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진화적 알고리즘을 이용한 비탄성방정식의 구성 파라미터 결정

Constitutive Parameter Identification of Inelastic Equations Using an Evolutionary Algorithm

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요 약

본 논문에서는 제안된 진화적 알고리즘을 바탕으로 한 비탄성 구성방정식의 파라미터를 결정하기 위한 방법을 제시한다. 이 방법의 장점은 오차를 갖고 있는 측정된 데이터들이나 모델 방정식들이 부정확하더라도 적절한 파라미터들이 결정되어 진다는 것이다. 실험설계는 단축하중과 일정 온도조건하의 샤보쉬 재료모델의 파라미터 결정에 적합하였다. 동시에 모델의 파라미터들은 실험데이터들과 제안한 방법에 의한 값들과 일치하였다. 다른 방법들에 의한 값들과 비교해 본 결과, 제안한 방법에 의한 응력-변형률 선도는 실제적인 재료거동에 비해 좋게 나타났다.

Abstract

This paper presents a method for identifying the parameter set of inelastic constitutive equations, which is based on an Evolutionary Algorithm. The advantage of the method is that appropriate parameters can be identified even when the measured data are subject to considerable errors and the model equations are inaccurate. The design of experiments suited for the parameter identification of a material model by Chaboche under the uniaxial loading and stationary temperature conditions was first considered. Then the parameter set of the model was identified by the proposed method from a set of experimental data. In comparison to those by other methods, the resultant stress-strain curves by the proposed method correlated better to the actual material behaviors.

Key Words : Evolutionary Algorithm, Inelastic Constitutive Equation, Inverse Analysis, Search Space

1. 서 론

A variety of theoretical models to describe a wide range of viscoplastic behaviors of metallic materials have been proposed and discussed in the referenced literature [1-3]. Viscoplastic constitutive equations derived from these theories involve many parameters, which significantly influence the behaviors of the constitutive equations. Appropriate parameters must be determined accordingly such that the accurate behaviors of materials can be expressed.

Every constitutive equations has its own method for the parameter identification. In conventional approaches, the model of interest is first approximated and its parameters are identified sequentially through the curve fitting approach [4]. However, the determination of its process is problem-dependent, and thus may not be easy if the model is complex. In addition, the process may yield significant errors due to the model approximation, particularly when the parameter space is high-dimension.

On the other hand, the advance of computer harware has increased the popularity of an approach where all the parameters are identified simultaneously and, most commonly, optimization methods are used to find the parameter set by adjusting them until they provide the best agreement between the measured data and the computed model response. As a result, a number of calculus-based optimization techniques were proposed and incorporated to solve this optimization problem [5]. These techniques, however, can fail in the actual situation, for example, when the measured data are noisy and the model equations are inaccurate, since they can cause the objective function to be complex such as nonconvex and multimodal. These techniques are thus practically useful only if some regularization technique [6] is incorporated properly.

On the other hand, Evolutionary Algorithms(EAs),

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which have come to represent Genetic Algorithms (GAs) [7], Evolution Programming [8], Evolution Strategies [9] and their recombined algorithms [10], have appeared as robust optimization techniques in the last frew decades. EAs are based on the collective learning process within a population of individuals, each of which represents a search point in the space of potential solutions to a given problem. Each of these algorithms has clearly demonstrated its capability to yield good approximate solutions even in case of complicated multimodal, discontinuous, non-differentiable, and even noisy or moving rsponse surface optimization problems. The popularity of the algorithms is primarily due to their probabilistic but efficient nature. In their comparison of the EAs, the authors previously showed that the algorithms having continuous individuals tend to converge faster for continuous search space problems, which is the subject of the paper, than the algorithms with discrete individuals.

In this paper, we therefore propose to use an EA with continuous individuals for identifying the parameter set of inelastic constitutive equations. The advantage of the proposed method is that stable parameters can be identified even in illposed situations. The EA described in this paper was proposed by the authors and their previous results show that the algorithm can be used as a robust optimization method effectively for a variety of complex continuous search space problems.

2. Inelastic Constitutive Equations

In general, constitutive relations are given in differential form for the strain ϵ, and a set of ζ internal variables $\xi \in R^{\zeta}$ and, typically, have the following form:

$$\dot{\epsilon} = \dot{\hat{\epsilon}}(\theta, \kappa, \sigma, \epsilon, \xi, \ldots), \quad (1)$$

$$\dot{\xi} = \dot{\hat{\xi}}(\theta, \kappa, \sigma, \epsilon, \xi, \ldots), \quad (2)$$

where σ and ξ are the stress and temperature respectively and $\kappa \in R^{\tau}$ presents a vector τ material parameters. The following initial conditions are given for their direct analysis:

$$\epsilon|_{t=0} = \epsilon_0, \quad (3)$$

$$\xi|_{t=0} = \xi_0. \quad (4)$$

CChaboche's viscoplastic model, for instance, is capable of describing cyclic hardening and softening behaviors with the yielding surface and appears to be capable of modeling a wide range of inelastic material behavior characteristics. Its formation under the uniaxial loading and stationary temperature conditions is given by

$$\sigma = E\epsilon^{e}, \quad (5)$$

$$\dot{\epsilon} = \dot{\epsilon}^{e} + \dot{\epsilon}^{p}, \quad (6)$$

$$\dot{\epsilon}^{p} = \left\langle \frac{|\sigma - Y| - R}{K} \right\rangle sgn(\sigma - Y), \quad (7)$$

$$\dot{Y} = H\dot{\epsilon}^{p} - DY|\dot{\epsilon}^{p}|, \quad (8)$$

$$\dot{R} = h|\dot{\epsilon}^{p}| - dR|\dot{\epsilon}^{p}|, \quad (9)$$

where state variables σ, ε^{e}, ε^{p}, Y and R are the uniaxial stress, the uniaxial strain, the uniaxial inelastic strain, the uniaxial back stress and the isotropic hardening variable respectively, and the vector $\kappa^{T}=[K, n, H, D, h, d, E]$ represents the material parameters. The notation <.> in equation (7) is zero if the value inside is negative. Initial conditions for the direct analysis of the model are given by equations (3), (10) and (11).

$$Y|_{t=0} = Y_0, \quad (10)$$

$$R|_{t=0} = R_0. \quad (11)$$

In the cyclic loading test, no external force is provided initially ($\varepsilon|_{t=0}=0$, $Y|_{t=0}=0$), and thus parameters, K, n, H, D, h, d and R0 must be determined to describe the performance of a specific material.

3. Evolutionary Algorithm for Continuous Search Space

EEAs are probabilistic optimization algorithms based on the model of natural evolution, and the algorithms has clearly demonstrated their capability to create good approximate solutions in complex optimization problems. The popularity of the algorithms is due to the following characteristics:

- less possibility to converge to a local minimum as the search starts from a number of points,
- compatibility with the parallel computer,
- robustness since only objective function in- f o r- mation is required.
- capability to find a solution in broad search space effectively through probabilistic operations.

Fig. 1 shows the fundamental structure of EAs. First, a population of individuals, each represented by a vector, is initially (generation t=0) generated at random, i.e.,

$$P^{0} = \{x_1^0, \ldots, x_i^0\} \in X^{\lambda}, \quad (12)$$

where $\lambda \in N$ represents the population size. The population then evolves towards better regions of the search space by means of randomized processes of recombination, mutation operator is not implemented in some algorithms. In the recombination operator r: $X^{\lambda} \rightarrow X^{\gamma}$, λ parental individuals breed $\gamma (\in N)$ offspring individuals by combining part of information from parental individuals. The mutation m: $X^{\gamma} \rightarrow X^{\gamma}$ forms new individuals by making large alterations with small possibility to the offspring individuals regardless of their inheritant information. With the evaluation of fitness for all the individuals, the selection operator s: $X^{\gamma} \cup X^{\gamma+\lambda} \rightarrow X^{\lambda}$, X favorably selects individuals of higher fitness to produce more often than those of lower fitness. These reproductive operations form one generation of the evolutionary process, which corresponds to one iteration in the algorithm, and the iteration is repeated until a given terminal criterion is satisfied.

```
t = 0 ;
Initialize P (t) ;
t = t + 1
do
    Recombinate P (t) ;
    Mutate P (t) ;
    Evaluate P (t) ;
    Select P (t) ;
While terminal condition is not satisfied
```

Fig. 1 Fundamental structure of evolutionary algorithms

Each EA consists of either binary or real individuals. As it is reported that the EAs whose individuals are of the continuous real vector form can search more rapidly and effectively than those with discontinuous binary individuals become of interest in the paper accordingly. In this case, the population at generation t is given by

$$P^t = \{x_1^t, \ldots, u_\lambda^t\} \in X^\lambda. \tag{13}$$

The EA proposed by the authors is presented in this paper as an example of such algorithms. The advantage of this algorithm is its simple operations, promising performance from the authors previous research [11], and compatibility with GENESIS, ver. 6 [12], which is one of the most popular GAs software. In this algorithm, recombination forms two offspring individuals from two randomly-selected parental individuals x_α^t and x_β^t according to the following scheme:

$$\begin{cases} r''(x_\alpha^t, x_\beta^t) = (1-\mu_\alpha^t) \cdot x_\alpha^t + \mu_\beta^t \cdot x_\beta^t \\ r''(x_\beta^t, x_\alpha^t) = \mu_\alpha^t \cdot x_\alpha^t + (1-\mu_\beta^t) \cdot x_\beta^t \end{cases} \tag{14}$$

where r'': $X^2 \rightarrow X$. The coefficient μ_i^t is defined by the normal distribution with mean 0 and standard deviation σ_i^t:

$$\mu_i^t = N(0, \sigma_i^{t^2}). \tag{15}$$

The standard deviation can be self-adaptive (variable with respect to t) or constant. The self-adaptive strategy has been reported to make the convergence rate required for each generation faster at the expense of the computation time and vice versa. The mutation is not incorporated in the algorithm since the recombination can allow individuals to make large alternations when the coefficient μ_i^t is large. The evaluation of the fitness is most commonly conducted with a linear scaling, which takes into account the best individual of the population:

$$\Phi(x_i^t) = \max\{\psi(x^t) | x^t \in P^t\} - \psi(x_i^t). \tag{16}$$

As for selection, the proportional selection [13] and ranking selection [14] are available in the software. In the proportional selection, the reproduction probabilities of individuals p_s : X →[0, 1] are given by their relative fitness,

$$p_s(x_i^t) = \frac{\Phi(x_i^t)}{\sum_{j=1}^{\lambda} \Phi(x_j^t)}. \tag{17}$$

4. Parameter Identification of Inelastic Constitutive Equations

4.1 Formulation for Parameter Identification of Chaboche's Model

There have been seven parameters to be determined for Chaboche's model described in section 2. Let the parameter set $x^T=[K, n, H, D, h, d, R_0]$, and represent the constitutive equations (5)~(9) with strain ε as the input variable with respect to time and stress σ as the output variable in the following form:

$$\sigma = \psi(x, \varepsilon), \tag{18}$$

where ψ: $R^7 \times R \rightarrow R$. If m pairs of stress-strain data $\{[\sigma_1^*, \varepsilon_1^*], \ldots, [\sigma_m^*, \varepsilon_m^*]\}$ are used to determine the parameter set, then the optimization problem to be formulated according to section 3 is:

$$\min \sum_{i=1}^{m} k_i (\sigma_i^* - \psi(x, \epsilon_i^*))^2, \tag{19}$$

where k_i represents a weighting factor.

4.2 Uniqueness of Solution

Before the actual parameter identification is conducted, we must confirm that the stress-strain data obtained from some experiments can uniquely determine the parameter set when the model responses and equations are not subject to errors. Fig. 2 illustrate the configuration of a typical cyclic loading test where the strain rate is constant. The design of a suitable set of experiments was investigated stepwise through the following three test cases:

Case Ⅰ: Tensile behavior ($m=m_T$)
Case Ⅱ: Ⅰ + Cyclic hysteresis behavior ($m=m_T+m_C$).
Case Ⅲ: Ⅱ with different strain rates ($m=m_R(m_T+m_C)$).

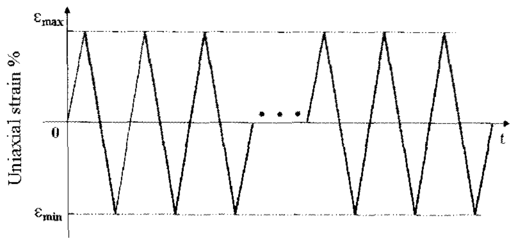

Fig. 2 Cyclic loading test

The proposed method was tested with the stress-strain data created from the parameter set x^T= [50, 3, 5000, 100, 300, 50, 0.6]. This parameter set, therefore, must be determined uniquely form the stress-strain data. Table 1 lists the number of cycles, the strain range, strain rate and the number of the stress-strain data used in the tests. The data of the tensile behavior (m_T=9) were obtained every 0.004% strain increment, while the data of the cyclic hysteresis behavior (m_C=10) were obtained at ε = ε_{max} for all cycles. In the test Case Ⅲ, cyclic hysteresis behavior with two strain rates was used (m_R=2). Internal parameters selected for the EA are listed in Table 2. The standard deviation was set to be constant for simplicity.

Table 1 Numerical example of uniqueness test

	εmax %	|ε̇| %/s	Material behavior	m	mT	mC
Case Ⅰ	0.36	8.0×10^{-3}	Tensile	9	9	0
Case Ⅱ	0.36	8.0×10^{-3}	Tensile + Cyclic loading	19	9	10
Case Ⅲ	0.36	8.0×10^{-3}	Tensile + Cyclic loading	19	9	10
	0.36	8.0×10^{-1}	Tensile + Cyclic loading	19	9	10

Table 2 Parameters for the evolutionary algorithm

Population size	50
Standard deviation	0.5 (constant)
Generation gap	5
Scaling window	1.0

The objective function values of Case Ⅰ-Ⅲ vs. generations are shown in Fig. 3 respectively. It can be first seen that the value of the objective function successfully converged close to zero for all the cases. Parameters identified in all the cases are listed in Table 3 in comparison to the exact solution.

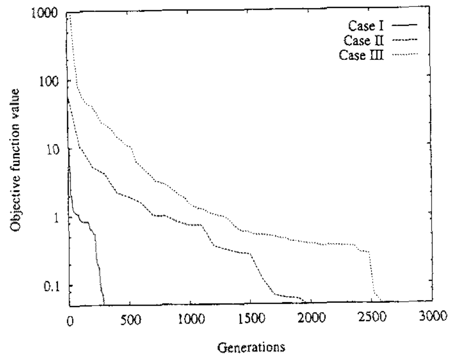

Fig. 3 Objective function values vs. generation

Table 3 Parameters identified in the uniqueness test

	K	n	H	D	h	d	R_0
Solution	50	3	5000	100	300	0.6	50
Case Ⅰ	98.3	2.46	4729	90	230	1.5	38.2
Case Ⅱ	98.8	1.83	5196	105	294	0.53	52.5
Case Ⅲ	49.2	2.97	5002	101	311	0.67	50.7

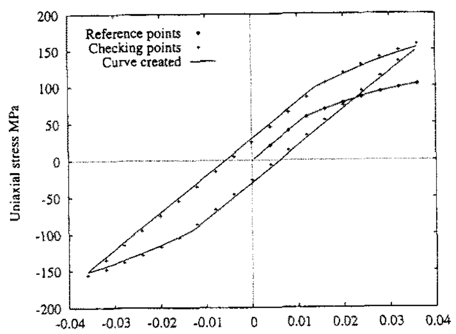

Fig. 4 Comparison between reference points and estimated curve for Case Ⅰ

Fig. 4 shows the curves of the tensile behavior and the 10th cyclic hysteresis behavior created from the parameter set identified. The points of the tensile behavior in the figure, termed reference points, were used to find the parameter set and the points of the 10th cyclic loading behavior, all derived from the exact solution, are also shown as checking data. Clearly, the checking data have some distance from the curve although the curve coincides with the reference data. Table 3 shows that only values of H and D are similar to the exact solution. The fact that the resultant objective function value is close to zero indicates that the solution is not unique. Fig. 5 shows the result fo the Case Ⅱ. The curve created is well along the reference points of both the tensile and cyclic loading behaviors. However, Table 3 indicates that parameters K and n were not similar to the exact solution Shown in Fig. 6 are the results of Case Ⅲ. Providing different strain rates, all the parameter set individual almost coincided with the exact solution, implying that the solution was uniquely determined.

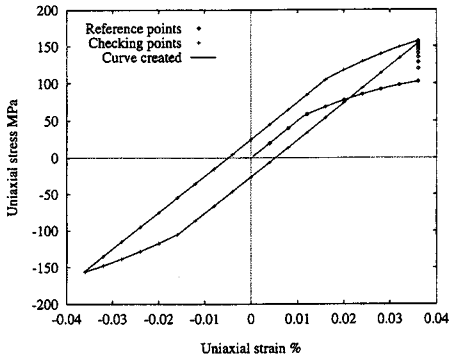

Fig. 5 Comparison between reference points and estimated curve for Case II

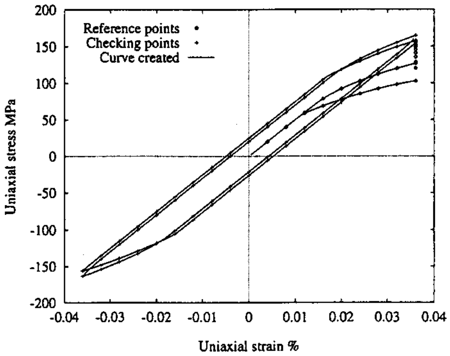

Fig. 6 Comparison between reference points and estimated curve for Case III

4.3 Identification under Model and Measurement Errors

In this section, the actual experimental data of 2 1/4Cr-1Mo Steel were used to investigate the capability of the proposed method. Parameters were identified with two other methods for comparison; one is a conventional stepwise technique and the other is a technique where a gradient-based optimization method [15] was used to minimize the objective function Eq. (20). In Hishida's technique, parameters *K, n, H* and *D* are first determined by means of the least square method after the constitutive law is simplified by letting the yield stress R be constant. Parameters *h, d* and R_0 are then determined in the second step.

Table 4 lists the resultant mean error value of each technique, which is defined as

$$\bar{e} \equiv \frac{\sum_{i=1}^{m} |[\sigma_i^* - \Psi(x, \epsilon_i^*)]/\sigma_i^*|}{m}, \tag{20}$$

together with the values of the initial parameter set. Note here that the initial parameter set for the EA is no described in the table as the initial parameter set has little influence on the performance of the EA by the fact that the EA starts with many randomly selected parameter sets. Curves with different strain rates, created from the proposed method, are shown in Fig. 7. Experimental data with strain rate 0.001%/s, which were not used for the identification, and their corresponding curve created are also shown in the figure to show the appropriateness of the parameter set identified.

Table 4 Initial parameter set and resultant mean error

	Initial parameter set							Mean error
	K	*n*	*H*	*D*	*h*	*d*	R_0	%
Proposed Method								6.7
Gradient-based technique	200	5	20,000	300	100	5	0	6.7
	50	5	20,000	300	100	5	0	∞
Stepwise Method	50	5	20,000	300	100	5	0	10.8

As shown in the table, the parameter set identified with the gradient-based technique from the initial parmeter set x^T= [200, 5, 20000, 300, 100, 5, 0] was almost identical to that with the proposed method. However, the cost functional by the gradient-based technique diverged when the initial parameter set was x^T= [50, 5, 20000, 300, 100, 5, 0]. Clearly this indicates that the successful performance of the technique largely depends on the initial parameter set to be chosen. The stepwise technique could successfully find a stable parameter set even when a different initial parameter set was selected. However, the technique left a larger mean error than did the proposed method. These results clearly indicated that the proposed technique is adequate for finding a parameter set which well describes the actual material behavior.

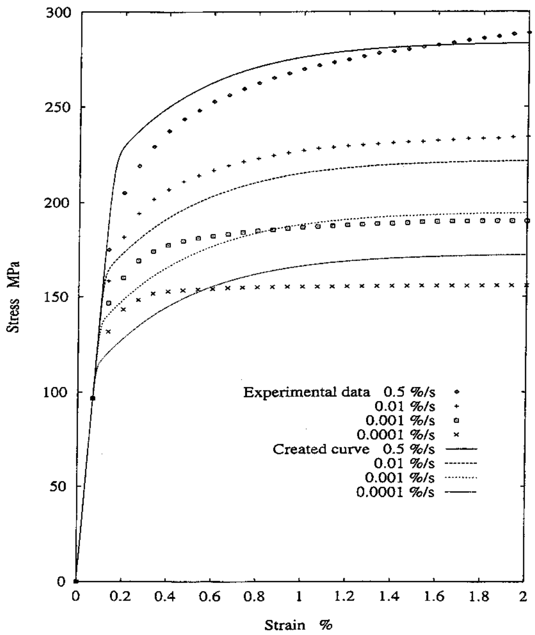

Fig. 7 Computed material curves with different strain rates

6. Conclusions

A method for identifying the parameter set of inelastic constitutive equations, which is based on an EA, has been proposed. The proposed method was tested for the parameter identification of Chaboche's model under the uniaxial loading and stationary temperature conditions, and a good approximate solution was obtained. The results of the test, compared to those by other techniques, indicate that the method is suitable for parameter identification of inelastic constitutive equations due to its robust nature.

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