoiding the costly backlogs in addition to the impact of the information distortions mentioned above, and then investigate the economical inventory policy. Further, through the sensitivity analysis, we show the impact of the customer order cancellation and the nonconforming items and also the effect of the customer order reservation on the inventory policy.

2. FORMULATION OF THE MODEL

We consider a retailer who sells a single item to customers. Demand is described by a Poisson process with parameter λ. Each demand is fulfilled at w time units after a customer places a reservation of an order, even if stocks are available at the time of the reservation. If the retailer has no on-hand inventory when handing over the item to the customer, the order of the customer becomes backlogged. In this case, a fixed penalty is incurred. Then, we consider the customer order cancellation in this article. The customer can cancel his order within w time units from the time of his reservation. In this case, the retailer incurs the administrative expense for cancelled demands, but the customer incurs no charge for the cancellation.

The retailer periodically reviews the inventory. The interval between successive reviews is τ time units. At each review epoch, if the inventory position is equal to or below s, a procurement order is issued to raise the inventory position to S. We assume that the procurement leadtime L is deterministic, where $w \le L \le \tau$. When the retailer receives the procurement items, the total inspection is executed for the purpose of removing all the nonconforming items included in the procurement completely.

The following notations are used throughout this article.

λ : average rate of customer demands.
p : probability that an order will be eventually cancelled.
q : probability that an item may be a nonconforming item, where whether the item is conforming or not is described by the Bernoulli distribution with parameter q.
L : lead-time of procurement.
w : fixed time period between the reservation and the delivery of customer demand.
G : elapsed time from the reservation to the cancellation of a demand, with the distribution function $F(G)$ and expectation $E[G]$, where $0 \le G \le w$.
τ : interval between successive reviews.
c_H : inventory holding cost per time unit.
c_B : penalty cost per unit of backorder per time unit.
c_K : setup cost per procurement.
c_A : administrative cost per unit of cancelled demand.
c_I : inspection cost per item.
c_D : loss per nonconforming item.
$I(t)$: inventory position at time t.
$D(t_1, t_2)$: total number of demands reserved between t_1 and t_2.
$D^{\dagger}(t_1, t_2)$: total number of demands reserved between t_1 and t_2 that are not eventually cancelled.
$N(t_1, t_2)$: total number of cancellations of demands between t_1 and t_2.

$N^{\dagger}(t_1, t_2)$: total number of cancellations of demands between t_1 and t_2 among the demands reserved between $t_1 - w$ and $t_2 - w$.

$U(t)$: total number of demands that are not cancelled yet at time t, where including demands cancelled after t.

$R(t)$: total number of nonconforming items included in the procurement which is issued at the time epoch t.

$M(t_1, t_2)$: total number of actual demands to be delivered between t_1 and t_2.

3. ANALYSIS OF THE MODEL

We determine the distribution of the inventory position $I(t)$ at a discrete time epoch t at first. Let T_i be the time epoch of the beginning of i th review interval, where $T_i = i\tau, i = 1, 2, \cdots$. All the procurements are issued immediately after these time epochs. Hence, $I(T_i)$ is the inventory position before the decision of the procurement. We obtain the following balance equation:

$$\begin{aligned} I(T_{i+1}) &= I(T_i) + O(T_i) - D(T_i, T_{i+1}) + N(T_i, T_{i+1}) \\ &= I(T_i) + O(T_i) - D^{\dagger}(T_i, T_{i+1}) \\ &\quad - N^{\dagger}(T_{i+1}, T_{i+1} + w) + N^{\dagger}(T_i, T_i + w) \\ &= I(T_i) + O(T_i) - D^{\dagger}(T_i, T_{i+1} - w) - U(T_{i+1}) \\ &\quad + N^{\dagger}(T_i, T_i + w), \end{aligned} \tag{1}$$

where $O(T_i)$ expresses the number of procurement items. In this article, we assume that the procurement items may include some nonconforming items. Hence, by considering the number of nonconforming items in the procurement, $O(T_i)$ is given as follows:

$$O(T_i) = \begin{cases} 0, & \text{if } I(T_i) > s \\ S - I(T_i) - R(T_i), & \text{if } I(T_i) \le s \end{cases} \tag{2}$$

Then, by letting $Y(t) \equiv S - I(t)$, we can rewrite the balance equation in Eq. (1) as

$$Y(T_{i+1}) = \begin{cases} Y(T_i) + D^{\dagger}(T_i, T_{i+1} - w) + U(T_{i+1}) \\ \quad - N^{\dagger}(T_i, T_i + w), \quad \text{if } Y(T_i) < S - s \\ R(T_i) + D^{\dagger}(T_i, T_{i+1} - w) + U(T_{i+1}) \\ \quad - N^{\dagger}(T_i, T_i + w),, \quad \text{if } Y(T_i) \ge S - s \end{cases} \tag{3}$$

$R(T_i)$ depends on $Y(T_i)$, but is independent of $Y(T_{i+1})$, $U(T_i)$, and $U(T_{i+1})$. On one hand, $U(T_{i+1})$ is independent of both $Y(T_i)$ and $U(T_i)$ due to $w \le \tau$, whereas $Y(T_{i+1})$ depends on $Y(T_i)$, $U(T_{i+1})$, $R(T_i)$, and the demands arrived after T_i from Eq. (3). Then, $Y(T_i)$ and $U(T_i)$ are dependent. Further, $N^{\dagger}(T_i, T_i + w)$ is determined by $U(T_i)$. Consequently, we define the state as $\{Y(T_i), U(T_i)\}$ and form a discrete time Markov chain based on the state space $\{Y(T_i), U(T_i)\}$ for the purpose of determining the distribution of $Y(T_i)$.

For convenience, we define the transition probability for Markov chain $\{Y(T_i), U(T_i)\}$ as

$$\begin{aligned} &\Pr\{Y(T_{i+1}) = c, U(T_{i+1}) = d \,|\, Y(T_i) = a, U(T_i) = b\} \\ &\equiv \Pr\{(a, b), (c, d)\}, \end{aligned}$$

for all integers a, c and nonnegative integer b, d. Since $U(T_{i+1})$ is independent of both $Y(T_i)$ and $U(T_i)$, we translate Eq. (3) into Eq. (4).

Out of the outstanding demands $U(T_i)$, a portion $N^{\dagger}(T_i, T_i + w)$ will be cancelled with $(T_i, T_i + w]$. Denote the probability that each order reserved between $T_i - w$ and T_i is cancelled between T_i and $T_i + w$ by β. β can be derived as the conditional probability of cancelling a demand within $(T_i, T_i + w]$, given that the demand remains outstanding at T_i (Cheung and Zhang, 1999). Then, let x be the time that an order reservation is made, where $T_i - w < x \le T_i$. Since order arrivals are Poisson, given that the demand has arrived in $(T_i - w, T_i]$, x is uniformly distributed in the same interval (Gross and Harris 1985). Therefore, β is obtained as follows:

$$\begin{aligned} \beta &= \frac{p\int_{T_i - w}^{T_i} \frac{1}{w}\left[1 - F(T_i - x)\right]dx}{(1-p) + p\int_{T_i - w}^{T_i} \frac{1}{w}\left[1 - F(T_i - x)\right]dx} \\ &= \frac{p\int_0^w \frac{1}{w}\left[1 - F(G)\right]dG}{(1-p) + p\int_0^w \frac{1}{w}\left[1 - F(G)\right]dG} \\ &= \frac{pE[G]}{(1-p)w + pE[G]}. \end{aligned} \tag{5}$$

Since the decision for the cancellation of an order is independent of the other orders, $N^{\dagger}(T_i, T_i + w)$ is binomially distributed with parameters $U(T_i)$ and β (Cheung and Zhang, 1999). Hence, we obtain the following equation:

$$\Pr\left\{N^{\dagger}(T_i, T_i + w) = n \,\middle|\, U(T_i) = b\right\} = \binom{b}{n}\beta^n (1-\beta)^{b-n}. \tag{6}$$

$$\begin{aligned} &\Pr\{(a, b), (c, d)\} = \Pr\left\{Y(T_{i+1}) = c \,\middle|\, Y(T_i) = a, U(T_i) = b, U(T_{i+1}) = d\right\}\Pr\{U(T_{i+1}) = d\} \\ &= \begin{cases} \Pr\left\{D^{\dagger}(T_i, T_{i+1} - w) = c - a - d + N^{\dagger}(T_i, T_i + w) \,\middle|\, U(T_i) = b\right\}\Pr\{U(T_{i+1}) = d\}, & \text{if } Y(T_i) < S - s \\ \Pr\left\{D^{\dagger}(T_i, T_{i+1} - w) = c - d - R(T_i) + N^{\dagger}(T_i, T_i + w) \,\middle|\, Y(T_i) = a, U(T_i) = b\right\}\Pr\{U(T_{i+1}) = d\}, & \text{if } Y(T_i) \ge S - s \end{cases} \end{aligned} \tag{4}$$

On one hand, $R(T_i)$ is binomially distributed with parameters $Y(T_i)$ and q. Hence, we obtain the following equation:

$$\Pr\{R(T_i)=r \mid Y(T_i)=a\}=\binom{a}{r}q^r(1-q)^{a-r}. \tag{7}$$

Since $U(T_{i+1})$ is the total number of demands that are reserved between $T_{i+1}-w$ and T_{i+1} and then are not yet cancelled at time T_{i+1}, $\Pr\{U(T_{i+1})=d\}$ is given by eq. (5) as follows (Cheung and Zhang, 1999) :

$$\begin{aligned}\Pr\{U(T_{i+1})=d\}&=\sum_{k=d}^{\infty}\frac{(\lambda w)^k}{k!}e^{-\lambda w}\binom{k}{d}\beta^{k-d}(1-\beta)^d\\&=\frac{[\lambda w(1-\beta)]^d}{d!}e^{-\lambda w(1-\beta)}.\end{aligned} \tag{8}$$

Further, $D^{\dagger}(T_i, T_{i+1}-w)$ is described by the Poisson distribution with parameter $\lambda(1-p)(\tau-w)$. As the above results, Eq. (4) can be translated into Eq. (9).

We compute the transition probability $\Pr\{(a, b), (c, d)\}$ based on Eq. (9) and form a transition probability matrix $\mathbf{P}$ by truncating the state space to $\{Y(T_i), U(T_i)\}$ $=\{\underline{a}, \underline{a}+1, \cdots, 0, 1, \cdots, \overline{a}\}\times\{0, 1, \cdots, \overline{b}\}$, where $\underline{a}<0, \overline{a}>0$ and $\overline{b}>0$ are lower or upper limits of the state variables to be numerically determined. Note that we normalize each row of the transition probability matrix by dividing each row by its row sum. Then, we obtain the stationary probability vector $\boldsymbol{\pi}=(\pi_{ab})$ by solving a set of linear equations as

$$\boldsymbol{\pi}\mathbf{P}=\boldsymbol{\pi}, \quad \boldsymbol{\pi}\cdot\mathbf{1}=\boldsymbol{\pi},$$

where $\mathbf{1}=(1, \cdots, 1)^t$. From Eq. (9), we find that this Markov chain is regular since the transition probability matrix $\mathbf{P}$ satisfies some conditions (Taylor and Karlin, 1998). Therefore, the solution $\boldsymbol{\pi}=(\pi_{ab})$ is unique.

As mentioned the above, we can obtain the stationary distribution of the inventory position provided that the inventory system is operated in the respective (*s*, *S*) inventory policies. Then, we consider a cost structure and define the optimization of the inventory policy. The elements for costs are considered to be holding inventory, backordered demands, setup of procurement, cancellation, nonconforming items, and inspection.

For the purpose of evaluating the holding cost for inventory and the backordered cost for demands, we compute the expected inventory position. Let $H(T_i+t)$ be the on-hand inventory level at the time epoch T_i+t, where $0\le t<\tau$. From Eq. (1)~Eq. (3), we obtain the following equation :

$$H(T_i+t)=\begin{cases}I(T_i)+U(T_i)-M(T_i, T_i+t), & \text{if } 0\le t<w\\ I(T_i)+U(T_i)-M(T_i, T_i+w)-D^{\dagger}(T_i, T_i+t-w), & \text{if } w\le t<L \text{ or } L\le t<\tau, I(T_i)>s.\\ S-R(T_i)+U(T_i)-M(T_i, T_i+w)-D^{\dagger}(T_i, T_i+t-w), & \text{if } L\le t<\tau, I(T_i)\le s\end{cases} \tag{10}$$

The random variable $M(T_i, T_i+t)$ represents the demand to be delivered within $(T_i, T_i+t]$ out of $U(T_i)$. Denote the delivery rate by $\alpha(t)$. Then, $M(T_i, T_i+t)$ is considered to be binomially distributed with parameters $\alpha(t)$ and $U(T_i)$. $\alpha(t)$ can be derived as the conditional probability of delivering the demand within $(T_i, T_i+t]$, given that the demand remains outstanding at T_i (Cheung and Zhang 1999). The derivation of $\alpha(t)$ is similar to that of β and it is as follows:

$$\begin{aligned}\alpha(t)&=\frac{(1-p)\frac{t}{w}}{(1-p)+p\int_{T_i-w}^{T_i}\frac{1}{w}[1-F(T_i-w)]dx}\\&=\frac{(1-p)t}{(1-p)w+p\int_0^w[1-F(G)]dG}\\&=\frac{(1-p)t}{(1-p)w+pE[G]}.\end{aligned} \tag{11}$$

Then, the expectation of $I(T_i)$ and $U(T_i)$ are computed as the following equations using the stationary probability π_{ab} :

$$\begin{aligned}&\Pr\{(a, b), (c, d)\}=\Pr\{Y(T_{i+1})=c \mid Y(T_i)=a, U(T_i)=b, U(T_{i+1})=d\}\Pr\{U(T_{i+1})=d\}\\
&=\begin{cases}\left\{\sum_{n=0}^{b}\Pr\{D^{\dagger}(T_i, T_{i+1}-w)=c-a-d+n\}\binom{b}{n}\beta^n(1-\beta)^{b-n}\right\}\Pr\{U(T_{i+1})=d\}, & \text{if } Y(T_i)<S-s\\ \left\{\sum_{r=0}^{a}\sum_{n=0}^{b}\Pr\{D^{\dagger}(T_i, T_{i+1}-w)=c-d-r+n\}\binom{a}{r}q^r(1-q)^{a-r}\binom{b}{n}\beta^n(1-\beta)^{b-n}\right\}\Pr\{U(T_{i+1})=d\}, & \text{if } Y(T_i)\ge S-s\end{cases}\\
&=\begin{cases}\left\{\sum_{n=0}^{b}\frac{[\lambda(1-p)(\tau-w)]^{c-a-d+n}}{(c-a-d+n)!}e^{-\lambda(1-p)(\tau-w)}\binom{b}{n}\beta^n(1-\beta)^{b-n}\right\}\frac{[\lambda w(1-\beta)]^d}{d!}e^{-\lambda w(1-\beta)}, & \text{if } Y(T_i)<S-s\\ \left\{\sum_{r=0}^{a}\sum_{n=0}^{b}\frac{[\lambda(1-p)(\tau-w)]^{c-d-r+n}}{(c-d-r+n)!}e^{-\lambda(1-p)(\tau-w)}\binom{a}{r}q^r(1-q)^{a-r}\binom{b}{n}\beta^n(1-\beta)^{b-n}\right\}\frac{[\lambda w(1-\beta)]^d}{d!}e^{-\lambda w(1-\beta)}, & \text{if } Y(T_i)\ge S-s\end{cases}\end{aligned} \tag{9}$$

$$E\left[I(T_i)\right] = S - \sum_{a=\underline{a}}^{\bar{a}} a\pi_a, \tag{12}$$

$$E\left[U(T_i)\right] = \sum_{b=0}^{\bar{b}} b\pi_b, \tag{13}$$

where

$$\pi_a = \sum_{b=0}^{\bar{b}} \pi_{ab}, \tag{14}$$

$$\pi_b = \sum_{a=\underline{a}}^{\bar{a}} \pi_{ab}. \tag{15}$$

Therefore, the expected on-hand inventory level $H(T_i + t)$ is given as Eq. (16), where it is confirmed that $1-\alpha(w)$ corresponds to β from both Eq. (5) and Eq. (11).

Then, denote $\max\{0, x\}$ by x^+. Hence, the expected on-hand inventory $E[H(T_i+t)^+]$ is evaluated as Eq. (17). When we denote $\max\{0, -x\}$ by x^-, it is obvious that $x^- = x^+ - x$. Hence, the expected backordered demands $E[H(T_i+t)^-]$ is evaluated as follows:

$$E\left[H(T_i+t)^-\right] = E\left[H(T_i+t)^+\right] - E\left[H(T_i+t)\right] \tag{18}$$

Then, the probability that the procurement order is issued, ξ, and the expected number of nonconforming items removed by inspection, $E[r]$, are computed as

$$\xi = \Pr\{Y(T_i) \geq S-s\} = 1 - \sum_{a=\underline{a}}^{S-s-1} \pi_a, \tag{19}$$

$$E[r] = \sum_{a=S-s}^{\bar{a}} \pi_a \sum_{r=0}^{a} r \binom{a}{r} q^r (1-q)^{a-r} = \sum_{a=S-s}^{\bar{a}} q a \pi_a. \tag{20}$$

Denote the expected cost per unit time about holding, backorder, setup, cancellation, nonconforming items, and inspection by *ECH*, *ECB*, *ECK*, *ECA*, *ECD*, and *ECI*, respectively. Then, every expected cost is obtained as follows:

$$ECH = \frac{1}{\tau} c_H \int_0^{\tau} E\left[H(T_i+t)^+\right] dt, \tag{21}$$

$$ECB = \frac{1}{\tau} c_B \int_0^{\tau} E\left[H(T_i+t)^-\right] dt, \tag{22}$$

$$ECK = \frac{c_K \xi}{\tau}, \tag{23}$$

$$ECA = c_A \lambda p, \tag{24}$$

$$ECD = \frac{c_D E[r]}{\tau}, \tag{25}$$

$$ECI = c_I \frac{\lambda}{1-q}. \tag{26}$$

Hence, the total expected cost per unit time in the respective (*s*, *S*) inventory policies is evaluated as the following equation:

$$TC_p(s, S) = ECH + ECB + ECK + ECA + ECD + ECI. \tag{27}$$

The optimization of $TC_p(s, S)$ involves the selection of the pair of (*s*, *S*) so that the sum of the costs are minimized. Note that we can ignore *ECA* and *ECI* in the optimization process in actual since *ECA* and *ECI* are constants which are independent of *s* and *S*. It is well known that $E[H(T_i+t)^+]$ and $E[H(T_i+t)^-]$ are convex in *S*. Hence, $TC_p(s, S)$ is convex in *S*. If we let $Q = S - s$,

$$E\left[H(T_i+t)\right] = \begin{cases} S - \sum_{a=\underline{a}}^{\bar{a}} a\pi_a + \left[1-\alpha(t)\right]\sum_{b=0}^{\bar{b}} b\pi_b, & \text{if } 0 \leq t < w \\ S - \sum_{a=\underline{a}}^{\bar{a}} a\pi_a + \left[1-\alpha(w)\right]\sum_{b=0}^{\bar{b}} b\pi_b - \lambda_t, & \text{if } w \leq t < L \\ S - \sum_{a=\underline{a}}^{S-s-1} a\pi_a - \sum_{a=S-s}^{\bar{a}} \pi_a \sum_{r=0}^{a} r\binom{a}{r} q^r (1-q)^{a-r} + \left[1-\alpha(w)\right]\sum_{b=0}^{\bar{b}} b\pi_b - \lambda_t, & \text{if } L \leq t < \tau \end{cases} \tag{16}$$

$$E\left[H(T_i+t)^+\right] = \begin{cases} \sum_{a=\underline{a}}^{\bar{a}} \sum_{b=0}^{\bar{b}} (S-a+b)^+ \pi_{ab}, & \text{if } t = 0 \\ \sum_{a=\underline{a}}^{\bar{a}} \sum_{b=0}^{\bar{b}} \pi_{ab} \sum_{n=0}^{b} (S-a+n)^+ \binom{b}{n} \left[1-\alpha(t)\right]^n \left[\alpha(t)\right]^{b-n}, & \text{if } 0 < t < w \\ \sum_{a=\underline{a}}^{\bar{a}} \sum_{b=0}^{\bar{b}} \pi_{ab} \sum_{n=0}^{b} \sum_{m=0}^{S-a+n} (S-a+n-m)^+ \binom{b}{n} \beta^n (1-\beta)^{b-n} \frac{\lambda_t^m}{m!} e^{-\lambda_t}, & \text{if } w \leq t < L \\ \sum_{a=\underline{a}}^{S-s-1} \sum_{b=0}^{\bar{b}} \pi_{ab} \sum_{n=0}^{b} \sum_{m=0}^{S-a+n} (S-a+n-m)^+ \binom{b}{n} \beta^n (1-\beta)^{b-n} \frac{\lambda_t^m}{m!} e^{-\lambda_t} & \\ \quad + \sum_{a=S-s}^{\bar{a}} \sum_{b=0}^{\bar{b}} \pi_{ab} \sum_{r=0}^{a} \sum_{n=0}^{b} \sum_{m=0}^{S-a+n} (S-r+n-m)^+ \binom{a}{r} q^r (1-q)^{a-r} \binom{b}{n} \beta^n (1-\beta)^{b-n} \frac{\lambda_t^m}{m!} e^{-\lambda_t}, & \text{if } L \leq t < \tau \end{cases} \tag{17}$$

ξ is dependent on *Q* but not *S*. The optimal *s* and *S* can be efficiently sought with a two phase approach. First, we fix *Q* and derive the optimal *S*. In the second phase, we search over *Q* to be the optimal *s* (Cheung and Zhang 1999).

4. NUMERICAL EXAMPLES

In this section, we show the impact of nonconforming items on (*s*, *S*) inventory model, and then we have a sensitivity analysis. At first, set some model parameters as follows:

$$\tau = 5, \quad L = 2, \quad \lambda = 5.0, \quad E[G] = 0.5.$$

And then, the respective coefficients for costs are given as

$$c_H = 0.5, \quad c_B = 5.0, \quad c_K = 100.0,$$
$$c_A = 5.0, \quad c_D = 10.0, \quad c_I = 1.0.$$

Table 1 shows the results of every cost in some q under the particular inventory policy (s, S) = (10, 40), where the rest model parameters are given as $w = 1$, $p = 0.25$. From Table 1, we have obtained some information as follows:

As the proportion of nonconforming items is increased,

a) *ECH* is reduced due to the removal of nonconforming items included in the procurement.
b) *ECB* is increasing because the removal of nonconforming items included in the procurement leads to the shortage of the stock.
c) *ECK* is increasing because the inventory position becomes frequently equal to or below s at the review epochs due to the removal of nonconforming items included in the procurement.
d) *ECA* is not changed since the customer order cancellation is assumed to be independent of the inclusion of nonconforming items.
e) *ECD* is increasing because the number of the nonconforming items becomes large.
f) *ECI* is increasing because more items are procured and inspected for the purpose of meeting demands.

These results are quite reasonable.

Then, we derive the optimal *s* and *S* through two phase approach mentioned in the previous section. At first, the change of $TC_p(s, S)$ is shown in *S* for given *Q* in Figure 1. The parameters in Figure1 are same as those of Table 1, where *q* = 0.05. Let $S^\dagger$ be such that $TC_p(s, S)$ is minimized for given *Q*. Then, we show the change of $TC_p(s, S^\dagger)$ in *Q* in Figure 2, where $S^\dagger$ is given for every *Q*. Consequently, we obtain the optimal inventory policy (*s*, *S*).

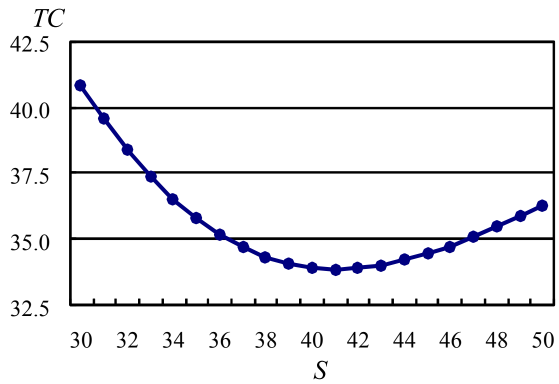

Figure 1. the change of $TC_p(s, S)$ in *S* for given *Q*.

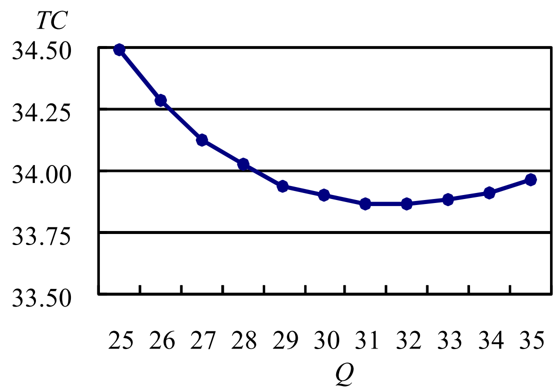

Figure 2. the change of $TC_p(s, S^\dagger)$ in *Q*.

Then, we show the impact of nonconforming items and some model parameters on the economical inventory

Table 1. every cost in some q under the condition as the inventory policy $(s, S) = (10, 40)$ and $w = 1$, $p = 0.25$.

q	*ECH*	*ECB*	*ECK*	*ECA*	*ECD*	*ECI*	*TC*
0.00	8.92	1.87	9.57	6.25	0.00	5.00	31.60
0.01	8.79	1.93	9.63	6.25	0.38	5.05	32.03
0.05	8.28	2.26	9.89	6.25	1.97	5.26	33.90
0.10	7.66	2.78	10.28	6.25	4.15	5.56	36.67
0.25	6.40	3.53	12.60	6.25	12.45	6.67	47.90

policy. Table 2 indicates the optimal inventory policy (s^*, S^*), and every cost in some q under the condition as $w=1$, $p=0.25$. From Table 2, it is shown that the increase of q leads to large S for the purpose of compensating the reduction of the stock due to the removal of nonconforming items.

Further, Table 3 is given in order to show the impact of the cancellation. Table 3 corresponds to the situation as $w=1$, $p=0.00$, in other words, the customer does not cancel. The other model parameters are the same as them in Table 2. Since the customer does not cancel, the actual demand is increased. Therefore, it is verified that the optimal S in Table 3 is large in comparison to the results in Table 2. In addition, the optimal s is large in comparison to the results in Table 2. When customers are allowed to cancel their orders, the item that is eventually cancelled is regarded as the stock in the optimal program. Consequently, optimal s becomes larger when customers are not allowed to cancel their orders. Also, it is conformed that the increase of q leads to large S in Table 3.

Furthermore, Table 4 is shown in order to research the effect of the customer order reservation. Table 4 corresponds to the situation as $w=0$, $p=0.00$, in other words, the customer does not cancel, and then the customer orders are needed to fulfill immediately after their requests. From Table 3 and Table 4, it is shown that the optimal s and S in Table 4 are large in comparison to the results in Table 3 for the purpose of avoiding the costly backorder.

5. CONCLUSION

Recently, some researches have focused upon an impact of distortions in the information, *e.g.*, customer order cancellation, on the economical inventory policy. On one hand, the nonconforming item should be considered a kind of distortions in the inventory information, because the nonconforming item is equivalent to a phony stock. We have considered the inventory model under the situation that customers can cancel their orders and the procurement of suppliers may include some nonconforming items. Then, the customer order reservation has been introduced into the inventory model for the purpose of avoiding the costly backlogs. In this article, we have formulated a periodic review (s, S) inventory model and investigate the economical operation under the situation mentioned above. Further, we have shown the impact of some distortions and the effect of the customer order reservation on the inventory policy.

Table 2. optimal model parameters (s^*, S^*) and every cost in some q under the condition as $w=1$, $p=0.25$.

q	s^*	S^*	ECH	ECB	ECK	ECA	ECD	ECI	TC
0.00	10	39	8.59	2.02	9.73	6.25	0.00	5.00	31.59
0.01	10	40	8.79	1.93	9.63	6.25	0.38	5.05	32.03
0.05	10	41	8.59	2.07	9.73	6.25	1.97	5.26	33.86
0.10	10	44	8.82	2.01	9.60	6.25	4.15	5.56	36.38
0.25	10	54	9.14	2.03	9.37	6.25	12.45	6.67	45.90

Table 3. optimal model parameters (s^*, S^*) and every cost in some q under the condition as $w=1$, $p=0.00$.

q	s^*	S^*	ECH	ECB	ECK	ECA	ECD	ECI	TC
0.00	13	44	9.01	2.04	9.76	0.00	0.00	5.00	25.81
0.01	13	44	8.87	2.14	9.82	0.00	0.40	5.05	26.28
0.05	13	46	8.96	2.13	9.78	0.00	2.11	5.26	28.24
0.10	13	49	9.15	2.08	9.69	0.00	4.44	5.56	30.92
0.25	13	59	9.36	2.17	9.55	0.00	13.33	6.67	41.08

Table 4. optimal model parameters (s^*, S^*) and every cost in some q under the condition as $w=0$, $p=0.00$.

q	s^*	S^*	ECH	ECB	ECK	ECA	ECD	ECI	TC
0.00	20	57	11.21	2.46	9.95	0.00	0.00	5.00	28.63
0.01	20	57	11.02	2.62	9.99	0.00	0.51	5.05	29.20
0.05	20	59	10.99	2.69	10.01	0.00	2.63	5.26	31.59
0.10	20	62	11.05	2.70	10.01	0.00	5.56	5.56	34.87
0.25	20	74	11.39	2.63	9.95	0.00	16.67	6.67	47.30

ACKNOWLEDGMENT

We would like to thank the Editor and referees very much for their helpful, useful, and constructive comments and suggestions regarding this article. Then, this research was partially supported by the Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research (C), No.20510138, 2008-2010.

REFERENCES

Cheung, K. L. and Zhang, A. X. (1999), The impact of inventory information distortion due to customer order cancellations, *Naval Research Logistics*, **46**, 213-231.

Gross, D. and Harris, C. M. (1985), *Fundamentals of queueing theory*, Wiley, New York.

Khouja, M. (2003), The impact of quality consideration on material flow in two-stage inventory systems, *International Journal of Production Research*, **41**, 1533-1547.

Lee, H. L., Padmanabhan, P., and Whang, S. (1997), Information distortion in a supply chain: The bullwhip effect, *Management Science*, **43**, 546-558.

Moinzadeh, K. and Lee, H. L. (1987), A continuous inventory model with constant resupply time and defective items, *Naval Research Logistics*, **34**, 457-467.

Taylor, H. M. and Karlin, S. (1998), *An introduction to stochastic modeling (3rd edition)*, Academic Press, Orlando, FL.

Wu, K. S. and Ouyang, L. Y. (2001), (Q, r, L) inventory model with defective items, *Computers and Industrial Engineering*, **39**, 173-185.

Yuan, X. M. and Cheung, K. L. (2003), Optimal inventory policy for systems with demand cancellation, *IIE Transactions*, **35**, 739-747.