

The Incremental Cost Matrix Procedure for Locating Repair Service Centers in Multinational Reverse Logistics

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Abstract. This study provides a heuristic algorithm to solve the locating problem of repair service centers (RSCs). To enhance the customer service level with more satisfaction and quicker responsiveness, the locating problem of RSCs has become one of the important issues in reverse supply chain management. This problem is formulated as a zero-one mixed integer programming in which an exiting distributor will be considered to be an un-capacitated repair service center for the objective of cost-minimizing. Since logistical costs are highly inter-related with the multinational location of distributors and RSCs, the fixed cost for setting a repair service center, variable cost, transportation cost, and exchange rates are considered in this study. Recognizing the selection of un-capacitated RSCs' locations is a combinatorial optimization problem and is a zero-one mixed integer programming with NP-hard complexity, we provide a heuristic algorithm named as incremental cost matrix procedure (ICMP) to simplify the solving procedure. By using the concise and structural cost matrix, ICMP can efficiently screen the potential location with cost advantage and effectively decide which distributor should be a RSC. Results obtained from the numerical experiments conducted in small scale problem have shown the fact that ICMP is an effective and efficient heuristic algorithm for solving the RSCs locating problem. In the future, using the extended ICMP to solve problems with larger industrial scale or problems with congestion effects caused by the variation of customer demand and the restriction of the RSC capacity is worth a further investigation.

Keywords: Locating Problem, Repair Service Centers, Multinational Reverse Logistics

1. INTRODUCTION

There are two major types of supply chains to be concerned within any production and distribution sys-

tem: the forward and the reverse supply chain (also called reverse logistics). International co-operative global complementary production systems (ICGCPS), which is a global production system with several production bases

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located in several countries, is on behalf of the forward supply chain system. ICGCPS have been widely developed in order to transfer product design, manufacturing and management technologies and to improve the international coordination and divisions of labor (Hiraki *et al.*, 2008). On the other hand, reverse logistics can take place through the original forward channel, through a separate reverse channel, or through combinations of the forward and reverse channel (Fleischmann *et al.*, 1997). Moreover, an effective reverse logistics operation can benefit both the organization and its customers. To heighten this critical dimension of competitive priorities, many successful organizations have encompassed multiple operating activities, including: customer support through valid training, product warranties, maintenance and repair, product upgrades, sales of complementary products, and product disposal (Amini *et al.*, 2005) for the purpose of improving the customer's relationship. One of the most important after sale activities is repair services. Blumberg (1999) has shown the fact that the demand for repair services is robust and increasing, both in the US and worldwide, and the existence, effectiveness, and efficiency of repair services are depend heavily on effective reverse logistics operations. Repair services management positively impact total costs of ownership (Tibben-Lembke, 1998), thereby affecting customer loyalty. The organization with well-established repair service system benefits because it has the opportunity to realize additional profit streams from after sale services as well as repeat purchases from loyal customers.

As mentioned above, locating the global repair service centers (GRSCs) for a multinational production-distribution alliance in the reverse logistics becomes one of the challenging problems. The locating problem aims at determining a set of repair service centers in such a way that the sum of fixed, variable, and transportation costs is minimized.

The design of logistics networks has long been the application of location models within operations research, such as the capacitated plant location problem (CPLP) versus simple or un-capacitated plant location problem (SPLP). The capacitated plant location problem has been a lot of research on the forward supply chain concerned with the strategy is distributed the products from manufacturer plants or factories to final customers. These problems are to find minimum total cost in the set of potential locations for plants with fixed cost, variable cost, and capacities. Finally, an extensive review of the capacitated plant location problem can be found in Sridharan (1995). Our study has some elements in common with the simple or un-capacitated plant location problem.

2. MATHEMATICAL FORMULATION

Since location and logistical costs are highly inter-related for locating the global repair service centers, this study provides an integrated model, and minimizes total

physical distribution costs by simultaneously determining optimal locations and shipment.

2.1 Basic Assumption

In order to formulate a mathematical model for the proposed model, we make the following assumptions:

- (1) Defective and repairable goods are currently dispersed to the regional market. An efficient strategy is made to ship defective and repairable goods from the regional market to specific regional repair service center or original factory if it needed to be.
- (2) For sharing the limited resource, the repair service center is attached to the existing regional agency. A repair service center departing form an existing regional distributor cannot be considered.
- (3) Each repair service center has a sufficient capacity to deal with returned defective and repairable goods.
- (4) All the parametric values of costs, such as fixed, variable, and transportation, are known.
- (5) Each defective and repairable good is not allowed to skip over the unopened repair service center and is sent back to original factory directly. That is, each returned defective and repairable goods should be first sent to a repair service center in distributor, and then be sent to the original factory if it needed to be.

2.2 Parameters and Variables

The following notation will be used to describe parameters and variables in developing the model:

2.2.1 Parameters

- N the number of the distribution region;
- i index of the distribution region ($i = 1, 2, \dots, N$) and the original factory ($i = 0$);
- j index for the regional distributor, $j = 1, 2, \dots, N$;
- $X_{i,j}$ fraction of units from distribution region i to repair service center j ;
- d_i units of defective and repairable goods should be sent to repair service center at distribution region i ;
- d_j units of defective and repairable goods should be sent to original factory at regional repair service center j ;
- m_i units of defective and repairable goods should be sent to original factory at distribution region i ;
- F_j fixed cost for setting repair service center j , local currency;
- $f_{i,j}$ unit transportation cost for a defective and repairable good from distribution region i to repair service center j , local currency;
- $f_{j,i}$ unit transportation cost for a defective and repairable good from repair service center j to distribution region i , local currency;
- $f_{j,0}$ unit transportation cost for a defective and repairable good from repair service center j to original factory 0 ;

- $f_{0,j}$ unit transportation cost for a defective and repairable good from original factory 0 to repair service center j ;
- VC_j unit variable cost for a defective and repairable good at repair service center j , local currency;
- VC_0 unit variable cost for a defective and repairable good at original factory;
- ε_j exchange rate of currency for the repair service center j .

2.2.2 Variables

$$R_j = \begin{cases} 1, & \text{if repairing center set;} \\ 0, & \text{otherwise.} \end{cases}$$

2.3 Model Structure

In the mathematical model that follows, the objective function comprises the following components: (1) fixed cost for setting the repair service center, (2) unit variable cost for a defective and repairable good for each repair service center, and (3) unit transportation cost for a defective and repairable good from a distribution region to the repair service center. In addition, the exchange rate is also considered.

Objective function:

$$\begin{aligned} \text{Minimize } Z = & \sum_{j=1}^n \sum_{i=1}^m \varepsilon_j (VC_j + f_{i,j} + f_{j,i}) d_i \cdot X_{ij} \\ & + \sum_{j=1}^n \varepsilon_j \cdot F_j \cdot R_j \\ & + \sum_{j=1}^n R_j (VC_0 + f_{j,0} + f_{0,j}) \cdot d_j. \end{aligned} \quad (1)$$

Subject to:

$$\sum_{j=1}^n X_{ij} = 1 \text{ for all } i; \quad (2)$$

$$0 \leq X_{ij} \leq R_j \leq 1 \text{ for all } i, j; \quad (3)$$

$$0 \leq d_j = \sum_{i=1}^n X_{ij} \cdot m_i \cdot d_i, \text{ when } R_j = 1, \text{ for all } j; \text{ and} \quad (4)$$

$$R_j \in \{0, 1\}. \quad (5)$$

The objective function minimizes the sum of cost components as noted before. Constraint (2) assures that a regional customer is assigned to a single repair service center. Constraint (3) prevents any returned defective and repairable good across the unopened repair service center to original factory directly. Constraint (4) assures that all repairing product need be sent to the original factory from each regional through the opening regional repair service center. Constraint (5) is used to explain the feature of variables.

2.4 The Optimal Solution Procedure

The proposed model is a combinatorial optimiza-

tion problem with the complexity of NP-hard (Jayaraman *et al.*, 2003), and can be solved by using zero-one mixed integer programming (MIP). Since the repair capacity of each repair service center is not made, the model is known as the simple or un-capacitated plant location problem (SPLP). All the un-capacitated problems can be solved by one of the following procedures:

- (1) The parallel interior point algorithm (Silva and Abramson, 1998).
- (2) A branch and bound phase is required to obtain 0 or 1 value for the binary variables.

Procedure of using the conventional MIP is limited to the complexity of the problem and the large number of variables and constraints (Jayaraman *et al.*, 2003).

3. HEURISTIC PROCEDURE

To improve the computational efficiency of the combinatorial optimization problem, developing feasible and heuristic procedures is always necessary. We modified the reliability, covering and balanced matrices () to be the Incremental Cost Matrices Procedure (ICMP) in order to simplify the solving procedure in this section.

Recognizing the fact that some of the regional distributors have cost advantage than others in specific cost items to set up a repair service center, the ICMP can be developed to locating repair service centers with the consideration of primary costs in proper order. The first priority of costs to be taken into account is the fixed cost of setting repair service center and transportation cost. The variable cost of repair is the second priority. The procedure is explained in the following steps:

Begin

Step 0 Initialize Matrix A which contains the first priority of costs, fixed cost of setting repair and service center transportation cost.

a_{ij} is defined as the relevant cost which are total transportation cost from regional market to initial repair service center.

F_{nn} is defined as the fixed cost of setting repair service center.

$$A = \begin{bmatrix} F_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & F_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & F_{nn} \end{bmatrix}, \quad (6)$$

$$\text{where } a_{ij} = \begin{cases} \varepsilon_j \cdot 2f_{i,j} \cdot d_i, & i \neq j; \\ \varepsilon_j \cdot F_{i,j}, & i = j. \end{cases}$$

Step 1 Create a matrix with secondary priority of costs, variable cost of repair.

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}, \quad (7)$$

where $b_{ij} = \varepsilon_j \cdot d_i \cdot VC_j$ is defined as the element of variable cost.

Step 2 Summate matrices \mathbf{A} and \mathbf{B}

$$\mathbf{C} = \mathbf{A} + \mathbf{B} \quad (8)$$

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}. \quad (9)$$

If $c_{ij} = \min\{c_{m1}, c_{m2}, \dots, c_{mn}\}$ and $i = j$, the repair service center at j is set up. Stop the procedure.

If $c_{ij} = \min\{c_{m1}, c_{m2}, \dots, c_{mn}\}$ but $i \neq j$, the repaired products should be sent from i to j . Stop the procedure.

If $c_{ij}|_{i \neq j} = c_{ij}|_{i=j}$ in row " m ", go to Step 3.

Step 3 Create the checking matrix " \mathbf{D} "

The checking matrix is considered of the third priority of costs, the transportation cost of repaired products sent back to original factory at distribution region.

d_{ij} is defined as the repaired product should be sent to the original factory at distribution region i .

$$\mathbf{D} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix}, \quad (10)$$

where $d_{ij} = d_i \cdot m_i \cdot 2f_{0j}$.

Step 4 Summate the matrix \mathbf{C} and matrix \mathbf{D}

$$\mathbf{E} = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1n} \\ e_{21} & e_{22} & \cdots & e_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ e_{m1} & e_{m2} & \cdots & e_{mn} \end{bmatrix}, \quad (11)$$

where $\mathbf{E} = \mathbf{C} + \mathbf{D}$.

If $e_{ij} = \min\{e_{m1}, e_{m2}, \dots, e_{mn}\}$ and $i = j$, the repair service center is set up at regional distributor j , or otherwise, Stop the procedure.

If $e_{ij} = \min\{e_{m1}, e_{m2}, \dots, e_{mn}\}$ but $i \neq j$, the repaired products should be sent from i to j . Stop the procedure.

If $e_{ij}|_{i \neq j} = c_{ij}|_{i=j}$ in row " m ", the repair service center is set up at regional distributor j . Stop the procedure.

End

4. COMPUTATIONAL STUDY

In the illustrative example, four regional distributors ($N = 4$) are considered to be set up as the repair service center and their parameters are shown in Table 1 and Table 2. A total of 41 test problems are generated for the combinations of four regional distributors and four repair service centers. The problems with and without the consideration of exchange rates are included. For the convenience of comparing, the total cost of the model has been transferred to the local currency of the original factory.

4.1 The Optimal Solution

The optimal solutions of the illustrative example with and without the consideration of exchange rates obtained by using the conventional MIP are shown in Table 3 and Table 4, respectively. The optimal solution shows that repair service centers are set up at regional distributors 2, 3, and 4. The repair requirement of distribution region 2 is severed by the repair service centers at regional distributors 2. This optimal locating policy costs \$615,500 in the currency of the original factory.

4.2 The Incremental Cost Matrices Procedure

By using ICMP, the procedure for determining the locations of repair service centers is shown as follows:

Begin

Step 0 Initial a matrix \mathbf{A}

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 250000 & 20000 & 60000 & 40000 \\ 2 & 200000 & 27000 & 160000 & 120000 \\ 3 & 1200000 & 320000 & 30000 & 160000 \\ 4 & 200000 & 60000 & 40000 & 37500 \end{bmatrix}.$$

Step 1 Create a matrix \mathbf{B}

$$\mathbf{B} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 250000 & 36000 & 40000 & 45000 \\ 2 & 500000 & 72000 & 80000 & 90000 \\ 3 & 1000000 & 144000 & 160000 & 180000 \\ 4 & 250000 & 36000 & 40000 & 45000 \end{bmatrix}.$$

Step 2 $\mathbf{C} = \mathbf{A} + \mathbf{B}$

$$\mathbf{C} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 500000 & 56000 & 100000 & 85000 \\ 2 & 540000 & 99000 & 240000 & 210000 \\ 3 & 1240000 & 464000 & 190000 & 340000 \\ 4 & 290000 & 96000 & 80000 & 825000 \end{bmatrix}.$$

Stop the procedure.

End

Table 4. Results of the numerical experiment with exchange rate.

	Center	1	2	3	4								
	Total Cost	(1, 1, 0) (2, 1, 0) (3, 1, 0) (4, 1, 0)	3,822,500	(1, 1, 0) (2, 1, 0) (3, 1, 0) (4, 1, 0)	873,500	(1, 3, 0) (2, 3, 0) (3, 3, 0) (4, 3, 0)	805,500	(1, 4, 0) (2, 4, 0) (3, 4, 0) (4, 4, 0)	1,106,500				
	Centers	1, 2		1, 3		1, 4		2, 3		2, 4	3, 4		
(4) (1)	Total Cost	(1, 1, 0) (3, 1, 0) (4, 1, 0) (2, 2, 0)	3,228,500	(1, 1, 0) (2, 1, 0) (4, 1, 0) (3, 3, 0)	1,828,500	(1, 1, 0) (2, 1, 0) (3, 1, 0) (4, 4, 0)	3,448,500	(1, 2, 0) (2, 2, 0) (3, 3, 0) (4, 2, 0)	631,500	(1, 2, 0) (2, 2, 0) (3, 2, 0) (4, 4, 0)	857,500	(1, 3, 0) (2, 3, 0) (3, 3, 0) (4, 4, 0)	799,500
	Total Cost	(1, 1, 0) (2, 2, 0) (3, 2, 0) (4, 2, 0)	1,118,500	(1, 1, 0) (2, 3, 0) (3, 3, 0) (4, 3, 0)	1,004,500	(1, 1, 0) (2, 4, 0) (3, 4, 0) (4, 4, 0)	1,298,500	(1, 3, 0) (2, 2, 0) (3, 3, 0) (4, 3, 0)	667,500	(1, 4, 0) (2, 2, 0) (3, 4, 0) (4, 4, 0)	930,500	(1, 4, 0) (2, 4, 0) (3, 3, 0) (4, 4, 0)	844,500
	Total Cost	(1, 1, 0) (2, 2, 0) (3, 2, 0) (4, 1, 0)	1,476,500	(1, 1, 0) (2, 1, 0) (3, 3, 0) (4, 3, 0)	1,460,500	(1, 1, 0) (2, 1, 0) (3, 4, 0) (4, 4, 0)	1,716,500	(1, 2, 0) (2, 2, 0) (3, 3, 0) (4, 3, 0)	621,500	(1, 2, 0) (2, 2, 0) (3, 4, 0) (4, 4, 0)	877,500	(1, 3, 0) (2, 4, 0) (3, 3, 0) (4, 4, 0)	837,500
	Total Cost	(1, 1, 0) (2, 2, 0) (3, 1, 0) (4, 2, 0)	2,870,500	(1, 1, 0) (2, 3, 0) (3, 3, 0) (4, 1, 0)	1,372,500	(1, 1, 0) (2, 4, 0) (3, 1, 0) (4, 4, 0)	3,030,500	(1, 3, 0) (2, 2, 0) (3, 3, 0) (4, 2, 0)	677,500	(1, 4, 0) (2, 2, 0) (3, 2, 0) (4, 4, 0)	910,500	(1, 4, 0) (2, 3, 0) (3, 3, 0) (4, 4, 0)	806,500
(4) (2)	Centers	1, 2, 3		2, 3, 4		1, 3, 4		1, 2, 4					
	Total Cost	(1, 1, 0) (2, 2, 0) (3, 3, 0) (4, 1, 0)	1,234,500	(1, 2, 0) (2, 2, 0) (3, 3, 0) (4, 4, 0)	615,500	(1, 1, 0) (2, 1, 0) (3, 3, 0) (4, 4, 0)	1,454,500	(1, 1, 0) (2, 2, 0) (3, 1, 0) (4, 4, 0)	2,854,500				
	Total Cost	(1, 1, 0) (2, 2, 0) (3, 3, 0) (4, 2, 0)	876,500	(1, 3, 0) (2, 2, 0) (3, 3, 0) (4, 4, 0)	661,500	(1, 1, 0) (2, 3, 0) (3, 3, 0) (4, 4, 0)	998,500	(1, 1, 0) (2, 2, 0) (3, 2, 0) (4, 4, 0)	1,102,500				
	Total Cost	(1, 1, 0) (2, 2, 0) (3, 3, 0) (4, 3, 0)	866,500	(1, 4, 0) (2, 2, 0) (3, 3, 0) (4, 4, 0)	668,500	(1, 1, 0) (2, 4, 0) (3, 3, 0) (4, 4, 0)	1,036,500	(1, 1, 0) (2, 2, 0) (3, 4, 0) (4, 4, 0)	1,122,500				
(4) (4)	Centers	1, 2, 3, 4											
	Total Cost	(1, 1, 0) (2, 2, 0) (3, 3, 0) (4, 4, 0)	860,500										

the model, the ICMP is provided to simplify the solving procedure. In our illustrative example, the computational results have indicated that the provided heuristic procedure is effective and efficient. To refine the algorithm and to evidence the performance of the ICMP should be conducted in the future.

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