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Variable Sampling Inspection with Screening When Lot Quality Follows Mixed Normal Distribution

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Abstract. The variable sampling inspection scheme with screening for the purpose of assuring the upper limit of maximum expected surplus loss after inspection has been proposed. In this inspection scheme, it has been assumed that a product lot consists of products manufactured through a single production line and lot quality characteristics follow a normal distribution. In the previous literature with respect to inspection schemes, it has been commonly assumed that lot quality characteristics obey a single normal distribution under the condition that all products are manufactured in the same condition. On the other hand, the production line is designed in order that the workload of respective processes becomes uniform from the viewpoint of line balancing. One of the solutions for the bottleneck process is to arrange the workshops in parallel. The lot quality characteristics from such a production line with the process consisting of some parallel workshops might not follow strictly the single normal distribution. Therefore, we expand an applicable scope of the above mentioned variable sampling inspection scheme with screening in this article. Concretely, we consider the variable sampling inspection with screening for the purpose of assuring the upper limit of average outgoing surplus quality loss in the production lots when the lot quality follows the mixed normal distribution.

Keywords: Maximum Expected Surplus Loss, Mixed Normal Distribution, Quality Assurance, Sampling Inspection Plan with Screening, Taguchi's Quality Loss Function

1. INTRODUCTION

The quality evaluation by the proportion of nonconforming products has been applied to traditional quality control techniques. Nowadays, a high quality manufacturing environment has been promoted and formed. Then, the reduction in the proportion of nonconforming products has been gradually achieved due to the innovation of technology. In case we aim at the further quality improvement, more strict quality evaluation based on a new concept is needed. Taguchi (1986) has presented the concept of the quality loss as the quality evaluation of products based on the variable property instead of the quality evaluation based on the attribute property such as the proportion of nonconforming products. The concept of the Taguchi's quality loss is used for the decision making in various scenes of development and design of the product and process (Ben-Daya and Duffuaa, 2003; Frestervand, Kethey, and Waller, 2001; Kobayashi, Arizono, and Takemoto, 2003; Maghsoodloo and Li, 2000; Wu, Shamuzzaman, and Pan, 2004). The Taguchi's quality loss is defined as a quadratic function

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based on the deviation from a target value of a product. Then, the quality loss is evaluated even if a manufactured product is judged to be conforming as a product. Therefore, the achievement of high quality can be expected when the quality loss is applied to the quality evaluation in the inspection scheme. In the previous researches, Arizono *et al*. (1997) have proposed a single sampling inspection plan based on operating characteristics from the viewpoint of assuring the Taguchi's quality loss. Then, Morita, Arizono, and Takemoto (2009) have proposed the variable sampling inspection with screening for the purpose of assuring the upper limit of maximum expected surplus loss after inspection.

Generally, the bottleneck process for the workload appears in the production system. According to the performance of the entire system, the improvement in the workload at the bottleneck process needs to be discussed. The production line is designed in order that the workload of respective processes becomes uniform from the viewpoint of line balancing. One of the solutions for the bottleneck process is to arrange the workshops in parallel. In the traditional inspection scheme, it has been assumed that the product lot consists of products manufactured through the single workshop and the lot quality characteristics follow a single normal distribution. When some workshops are arranged in parallel at the process, the quality characteristics of the products manufactured via such a process might not strictly follow the single normal distribution. In the case that the quality characteristics of the products yielded in the respective workshop follow the normal distribution, the quality characteristics of the products in the entire process might follow the mixed normal distribution. When the quality evaluation is provided by the proportion of nonconforming products, it is basically important whether the product specification is satisfied or not in spite of the difference in the distribution of the quality characteristics. On one hand, when the quality evaluation is provided by Taguchi's quality loss, the difference in the distribution of the quality characteristics makes a great impact on the quality loss.

In this article, we propose the design procedure of the variable sampling inspection with screening for the purpose of assuring the upper limit of average outgoing surplus quality loss in the production lots when the lot quality follows the mixed normal distribution consisting of some normally distributed quality characteristics.

2. CONCEPT OF ASSURING AVERAGE OUTGOING SURPLUS QUALITY LOSS LIMIT

According to the concept of the Taguchi's quality loss, the quality loss function is expressed as the quadratic form with respect to the difference between an actual value x and a target value μ_T. Then, it is expressed as $k(x-\mu_T)^2$. Simply, we denote $k=1$ without the loss of generality because k is a constant. When the quality characteristics x obeys a normal distribution $N(\mu, \sigma^2)$, the expected loss per product has been obtained as follows.

$$\tau^2 = E\left[(x-\mu_T)^2\right] = \sigma^2 + (\mu-\mu_T)^2 . \tag{1}$$

Assume a normal distribution $N(\mu_i, \sigma_i^2)$ with mean μ_i and variance $\sigma_i^2, i=1,2,\cdots,m,$ where m is the number of workshops. A lot is composed of the products of the quality characteristics following the mixture distribution of those normal distributions under the situation mentioned the above. In this article we call such a lot "the mixture lot." Then, p_i denote the respective mixture rates, where $p_1 \le p_2 \le \cdots \le p_m$ for convenience.

Assume that p_i is known, where the number of products manufactured at the respective workshops is not always equivalent. The inspection is also assumed to be executed for the mixture lot composed of the products manufactured at some workshops, where the workshop can not be distinguished by products. If not, the inspection equipment should be needed at the end of the respective workshops, or the small lots at each workshop need to be respectively inspected.

From Eq. (1), the expected loss of respective products following the normal distribution $N(\mu_i, \sigma_i^2)$ $i=1,2,\cdots,m,$ is expressed as

$$\tau_i^2 = E\left[(x_i-\mu_T)^2\right] = \sigma_i^2 + (\mu_i-\mu_T)^2 . \tag{2}$$

Therefore, the expected loss of the mixture lot is expressed as

$$\theta^2 = \sum_{i=1}^{m} p_i \tau_i^2 . \tag{3}$$

Then, assume that the normal distribution $N(\mu_T, \sigma_T^2)$ is defined as the ideal distribution of quality characteristics. When the respective lots $N(\mu_i, \sigma_i^2), i=1,2,\cdots,m,$ are the ideal quality characteristic distributions, the mean μ_i and the variance σ_i^2 of the quality characteristics of each product are $(\mu_i, \sigma_i^2) = (\mu_T, \sigma_T^2),\ i=1,2,\cdots,m.$ Then, the expected loss of the mixture lot θ^2 is expressed as $\theta^2 = \sigma_T^2 \equiv \theta_T^2$. θ_T^2 can be interpreted as an unavoidable loss in the ideal quality characteristic distribution. Since the variance in the ideal state where the quality loss is minimized is defined as σ_T^2, the variance σ_i^2 is $\sigma_i^2 \ge \sigma_T^2,\ i=1,2,\cdots,m.$ The expected loss of a product in lots θ^2 is expressed as

$$\theta^2 = \sum_{i=1}^{m} p_i \tau_i^2 \ge \theta_T^2 (= \sigma_T^2) . \tag{4}$$

Let $P(\theta^2)$ be the probability that the mixture lot on θ^2 is accepted in this sampling inspection. We consider the sampling inspection plan in order that the probability $1-P(\theta_T^2)$ where the lot on θ_T^2 is rejected is less than the specified producer's risk α. While, with respect to the lot which is rejected in this sampling inspection plan, we inspect totally the products in the rejected lot. Then, throughout a screening rule, some products are exchanged. Concretely, we consider the screening with replacing rule in order that the expected loss after screening is less than θ_T^2.

We discuss the replacing rule in the screening. If the actual value x of the product within the following range:

$$(x-\mu_T)^2 \le \theta_T^2 (=\sigma_T^2), \tag{5}$$

that is,

$$\mu_T - \theta_T \le x \le \mu_T + \theta_T,$$

the product is shipped directly, otherwise, it is replaced by a product within the range of Eq. (5). When let Θ^2 and L be the expected loss per product on the screened lot and the expected loss per product on all lots after screening, the following relation is derived

$$L = \theta^2 P(\theta^2) + \Theta^2 \left\{1 - P(\theta^2)\right\}.$$

It is clearly that the value Θ^2 is less than or equal to θ_T^2 from Eq. (5). Let L_{upper} represent the expected loss limit of L, then we have

$$L_{upper} = \theta^2 P(\theta^2) + \theta_T^2 \left\{1 - P(\theta^2)\right\}. \tag{6}$$

Further, we obtain the following relation:

$$L_{upper} - \theta_T^2 = (\theta^2 - \theta_T^2) P(\theta^2) \equiv S. \tag{7}$$

Then, the value S in Eq. (7) is interpreted to be the maximum value of the surplus expected quality loss (in what follows , we call "average outgoing surplus quality loss limit (AOSQLL)"). We consider the acceptance sampling plan in order that the value S is less than or equal to a specified permissible average outgoing surplus quality loss limit (PAOSQLL) S_{limit}. Consequently, we develop the algorithm for deciding the sampling plan in order that the following relation is satisfied:

$$(\theta^2 - \theta_T^2) P(\theta^2) \le S_{\text{limit}}. \tag{8}$$

3. PROPOSAL OF DESIGN PROCEDURE FOR VARIABLE SAMPLING PLANS WITH SCREENING INSPECTION

Let $x_{ij},\ j = 1, 2, \cdots, r_i$, be random samples from a normal distribution $N(\mu_i, \sigma_i^2),\ i = 1, 2, \cdots, m,$ and r_i be sample size from the quality characteristics following the normal distribution $N(\mu_i, \sigma_i^2)$, where μ_i and σ_i^2 are unknown.

While the mixture rate p_i is known, a product can not distinguish the workshop where that is manufactured. Also, r_i is a random variable of multinomial distribution $Mult(n, p_i)$.

The estimator $\hat{\theta}^2$ of the expected loss is defined by

$$\hat{\theta}^2 = \frac{1}{n} \sum_{i=1}^{m} \sum_{j=1}^{r_i} (x_{ij} - \mu_T)^2.$$

and the distribution of the estimator $\hat{\theta}^2$ follows

$$\hat{\theta}^2 \sim \frac{1}{n} \sum_{i=1}^{m} \sigma_i^2 \chi^2_{r_i, r_i \xi_i}, \tag{9}$$

$$\xi_i = \frac{(\mu_i - \mu_T)^2}{\sigma_i^2},\ i = 1, 2, \cdots, m.$$

$\chi^2_{\varphi, \varphi\xi}$ denotes a noncentral chi-square distribution with φ degrees of freedom and noncentrality parameter $\varphi\xi$. Because the distribution of r_i obeys the multinomial distribution $Mult(n, p_i)$, the mean and variance of the estimator $\hat{\theta}^2$ are respectively given by

$$E\left[\hat{\theta}^2\right] = \sum_{i=1}^{m} p_i \tau_i^2, \tag{10}$$

$$V\left[\hat{\theta}^2\right] = \frac{3}{n} \sum_{i=1}^{m} p_i (\tau_i^2 - \theta^2)^2 + \frac{2}{n}\theta^4 - \frac{2}{n} \sum_{i=1}^{m} p_i (\tau_i^2 - \sigma_i^2)^2. \tag{11}$$

We formulate the inspection plan. Let D be the acceptable value. The acceptance rule is constructed as

$$\begin{cases} \text{if } \hat{\theta}^2 < D, \text{then accept the lot} \\ \text{otherwise, rejected the lot} \end{cases}$$

And the rejected lot is totally inspected. When the expected loss in the ideal quality characteristic distribution is θ_T^2, we design the acceptance sampling plan in order that the producer's risk is less than or equal to the specified value α. Then, let us consider two cases, $\theta^2 = \theta_T^2$ and $\theta^2 > \theta_T^2$. In the case of $\theta^2 = \theta_T^2$, the respective lots are composed of products in the ideal quality characteristic distribution. $N(\mu_i, \sigma_i^2),\ i = 1, 2, \cdots, m,$ is the ideal state. Therefore, we obtain the following relations such as $\mu_i = \mu_T$ and $\tau_i^2 = \theta_T^2 (= \sigma_i^2),\ i = 1, 2, \cdots, m.$ From $\xi_i = 0$, $\hat{\theta}^2$ is explained by the central chi-square distribution. Then, from Eq. (9), we obtain the following equation regardless of r_i:

$$\hat{\theta}^2 \sim \frac{\sigma_T^2}{n} \sum_{i=1}^{m} \chi^2_{r_i} = \frac{\theta_T^2}{n} \chi_n^2. \tag{12}$$

Accordingly, the acceptance rule is translated into

$$\text{if } \hat{\theta}^2 \sim \frac{\chi_n^2(\alpha)}{n} \theta_T^2, \text{ then accept the lot,} \tag{13}$$

where $\chi_n^2(\alpha)$ denotes the upper 100α percentile of the central chi-square distribution with n degrees of freedom.

In the case of $\theta^2 > \theta_T^2$, the probability where the lot on θ^2 is accepted, $P(\theta^2)$ should be given from Eq. (8) as

$$P(\theta^2) \le \frac{S_{\text{limit}}}{\theta^2 - \theta_T^2}. \tag{14}$$

There are innumerable pairs of $(\mu_i, \sigma_i^2),\ i = 1, 2, \cdots,$

m with same θ^2 due to the relation of Eq. (2) and Eq. (3). Naturally, $P(\theta^2)$ is a function of (μ_i, σ_i^2), $i = 1, 2, \cdots, m$ because τ_i^2, $i = 1, 2, \cdots, m$ is a function of (μ_i, σ_i^2). Since the relation of Eq. (14) should be satisfied for all pairs of $(\mu_i, \sigma_i^2), i = 1, 2, \cdots, m$ with same θ^2, Eq. (14) needs to be translated into:

$$\max_{\{\mu_i, \sigma_i^2 \in \theta^2\}} P(\theta^2) \le \frac{S_{\text{limit}}}{\theta^2 - \theta_T^2}, \; i = 1, 2, \cdots, m. \tag{15}$$

With respect to the probability where the lot with the quality loss θ^2 is rejected by the acceptance rule in Eq. (13), the following relation has to be satisfied:

$$\min_{\{\mu_i, \sigma_i^2 \in \theta^2\}} \Pr\left(\hat{\theta}^2 > \frac{\chi_n^2(\alpha)}{n}\right) \ge 1 - \frac{S_{\text{limit}}}{\theta^2 - \theta_T^2}. \tag{16}$$

When the distribution of $\hat{\theta}^2$ is obtained by Eq. (9), a distribution function of a statistic $n\hat{\theta}^2$ is presented as

$$R(z \mid r_1, \cdots, r_m) = \sum_{k=0}^{\infty} b_k Q_{\chi^2}\left(z / \rho; n + 2k\right), \tag{17}$$

where respective r_i are given. Then, $Q_{\chi^2}(z;n)$ denotes a distribution function of the chi-square distribution with n degrees of freedom. ρ is a constant and it is given as $0 < \rho < 2\min\{\sigma_1^2, \sigma_2^2, \cdots, \sigma_m^2\}$. And b_k is defined by

$$a_k = \sum_{i=1}^{m} \frac{r_i}{2}\left(1 - \frac{\rho}{\sigma_i^2}\right)^k + \sum_{i=1}^{m} \frac{k r_i \xi_i}{2} \frac{\rho}{\sigma_i^2}\left(1 - \frac{\rho}{\sigma_i^2}\right)^k,$$

$$b_0 = \exp\left[\sum_{i=1}^{m} -\frac{r_i \xi_i}{2}\right] \times \prod_{i=1}^{m}\left(\frac{\rho}{\sigma_i^2}\right)^{\frac{r_i}{2}},$$

$$b_k = \frac{1}{k}\sum_{j=0}^{k-1} a_{k-j} b_j.$$

Therefore, by considering the distribution of r_i on the distribution function of Eq. (17), Eq. (16) is transformed into

$$1 - \varepsilon \ge \max_{\mu_i, \sigma_i^2 \in \theta^2} \sum_{r_i=0}^{n} \left\{ R(\theta_T^2 \chi_n^2(\alpha) \mid r_1, \cdots, r_m) \frac{n!}{r_1! \cdots r_m!} \prod_{i=1}^{m} p_i^{r_i} \right\}, \tag{18}$$

$$\varepsilon \equiv 1 - \frac{S_{\text{limit}}}{\theta^2 - \theta_T^2}.$$

There are innumerable pairs of $(\mu_i, \sigma_i^2), i = 1, 2, \cdots, m$, with same θ^2 as already explained. It is very difficult to solve such a pair of (μ_i, σ_i^2), $i = 1, 2, \cdots, m$, as to maximize the right side of Eq. (18). In order to solve the pair of $(\mu_i, \sigma_i^2), i = 1, 2, \cdots, m$ as to maximize the right side of Eq. (18), we apply the Patnaik's approximation (Patnaik, 1949). Let the mean and the variance of the statistic $2E[\hat{\theta}^2]\hat{\theta}^2 / V[\hat{\theta}^2]$ correspond with those of the chi-square distribution with ϕ degrees of freedom, where

$$\phi = \frac{2\left\{E\left[\hat{\theta}^2\right]\right\}^2}{V\left[\hat{\theta}^2\right]}. \tag{19}$$

Therefore, we obtain the approximation model:

$$\frac{2E\left[\hat{\theta}^2\right]}{V\left[\hat{\theta}^2\right]}\hat{\theta}^2 \sim \chi_\phi^2. \tag{20}$$

And from Eq. (3) and Eq. (10), the mean of $\hat{\theta}^2$ is given as

$$E\left[\hat{\theta}^2\right] = \theta^2. \tag{21}$$

From Eq. (20) and Eq. (21), we obtain the relation as

$$\hat{\theta}^2 \sim \frac{\theta^2}{\phi}\chi_\phi^2. \tag{22}$$

Then, Eq. (16) is transformed into

$$\frac{\chi_n^2(\alpha)}{n}\theta_T^2 \le \min_{\{\mu_i, \sigma_i^2 \in \theta^2\}} \frac{\chi_\phi^2(\varepsilon)}{\phi}\theta^2. \tag{23}$$

In order to obtain the pair of (μ_i, σ_i^2), $i = 1, 2, \cdots, m$, minimizing the right side of Eq. (23), we have further three cases as follows:

i) $\theta^2 \le \theta_T^2 + S_{\text{limit}}$

It means $\varepsilon \le 0$. It isn't necessary to consider this case on designing the sampling plan because the expected loss is always less than or equal to S_{limit}.

ii) $\theta_T^2 + S_{\text{limit}} < \theta^2 \le \theta_T^2 + 2S_{\text{limit}}$

In this case, $0 < \varepsilon \le 0.5$. Using Wilson-Hilferty approximation (Johnson, 1994), we obtain the following property for the function $\chi_\phi^2(\varepsilon)/\phi$: 1) in the following range:

$$0 \le u_\varepsilon < \sqrt{\frac{8}{9n}},$$

we know that the function $\chi_\phi^2(\varepsilon)/\phi$ is concave, and 2) in the range:

$$\sqrt{\frac{8}{9n}} < u_\varepsilon \le \infty,$$

the function $\chi_\phi^2(\varepsilon)/\phi$ is monotonously decreasing function for ϕ, where u_ε represent the upper 100ε percentile of the standard normal distribution (for details, see Appendix A). Therefore, we must have further two subcases as follows, where γ is defined by the following equation:

$$u_\gamma = \sqrt{\frac{8}{9n}}.$$

ii-1) $\theta_T^2 + S_{\text{limit}} < \theta^2 \le \theta_T^2 + S_{\text{limit}}/\gamma$

Then, $0 < \varepsilon \le 1 - \gamma$. Under this condition, from Appendix A, $\chi_\phi^2(\varepsilon)/\phi$ is monotonously decreasing function in ϕ. Therefore, $\chi_\phi^2(\varepsilon)/\phi$ is the minimum value when ϕ has the maximum value $\phi_{\max}$. Then, because $E[\hat{\theta}^2]$ is the fixed value θ^2, we have to obtain the condition of minimizing the variance $V[\hat{\theta}^2]$ from Eq. (11) in order to obtain the maximum value $\phi_{\max}$. In Eq. (11), we obtain $\tau_i^2 = \theta^2, i = 1, \cdots, m,$ for minimizing the variance $V[\hat{\theta}^2]$. Then, note that $\sigma_T^2 \le \sigma_i^2 \le \tau_i^2$. Hence, the variance $V[\hat{\theta}^2]$ is minimized when $\sigma_i^2 = \sigma_T^2$. Therefore, in order to minimize the variance $V[\hat{\theta}^2]$ in Eq. (11), we obtain the following relation:

$$V\left[\hat{\theta}^2\right] \ge \frac{2}{n}\sigma_T^2(2\theta^2 - \sigma_T^2). \tag{24}$$

In this case, the following combination of (μ_i, σ_i^2) for the maximum value $\phi_{\max}$ is obtained as

$$(\mu_i, \sigma_i^2) = (\mu_T \pm \sqrt{\theta^2 - \sigma_T^2}, \sigma_T^2),\ \ i = 1, 2, \cdots, m. \tag{25}$$

ii-2) $\theta_T^2 + S_{\text{limit}}/\gamma < \theta^2 \le \theta_T^2 + 2S_{\text{limit}}$

In this case, $1 - \gamma < \varepsilon \le 0.5$. Under this condition, from Appendix A, $\chi_\phi^2(\varepsilon)/\phi$ is concave in ϕ. Then, let $\phi_{\min}$ and $\phi_{\max}$ be the minimum value and maximum value of ϕ, respectively. Then, $\chi_\phi^2(\varepsilon)/\phi$ is minimized when ϕ is either $\phi_{\min}$ or $\phi_{\max}$. Then, we have to obtain the condition of minimizing and maximizing the variance $V[\hat{\theta}^2]$ from Eq. (11). $\phi_{\max}$ is already given in Eq. (25). While, it is necessary to obtain the condition to maximize the variance $V[\hat{\theta}^2]$ in order that $\phi_{\min}$ is derived. From Appendix B, $\phi_{\min}$ is given when

$$\begin{cases} (\mu_1, \sigma_1^2) = \left(\mu_T, \dfrac{\theta^2 - (1 - p_1)\sigma_T^2}{p_1}\right) \\ (\mu_i, \sigma_i^2) = (\mu_T, \sigma_T^2), i = 2, 3, \cdots, m \end{cases} \tag{26}$$

iii) $\theta_T^2 + 2S_{\text{limit}} < \theta^2$

In this case, it is obvious that $0.5 < \varepsilon$. Under this condition, $\chi_\phi^2(\varepsilon)/\phi$ is the monotonous increasing function in ϕ from Appendix C. Therefore, $\chi_\phi^2(\varepsilon)/\phi$ is minimized when ϕ is the minimum value $\phi_{\min}$. The combination of (μ_i, σ_i^2) to obtain the minimum value $\phi_{\min}$ is given as Eq. (26).

As mentioned the above, the purpose of this article is to provide the design procedure for the sampling inspection plan that S is less than or equal to S_{limit} for any $\theta^2 (> \theta_T^2)$. Therefore, we should employ the maximum value among sample sizes obtained from the above cases. We denote the adopted sample size by $n_{\max}$. We can obtain the following acceptance rule:

$$\text{if } \hat{\theta}^2 \le \frac{\chi_{n_{\max}}^2(\alpha)}{n_{\max}} \equiv D, \text{ the lot is accepted.} \tag{27}$$

Then, we can decide the acceptance sampling plan for assuring that AOSQLL is less than or equal to PAOSQLL for a given producer's risk α on θ_T^2.

4. NUMERICAL EXAMPLES

In order to illustrate the validity of the proposed procedure, we show some numerical examples. Let $N(\mu_0, \sigma_0^2) = N(0.0, 1.0^2)$, and then, the ideal expected loss is given as $\theta_T^2 = 1.0$. Then, let $\alpha = 0.05$ and $S_{\text{limit}} = 0.25$. First, we consider the situation of $m = 4$, where $p_i = 1/m, i = 1, \cdots, m,$ for convenience. Figure 1 shows the required sample size for each of the expected loss θ^2. In this case, we obtain sample size $n_{\max} = 34$ and the acceptance value $D = 1.42$ for satisfying that AOSQLL is less than or equal to a given value of PAOSQLL. Then, Table 1 shows the required sample size and the acceptance value in each of the number of workshops. We confirm that the result of $m = 1$ corresponds with the result of the previous literature by Morita, Arizono, and Takemoto (2009). Moreover, we confirm that the required sample size increases as the number of workshops is larger. Then, we verify that the AOSQLL for each θ^2 is satisfied. As an example, AOSQLL for each θ^2 is investigated when $m = 4$. In Figure 2, we show that the AOSQLL is less than or equal to the PAOSQLL S_{limit} on the inspection plan.

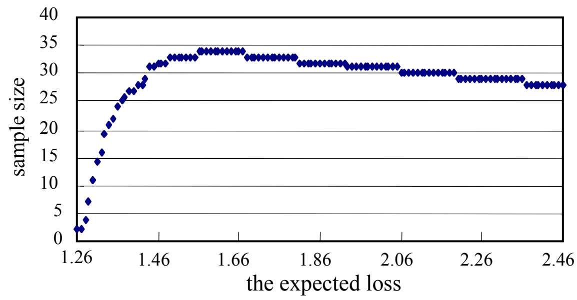

Figure 1. required sample size for each of expected loss θ^2 ($m = 4, \alpha = 0.05, S_{\text{limit}} = 0.25$).

Table 1. required sample size and acceptance value for each of the number of workshops ($\alpha = 0.05, S_{\text{limit}} = 0.25$).

m	n	D
1	30	1.45
2	31	1.45
3	32	1.44
4	34	1.42
5	36	1.41

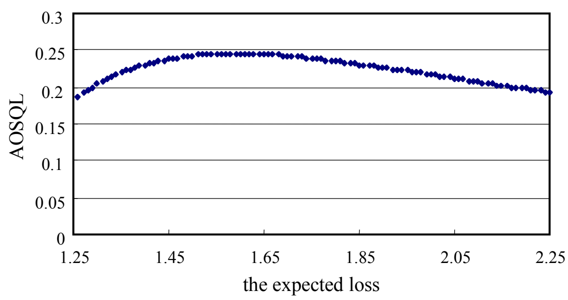

Figure 2. AOSQL for θ^2 under derived inspection plan ($m = 4, \alpha = 0.05, S_{\text{limit}} = 0.25$).

Then, we show Table 2 and Table 3, where the results are given for $\alpha = 0.05$ and $S_{\text{limit}} = 0.15$ in Table 2, and $\alpha = 0.10$ and $S_{\text{limit}} = 0.25$ in Table3, respectively. The condition in Table 2 is strict with manufacturers in comparison with the condition in Table 1. Hence, the required sample size is larger than that of Table 1 in each m. The condition in Table 3 is lenient with manufacturers in comparison with the condition in Table 1. Therefore, the required sample size is smaller than that of Table 1 in each m. Moreover, from Figure 3 and Table 4, we also find that the AOSQLL is less than or equal to the PAOSQLL S_{limit} on the derived inspection plan when $m = 4$. And we confirm the above result in each m.

5. CONCLUDING REMARKS

In this article, we have proposed the acceptance sampling scheme with screening to assure that the AOSQLL based on Taguchi's quality loss is always less than or equal to the specified PAOSQLL when the lot quality follows the mixed normal distribution. Then, the design procedure for this sampling plan has been provided. In the Taguchi's quality loss, there are innumerable pairs of the mean and variance in the quality characteristic distribution for a given expected quality loss. Then, we have defined the AOSQL, and further the AOSQLL has been derived as the upper limit of AO SQL. At last, we have verified that the AOSQLL is less than or equal to the specified PAOSQLL through some numerical examples.

On one hand, the PAOSQLL is specified in this article. PAOSQLL is a target value for assuring the quality level. When the Taguchi's quality loss is adopted, PAOSQLL is a monetary index. Therefore, it is interesting how to decide PAOSQLL. This will be our future research.

Table 2. required sample size and acceptance value for each of the number of workshops ($\alpha = 0.05$, $S_{\text{limit}} = 0.15$).

m	n	D
1	73	1.28
2	74	1.28
3	75	1.28
4	75	1.28
5	77	1.27

Table 3. required sample size and acceptance value for each of the number of workshops ($a = 0.10$, $S_{\text{limit}} = 0.25$).

m	n	D
1	19	1.43
2	21	1.40
3	22	1.40
4	25	1.37
5	27	1.36

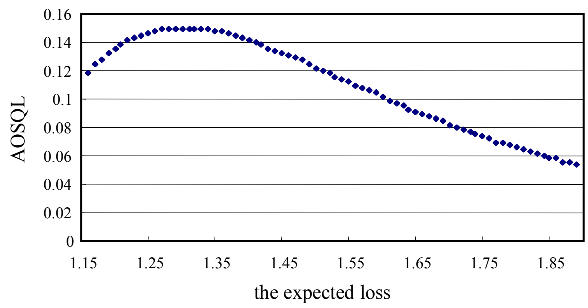

Figure 3. AOSQL for θ^2 under derived inspection plan ($m = 4, \alpha = 0.05$, $S_{\text{limit}} = 0.15$).

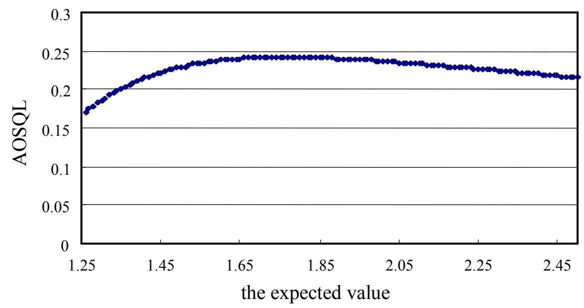

Figure 4. AOSQL for θ^2 under derived inspection plan ($m = 4$, $\alpha = 0.10$, $S_{\text{limit}} = 0.25$).

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APPENDIX A

Appendix A shows the behavior of $\chi_\phi^2(\varepsilon)/\phi$ in ϕ on $\theta_T^2 + S_{\text{limit}} < \theta^2 \le \theta_T^2 + 2S_{\text{limit}}$. We employ the Wilson-Hil ferty approximation for the upper percentile of central chi-square distribution:

$$\chi_\phi^2(1-\omega) = \phi\left\{1 - \frac{2}{9\phi} + u_{1-\omega}\sqrt{\frac{2}{9\phi}}\right\}^3,$$

where $u_{1-\omega}$ for $\omega(\ge 0.5)$ denotes the upper $100(1-\omega)$ percentile of the standard normal distribution. We consider the function;

$$\frac{\chi_\phi^2(1-\omega)}{\phi} = \left\{1 - \frac{2}{9\phi} + u_{1-\omega}\sqrt{\frac{2}{9\phi}}\right\}^3.$$

Then, we have

$$\frac{d}{d\phi}\frac{\chi_\phi^2(1-\omega)}{\phi} = \frac{\sqrt{2}}{6\phi^{\frac{3}{2}}}\left\{\sqrt{\frac{8}{9\phi}} - u_{1-\omega}\right\}\left\{1 - \frac{2}{9\phi} + u_{1-\omega}\sqrt{\frac{2}{9\phi}}\right\}^2.$$

Further, it is obvious that $\sqrt{8/9\phi} - u_{1-\omega} < 0$ in the case that $u_{1-\omega} > \sqrt{8/9n}$ due to $\phi \ge n$. Then, we know that the function of $\chi_\phi^2(1-\omega)/\phi$ is the monotonous decreasing function in ϕ. On the other hand, in the case that $u_{1-\omega} \le \sqrt{8/9n}$, we know that the function $\chi_\phi^2(1-\omega)/\phi$ is concave in ϕ.

APPENDIX B

Appendix B shows the derivation of Eq. (26) on maximizing the variance $V[\hat{\theta}^2]$ in order to obtain the minimum value $\phi_{\min}$. We consider the condition of maximizing Eq.(11). By $\sigma_i^2 = \tau_i^2$, we obtain the following equation:

$$V\left[\hat{\theta}^2\right] \le \frac{3}{n}\sum_{i=1}^{m} p_i(\tau_i^2 - \theta^2)^2 + \frac{2}{n}\theta^4. \quad \text{(B.1)}$$

Then, maximizing the variance is given by the following formation:

$$\max f(\tau_1^2, \cdots, \tau_m^2) = \frac{3}{n}\left\{\sum_{i=1}^{m} p_i(\tau_i^2 - \theta^2)^2\right\} + \frac{2}{n}\theta^4$$
$$s.t.\quad \theta^2 = \sum_{i=1}^{m} p_i\tau_i^2,\quad \tau_i^2 - \sigma_T^2 \ge 0.$$

Then, we formulate the Lagrange function. For convenience, let be $x_i = \tau_i^2 - \sigma_T^2 \ge 0$. Consequently, we formulate the following equations.

$$g(x_1, \cdots, x_m) = \frac{3}{n}\left\{\sum_{i=1}^{m} p_i(x_i - \theta^2 + \sigma_T^2)^2\right\} + \frac{2}{n}\theta^4 + \lambda\left\{\theta^2 - \sigma_T^2 - \sum_{i=1}^{m} p_i\tau_i^2\right\},$$
$$s.t.\quad x_i \ge 0.$$

From Karush-Kuhn-Tucker condition, we obtain the following equations:

$$\frac{\partial g(x_1^*, \cdots, x_m^*, \lambda^*)}{\partial x_i} = \frac{6}{n}p_i(x_i^* + \sigma_T^2 - \theta^2) - \lambda p_i \le 0,$$
$$x_i^*\frac{\partial g(x_1^*, \cdots, x_m^*, \lambda^*)}{\partial x_i} = x_i^*\left\{\frac{6}{n}p_i(x_i^* + \sigma_T^2 - \theta^2) - \lambda p_i\right\} = 0,$$
$$\frac{\partial g(x_1^*, \cdots, x_m^*, \lambda^*)}{\partial \lambda} = \theta^2 - \sigma_T^2 - \sum_{j=1}^{m} p_j x_j^* = 0.$$

By solving the above equations, the combination of (μ_i, σ_i^2) for maximizing the variance $V[\hat{\theta}^2]$ is as follows:

$$\begin{cases} (\mu_1,\ \sigma_1^2) = \left(\mu_T,\ \dfrac{\theta^2 - (1-p_1)\sigma_T^2}{p_1}\right). \\ (\mu_i,\ \sigma_i^2) = (\mu_T,\ \sigma_T^2),\ i = 2, 3, \cdots, m \end{cases}$$

APPENDIX C

Appendix C shows the behavior of $\chi_\phi^2(\varepsilon)/\phi$ for ϕ in $\theta_T^2 + 2S_{\text{limit}} < \theta^2$. Based on the approximation of the upper percentile of central chi-square distribution for $\omega < 0.5$, we have also

$$\frac{\chi_\phi^2(1-\omega)}{\phi} = \left\{1 - \frac{2}{9\phi} - u_{1-\omega}\sqrt{\frac{2}{9\phi}}\right\}^3.$$

Then, the differential coefficient for ϕ is derived as

$$\frac{d}{d\phi}\frac{\chi_{\phi}^{2}(1-\omega)}{\phi}=\frac{\sqrt{2}}{6\phi^{\frac{3}{2}}}\left\{\sqrt{\frac{8}{9\phi}}+u_{1-\omega}\right\}\left\{1-\frac{2}{9\phi}-u_{1-\omega}\sqrt{\frac{2}{9\phi}}\right\}^{2}.$$

Since $\phi,\ u_{\omega}>0,$ it is obvious that

$$\frac{d}{d\phi}\frac{\chi_{\phi}^{2}(1-\omega)}{\phi}>0.$$

Therefore, we know that the function $\chi_{\phi}^{2}(1-\omega)/\phi$ is the monotonous increasing function in ϕ.