Interval Valued Solution of Multiobjective Problem with Interval Cost, Source and Destination Parameters

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Abstract

Das et al. [EJOR 117(1999) 100-112] discussed the real valued solution procedure of the multiobjective transportation problem(MOTP) where the cost coefficients of the objective functions, and the source and destination parameters have been expressed as interval values by the decision maker. In this note, we consider the interval valued solution procedure of the same problem. This problem has been transformed into a classical multiobjective transportation problem where the constraints with interval source and destination parameters have been converted into deterministic ones. Numerical examples have been provided to illustrate the solution procedure for this case.

Key words : Multiobjective programming, Transportation problem, Interval numbers. stability of estimation.

1. Introduction

Since 1979, many authors[1, 2, 3] studied the multiobjective transportation problems. Various effective algorithms were developed for solving transportation problems with the assumption that the coefficients of the objective function, and the supply and demand values are specified in a precise way. Many authors [2, 4-11] proposed fuzzy and interval programming techniques to deal with ambiguous coefficients in mathematical programming. In a recent paper, Chanas and Kuchta [6] have generalized the known concept of the solution of the linear programming problem with interval coefficients in the objective function based on preference relations between intervals. And Das et al. [17] proposed a method to solve the multiobjective transportation problem in which the coefficients of the objective functions as well as the source and destination parameters are in the form intervals.

In the present paper, we consider the interval valued solution procedure of the multiobjective transportation problem in which the coefficients of the objective functions as well as the source and destination parameters are in the form intervals. This problem has been transformed into a classical multiobjective transportation problem where the constraints with interval source and destination parameters have been converted into deterministic ones. Numerical examples have been provided to illustrate the solution procedure for this case.

Manuscript received May. 30, 2008; revised Sep. 19, 2008.
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This work was supported by Grant 01-2008-000-10226-0 from the Korea Science and Engineering Foundation.

2. Preliminaries

Throughout this paper upper case letters, e.g., A, B, etc. denote closed intervals and lower case letters, e.g., a, b, etc. denote real numbers. The set of all real numbers is denoted by R. An interval is defined by an ordered pair of brackets as

$$A = [a_L, a_R] = \{a : a_L \leq a \leq a_R, a \in R\},$$

where a_L and a_R are, respectively, the left and right limits of A.

Definition 2.1 Let $* \in (+, -, \cdot, \div)$ be a binary operation on the set of real numbers. If A and B are closed intervals, then

$$A * B = \{a * b : a \in A, b \in B\}$$

defines a binary operation on the set of closed intervals. In the case of division, it is assumed that $0 \neq B$.

The interval operations used in this paper may be explicitly elicited from Definition 2.1 as:

$$A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R],$$

$$kA = k[a_L, a_R] = \begin{cases} [ka_L, ka_R] & \text{for } k \geq 0, \\ [ka_R, ka_L] & \text{for } k < 0, \end{cases}$$

where k is a real number.

If $a_L \geq 0$ and $b_L \geq 0$, then

$$A \cdot B = [a_L, a_R][b_L, b_R] = [a_L b_L, a_R b_R].$$

The order relations which represent the decision maker's preference between interval costs are defined for minimization problems. Let the uncertain costs from two alternatives be represented by intervals A and B, respectively. It is assumed that the cost of each alternative is known only to lie in the corresponding interval.

Definition 2.2 The order relation $\leq_{LR}$ between $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is defined as

$$A \leq_{LR} B \text{ iff } a_L \leq b_L \text{ and } a_R \leq b_R,$$
$$A <_{LR} B \text{ iff } A \leq B \text{ and } a_R \neq b_R.$$

This order relation $\leq_{LR}$ represents the decision maker's preference for the alternative with lower minimum cost and maximum cost, that is, if $A \leq_{LR} B$, then A is preferred to B.

3. Multiobjective interval transportation problem

In this section, we briefly review the multiobjective interval transportation problem (MITP) in [17]. The multiobjective interval transportation problem (MITP) is the problem of minimizing K interval valued objective functions with interval source and interval destination parameters which can be stated as follows:

Problem:

minimize

$$Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}[c^k_{L_{ij}}, c^k_{R_{ij}}]x_{ij}$$
$$\text{where } k = 1, 2, \cdots, K,$$

subject to

$$\sum_{j=1}^{n} x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, 2, \cdots, m,$$
$$\sum_{i=1}^{m} x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, 2, \cdots, n,$$
$$x_{ij} \geq 0, \quad i = 1, 2, \cdots, m, j = 1, 2, \cdots, n,$$

with

$$\sum_{i=1}^{m} a_{Li} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j},$$

where $[c^k_{L_{ij}}, c^k_{R_{ij}}](k = 1, 2, \cdots, k)$ is an interval representing the uncertain cost for the transportation problem; it can represent delivery time, quantity of goods delivered, under used capacity, etc. The source parameter lies between left limit a_{L_i} and right limit a_{R_i}. Similarly, destination parameter lies between left limit b_{L_j} and right limit b_{R_j}.

Definition 3.1. $x^0 \in S$ is an optimal solution of the Problem iff there is no other solution $x \in S$ which satisfies $Z(x) <_{LR} Z(x^0)$.

Three major cases that may arise in a multiobjective interval transportation problem can be described as:

I. The coefficients c^k_{ij} are in the form of interval, whereas source and destination parameters are deterministic.

II. The source and destination parameters, i.e., a_i and b_j, are in the form of intervals but the coefficients c^k_{ij} of objective functions are deterministic.

III. All parameters, i.e., coefficients of objective functions, the source (a_i) and destination (b_i) parameters are in the form of interval.

3.1. Case I

When the coefficients c^k_{ij} objective functions are in the form of interval, i.e., $c^k_{ij} = [c^k_{L_{ij}}, c^k_{R_{ij}}]$, and the constraints are deterministic, i.e., the parameters a_i and b_j are deterministic, the multiobjective transportation problem is as follows:

minimize

$$Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}[c^k_{L_{ij}}, c^k_{R_{ij}}]x_{ij}$$
$$\text{where } k = 1, 2, \cdots, K,$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \sum_{i=1}^{m} x_{ij} = b_j,$$
$$x_{ij} \geq 0, \quad i = 1, 2, \cdots, m, j = 1, 2, \cdots, n,$$
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$$

3.2. Case II

When the coefficients c^k_{ij} objective functions are crisp but the source parameters a_i and destination parameters b_j are in the form of interval, the multiobjective transportation problem can be stated as follows:

minimize

$$Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c^k_{ij} x_{ij}$$
$$\text{where } k = 1, 2, \cdots, K,$$

subject to

$$\sum_{j=1}^{n} x_{ij} \in [a_{L_i}, a_{R_i}], \quad i=1,2,\cdots,m,$$
$$\sum_{i=1}^{m} x_{ij} \in [b_{L_j}, b_{R_j}], \quad j=1,2,\cdots,n,$$
$$x_{ij} \geq 0, \quad i=1,2,\cdots,m, j=1,2,\cdots,n,$$
$$\sum_{i=1}^{m} a_{Li} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}.$$

3.3. Case III

When the coefficients c_{ij}^k objective functions , the source parameters a_i, and destination parameters b_j are in the form of interval, the multiobjective interval transportation problem can be formulated as :

minimize

$$Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{L_{ij}}^k, c_{R_{ij}}^k]x_{ij}$$
$$\text{where } k=1,2,\cdots,K.$$

subject to

$$\sum_{j=1}^{n} x_{ij} \in [a_{L_i}, a_{R_i}], \quad i=1,2,\cdots,m,$$
$$\sum_{i=1}^{m} x_{ij} \in [b_{L_j}, b_{R_j}], \quad j=1,2,\cdots,n,$$
$$x_{ij} \geq 0, \quad i=1,2,\cdots,m, j=1,2,\cdots,n,$$
$$\text{with } \sum_{i=1}^{m} a_{Li} = \sum_{j=1}^{n} b_{L_j}, \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}.$$

4. Interval valued solution of MITP

In this section, we propose an interval valued solution procedure of the multiobjective interval transportation problem, where the co-coefficient of the objective functions and the source and destination parameters have been considered as interval. When the coefficients c_{ij}^k objective functions, the source parameters a_i, and destination parameters b_j are in the form of interval, the multiobjective interval transportation problem for interval valued solution can be formulated as :

minimize

$$Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{L_{ij}}^k, c_{R_{ij}}^k][x_{L_{ij}}, x_{R_{ij}}] \quad (1)$$
$$\text{where } k=1,2,\cdots,K,$$

subject to

$$\sum_{j=1}^{n}[x_{L_{ij}}, x_{R_{ij}}] \subset [a_{L_i}, a_{R_i}],$$
$$\sum_{i=1}^{m}[x_{L_{ij}}, x_{R_{ij}}] \subset [b_{L_j}, b_{R_j}], \quad (2)$$
$$x_{L_{ij}} \geq 0, \quad i=1,2,\cdots,m, \quad j=1,2,\cdots,n, \quad (3)$$
$$\text{with } \sum_{i=1}^{m} a_{Li} = \sum_{j=1}^{n} b_{L_j},$$
$$\text{and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}. \quad (4)$$

Without loss of generality, we assume that

$$c_{L_{ij}}^k \geq 0, \quad k=1,2,\cdots,K, \quad i=1,2,\cdots,m, \quad j=1,2,\cdots,n.$$

Then, we can rewrite Z^k as for $k=1,2,\cdots,K$,

$$Z^k = [\sum_{i=1}^{m}\sum_{j=1}^{n} c_{L_{ij}}^k x_{R_{ij}}, \sum_{i=1}^{m}\sum_{j=1}^{n} c_{R_{ij}}^k x_{L_{ij}^k}] = [z_L^k(x_L), z_R^k(x_R)].$$

The solution set of Eq.(1) defined by Definition 2.3 can be obtained as the following multiobjective interval transportation problem

$$\text{minimize } \{Z_L^k, Z_R^k\}, \quad k=1,2,\cdots,K,$$

subject to the constraints (2)-(4).

This problem is a new type of multiobjective interval transportation problem and may be restated as

$$\text{minimize } Z_L^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{R_{ij}}^k x_{L_{ij}},$$

$$\text{minimize } Z_R^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{L_{ij}}^k x_{R_{ij}},$$

subject to

$$\sum_{j=1}^{n} x_{L_{ij}} \in [a_{L_i}, a_{R_i}], \sum_{i=1}^{m} x_{L_{ij}} \in [a_{L_j}, a_{R_j}], \quad (5)$$
$$x_{L_{ij}} \geq 0, x_{R_{ij}} \geq 0, x_{L_{ij}} \leq x_{R_{ij}}$$
$$i=1,2,\cdots,m, \quad j=1,2,\cdots,n,$$
$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j}, \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}. \quad (6)$$

If we restate (5) as

$$\sum_{j=1}^{n} x_{L_{ij}} \geq a_{L_i}, \quad \sum_{j=1}^{n} x_{L_{ij}} \leq a_{R_i},$$

$$\sum_{i=1}^{n} x_{L_{ij}} \geq a_{L_j}, \quad \sum_{i=1}^{n} x_{L_{ij}} \leq a_{R_j}$$

and similarly (6) as

$$\sum_{j=1}^{n} x_{R_{ij}} \geq a_{L_i}, \quad \sum_{j=1}^{n} x_{R_{ij}} \leq a_{R_i},$$

$$\sum_{i=1}^{n} x_{R_{ij}} \geq a_{L_j}, \quad \sum_{i=1}^{n} x_{R_{ij}} \leq a_{R_j},$$

the above two problems are deterministic transportation problems.

Example 4.1.

$$\text{minimize } Z^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} [c^1_{L_{ij}}, c^1_{R_{ij}}][x_{L_{ij}}, x_{R_{ij}}],$$

$$\text{minimize } Z^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} [c^2_{L_{ij}}, c^2_{R_{ij}}][x_{L_{ij}}, x_{R_{ij}}],$$

subject to

$$\sum_{j=1}^{4} [x_{L_{1j}}, x_{R_{1j}}] = [7, 9],$$

$$\sum_{j=1}^{4} [x_{L_{2j}}, x_{R_{2j}}] = [17, 21],$$

$$\sum_{j=1}^{4} [x_{L_{3j}}, x_{R_{3j}}] = [16, 18],$$

$$\sum_{i=1}^{3} [x_{L_{i1}}, x_{R_{i1}}] = [10, 12],$$

$$\sum_{i=1}^{3} [x_{L_{i2}}, x_{R_{i2}}] = [2, 4],$$

$$\sum_{i=1}^{3} [x_{L_{i3}}, x_{R_{i3}}] = [13, 15],$$

$$\sum_{i=1}^{3} [x_{L_{i4}}, x_{R_{i4}}] = [15, 17],$$

$$x_{R_{ij}} \geq x_{L_{ij}} \geq 0, \ i = 1, 2, 3, \ j = 1, 2, 3, 4$$

where

$$c^1 = \begin{pmatrix} [1,2] & [1,3] & [5,9] & [4,8] \\ [1,2] & [7,10] & [2,6] & [3,5] \\ [7,9] & [7,11] & [3,5] & [5,7] \end{pmatrix},$$

$$c^2 = \begin{pmatrix} [3,5] & [2,6] & [2,4] & [1,5] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \end{pmatrix},$$

We write the equivalent multiobjective deterministic transportation problem as;

$$\text{minimize } Z_L^k = \sum_{i=1}^{3}\sum_{j=1}^{4} c^1_{R_{ij}} x_{L_{ij}}, \ k = 1, 2$$

$$\text{minimize } Z_R^K = \sum_{i=1}^{3}\sum_{j=1}^{4} c^1_{L_{ij}} x_{R_{ij}}, \ k = 1, 2$$

subject to

$$\sum_{j=1}^{4} x_{L_{1j}} \leq 9, \ \sum_{j=1}^{4} x_{L_{1j}} \geq 7, \ \sum_{j=1}^{4} x_{L_{2j}} \leq 21,$$

$$\sum_{j=1}^{4} x_{L_{2j}} \geq 17, \ \sum_{j=1}^{4} x_{L_{3j}} \leq 18, \ \sum_{j=1}^{4} x_{L_{3j}} \geq 16,$$

$$\sum_{i=1}^{3} x_{L_{i1}} \leq 12, \ \sum_{i=1}^{3} x_{L_{i1}} \geq 10, \ \sum_{i=1}^{3} x_{L_{i2}} \leq 4,$$

$$\sum_{i=1}^{3} x_{L_{i2}} \geq 2, \ \sum_{i=1}^{3} x_{L_{i3}} \leq 15, \ \sum_{i=1}^{3} x_{L_{i3}} \geq 13,$$

$$\sum_{i=1}^{3} x_{L_{i4}} \leq 17, \ \sum_{i=1}^{3} x_{L_{i4}} \geq 15,$$

$$x_{R_{ij}} \geq x_{L_{ij}} \geq 0, \ i = 1, 2, 3, \ j = 1, 2, 3, 4$$

where

$$c_{L^1_{ij}} = \begin{pmatrix} 1 & 1 & 5 & 4 \\ 1 & 7 & 2 & 3 \\ 7 & 7 & 3 & 5 \end{pmatrix}, \quad c_{L^2_{ij}} = \begin{pmatrix} 3 & 2 & 2 & 1 \\ 4 & 7 & 7 & 9 \\ 4 & 1 & 3 & 1 \end{pmatrix},$$

and

$$c_{R^1_{ij}} = \begin{pmatrix} 2 & 3 & 9 & 8 \\ 2 & 10 & 6 & 5 \\ 9 & 11 & 5 & 7 \end{pmatrix}, \quad c_{R^2_{ij}} = \begin{pmatrix} 5 & 6 & 4 & 5 \\ 6 & 9 & 10 & 11 \\ 8 & 3 & 6 & 2 \end{pmatrix}.$$

The optimal solution of the problem is obtained as

$$x_{11} = [1.04, 2.50], \ x_{12} = [2.00, 2.00],$$
$$x_{13} = [2.24, 2.24], \ x_{14} = [1.71, 1.90],$$
$$x_{21} = [8.96, 9.05], \ x_{22} = [0.00, 0.70],$$
$$x_{23} = [4.84, 4.84], \ x_{24} = [3.21, 3.75],$$
$$x_{31} = [0.00, 0.08], \ x_{32} = [0.00, 0.24],$$
$$x_{33} = [5.92, 6.46], \ x_{34} = [10.08, 10.08],$$
$$Z^1 = [130.19, 205.11], \ \text{and } Z^2 = [156.61, 227.82].$$

5. Conclusion

The present paper proposes an interval valued solution procedure of the multiobjective interval transportation problem, where the co-coefficient of the objective functions and the source and destination parameters have been considered as interval. Initially, the problem has been converted into a classical multiobjective transportation problem where the objectives which are the right limit and left limit of the interval objective functions' are minimized. These objective functions can be considered as the minimization of the worst case and the best case. Lastly, the solution procedure have been illustrated by one example which may occur in the multiobjective interval transportation problem.

The future study may guess is to propose a fuzzy valued solution procedure of the fuzzy multiobjective transportation problem, where the coefficient of the objective functions and the source destination parameters have been considered as fuzzy number.

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