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Uncertainty in Operational Modal Analysis of Hydraulic Turbine Components

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Abstract

Operational modal analysis (OMA) allows modal parameters, such as natural frequencies and damping, to be estimated solely from data collected during operation. However, a main shortcoming of these methods resides in the evaluation of the accuracy of the results. This paper will explore the uncertainty and possible variations in the estimates of modal parameters for different operating conditions. Two algorithms based on the Least Square Complex Exponential (LSCE) method will be used to estimate the modal parameters. The uncertainties will be calculated using a Monte-Carlo approach with the hypothesis of constant modal parameters at a given operating condition. In collaboration with Andritz-Hydro Ltd, data collected on two different stay vanes from an Andritz-Hydro Ltd Francis turbine will be used. This paper will present an overview of the procedure and the results obtained.

Keywords: Flow induced vibration; modal analysis; system identification; uncertainty; modal parameters.

1. Introduction

Modal properties are needed to predict the dynamic behavior of a structure. For a hydraulic turbine component, this is a major source of uncertainty because of the presence of water. The forces generated by the fluid flow modify the structure behavior. The force components in phase with acceleration and displacement will act as added mass and the forces components in phase with velocity will modify the damping. Even if analytical tools exist to predict these parameters, there is a need for validation with experimental data and experimental modal analysis may not be sufficient. The main issue with this approach is the difference between the experimental conditions and the real conditions occurring during operation. Extracting modal parameters directly from operational data is a good option when measurements are available.

Operational Modal Analysis (OMA) allows the identification of modal parameters directly from the response of a system. OMA relies upon the assumption that the system input could be assimilated to white noise. The white noise excitation assumption however is not totally true. The input will almost always contain some harmonic excitations in addition to random input. But since harmonic excitation can be regarded as virtual mode with no damping, the real mode can be differentiated from the virtual modes generated by harmonic excitation. In this paper, the Natural Excitation Technique (NExT) combined with the Least Square Complex Exponential (LSCE) algorithm will be used [1, 2]. The technique is fast and simple to use [3]. Furthermore, the LSCE algorithm can be modified to explicitly include the harmonic excitations in the identification procedure, therefore leading to a more robust approach [4].

With OMA algorithm, a change in the algorithm parameters may modify the results. Therefore, in this paper, a sensitivity analysis will be performed to validate the chosen parameters. The results uncertainty will be estimated using a Monte-Carlo approach with the hypothesis of constant modal parameters within a given operating condition. Then, using the validated set of parameter, the results and their associated uncertainty will be calculated. In addition to an overview of the procedure, this paper will also present results obtained from field test data.

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2. Field Test Data

The data gathered on two different stay vanes from an Andritz-Hydro Ltd Francis turbine has been used. Each stay vane was instrumented with four strain gauges, 2 on each side, located at each extremity as shown in Fig. 1.

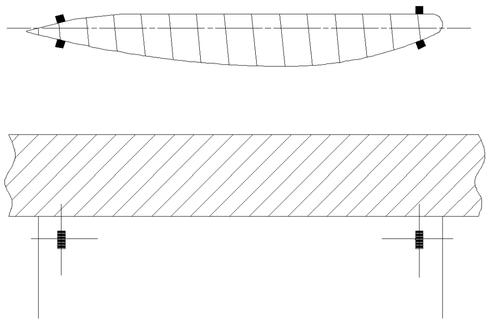

Fig. 1 Strain gauges position

The turbine has 13 blades and 20 guide vanes with a rotating speed of 100 RPM. The #2 stay vanes, close to the inlet, and #18, toward the end of the spiral case, have been instrumented. Their relative shapes are presented in Fig. 2.

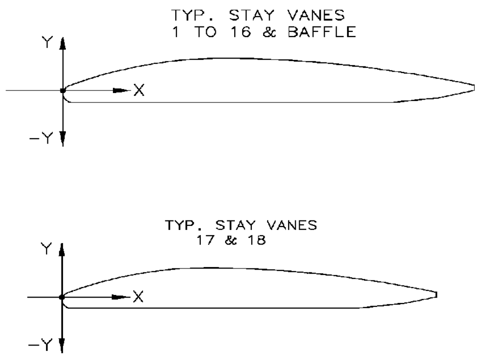

Fig. 2 Instrumented stay vanes relative shapes

Data from 6 operating conditions, ranging from 0 % to 106 % of the Best Efficiency Point (BEP), have been analyzed. The acquisition sampling rate was 2048 Hz. All the data presented here has been extracted from the same field test. The signal lengths used for each operating condition are presented in Table 1.

Table 1 Signal length

% of BEP	Length of signal [s]
0	250
24	150
39	250
57	250
84	250
106	250

Three harmonic excitations were expected in the stay vanes signal: the rotating speed, runner blade passing frequency from rotor-stator interaction, and generator electrical interference. The calculated frequencies are shown in Table 2.

Table 2 Generated excitation

Frequency	Equation	Description
1.67 Hz	RPM / (60 sec)	Rotating speed
21.67 Hz	(# Blades) x (Rotating speed)	Rotor-stator interaction
60.00 Hz		Generator

The expected natural frequencies in water obtained using analytical tools are presented in Table 3 for each stay vane.

Table 3 Expected natural frequency

Stay Vane	Mode 1	Mode 2	Mode 3
#2	87 Hz	188 Hz	281 Hz
#18	82 Hz	187 Hz	214 Hz

3. Methodology

The underlying principle of the NExT method is that the correlation function $R_{ij}(t)$ between the responses at positions *i* and *j* is similar to the response at *i* due to an impulse at *j*.

$$R_{ij}(t) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} q_i(\tau) q_j(\tau - t)\, d\tau = \sum_{r=1}^{N} \frac{\phi_{ri} A_{ri}}{m_r^d \omega_r^d} e^{-\zeta_r \omega_r^n t} \sin(\omega_r^d t + \theta_r) \tag{1}$$

$$\omega_r^d = \omega_r^n \sqrt{1-\zeta_r^{\;2}} \tag{2}$$

Where

ϕ_{ri} The i^{th} component of the eigenmode number *r*

A_{ri} Constant associated to the j^{th} response signal

m_r Modal mass

ω_r^n Eigenfrequency

ζ_r Modal damping ratio

θ_r Phase angle associated with the r^{th} modal response

The correlation between signals is the superposition of decaying oscillations having the same damping and natural frequencies as the structural mode; they can thus be identified by a time domain method like the LSCE method. The correlation function can also be written as eq. (3).

$$R_{ij}(k\Delta t) = \sum_{r=1}^{N} e^{s_r k\Delta t} C_{rj} + \sum_{r=1}^{N} e^{s_r^{*} k\Delta t} C_{rj}^{\;*} \tag{3}$$

$$s_r = \omega_r \zeta_r + i\omega_r \sqrt{1-\zeta_r^{\;2}} \tag{4}$$

Where C_{rj} is a constant associated with the r^{th} mode for the j^{th} response signal. By numbering complex modes and eigenvalues in sequence, the eq. (3) becomes eq. (5).

$$R_{ij}(k\Delta t) = \sum_{r=1}^{2N} C_{rij}' e^{s_r k\Delta t} \tag{5}$$

S_r is in complex conjugate form for which a polynomial of order 2N exists with $e^{s_r \Delta t}$ being the root as shown in eq. (6).

$$\beta_0 + \beta_1 V_r^{\;1} + \beta_2 V_r^{\;2} + \ldots + \beta_{2N-1} V_r^{\;2N-1} + V_t^{\;2N} = 0 \tag{6}$$

With $V_r = e^{s_r \Delta t}$ and $\beta_{2N} = 1$

Multiplying the impulse response for each sample k by β_k and summing every sample, the following equation is obtained:

$$\sum_{k=0}^{2N} \beta_k R_{ij}(k\Delta t) = \sum_{k=0}^{2N} \left(\beta_k \sum_{r=1}^{2N} C_{rij}' V_r^{\;k} \right) = \sum_{r=1}^{2N} \left(C_{rij}' \sum_{k=0}^{2N} \beta_k V_r^{\;k} \right) = 0 \tag{7}$$

The coefficients β_k can be found by solving eq. (8).

$$\beta_0 R_0 + \beta_1 R_1 + \ldots + \beta_{2N-1} R_{2N-1} = -R_{2N} \tag{8}$$

Rearranged as a linear system, giving

$$\left[R\right]_{ij} \left\{\beta\right\} = -\left\{R'\right\}_{ij} \tag{9}$$

Since the parameters β_k are related to modal parameters, they can be derived from any correlation or auto correlation within the system. Using all correlation functions, eq. (9) becomes

$$\begin{bmatrix} [R]_{11} \\ [R]_{12} \\ \vdots \\ [R]_{qp} \end{bmatrix} \{\beta\} = -\begin{Bmatrix} [R']_{11} \\ [R']_{12} \\ \vdots \\ [R']_{qp} \end{Bmatrix} \tag{10}$$

Where q is the response and p is the reference. The solution for this system equation can be found using pseudo inverse technique. The modal parameters are then extracted from the root of eq. (8).

A modified version of the LSCE method has been developed by Mohanty and Rixen [4]. This new method is able to deal with the presence of harmonic excitations by including them explicitly in the identification algorithm using two new terms for each harmonic excitation. These terms are eigenvalue with no damping as shown in eq. (11).

$$s_r = \pm\, i\omega_r \tag{11}$$

$$V_r = e^{\pm\, i\omega_r \Delta t} = \cos(\omega_r \Delta t) \pm i \sin(\omega_r \Delta t) \tag{12}$$

Equation (12) being the root of eq. (6) the system of eq. (13) can be formed. Adding the linear eq. (13) to eq. (10), the solution will include the exact frequency of the harmonic excitation.

$$\begin{bmatrix} 0 & \sin(\omega_r \Delta t) & \ldots & \sin(\omega_r (2N-1)\Delta t) \\ 1 & \cos(\omega_r \Delta t) & \ldots & \cos(\omega_r (2N-1)\Delta t) \end{bmatrix} \{\beta\} = -\begin{Bmatrix} \sin(\omega_r (2N)\Delta t) \\ \cos(\omega_r (2N)\Delta t \end{Bmatrix} \tag{13}$$

On stay vane #2, the regular LSCE method failed to identify the first eigenvalue as shown in the stability diagrams Fig. 3. We were not able to obtain stable results because of the presence of the 4th harmonic of the blade passing frequency (86.7 Hz) near the first eigenvalue.

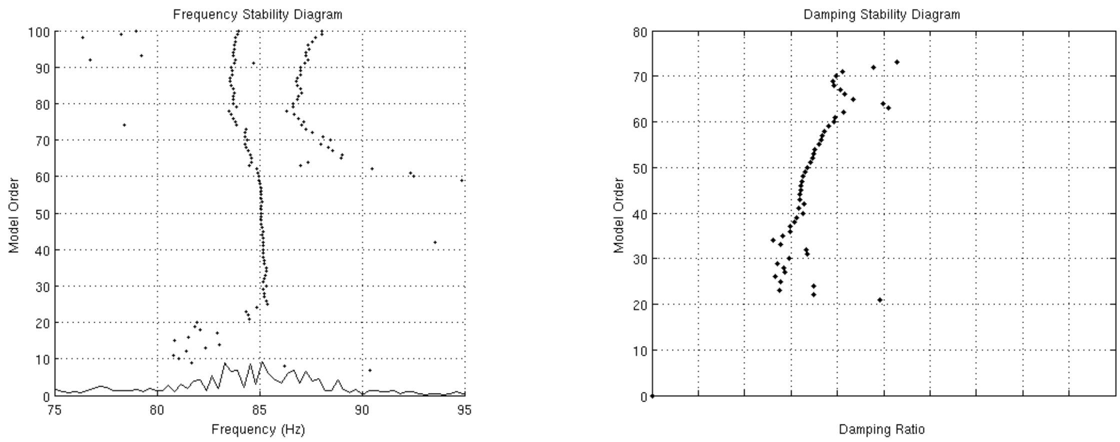

Fig. 3 Stay vane #2 – LSCE – 39% BEP

The modified LSCE method was used to deal with the 4th harmonic of the blade passing frequency on stay vane #2. The results using the modified LSCE method with the fourth harmonic of the blade passing frequency (86.7 Hz) are shown in Fig. 4.

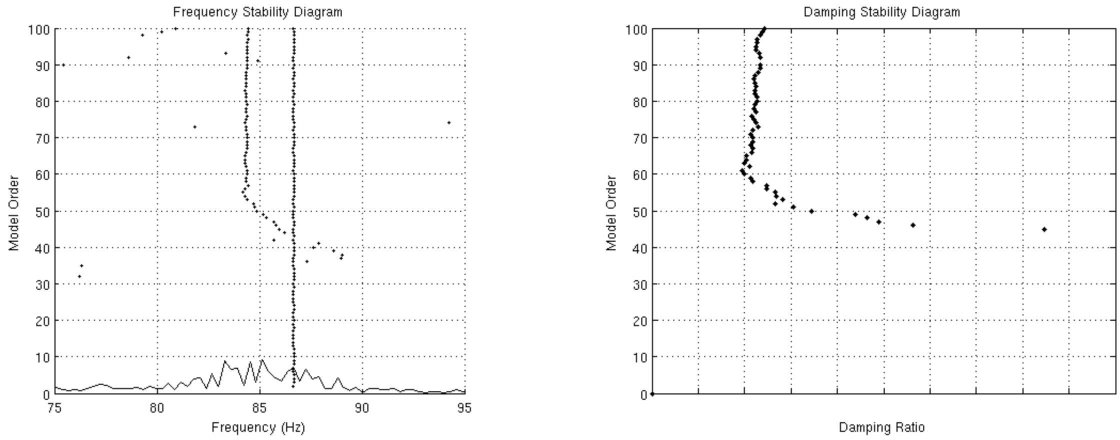

Fig. 4 Stay vane #2 – Modified LSCE – 39% BEP

The results obtained with the regular LSCE method were stable on stay vane #18. For this stay vane, the modified LSCE method did not improve the results and therefore was not used.

4. Sensitivity analysis

To gain confidence in the results obtained, a sensitivity analysis was performed for each stay vane. The modified LSCE algorithm was used on stay vane #2 and the regular LSCE algorithm was used on stay vanes #18. For stay vanes #2, the fourth harmonic of the blade passing frequency 86.7 Hz was included in the algorithm. For each stay vane, the results from a defined window length were bootstrapped using a Monte Carlos approach [5-7]. A section of the defined data length was therefore randomly selected within the available signal for each estimate. The influence of each selected parameter was analyzed individually while the others remained constant. No mixed effect was evaluated. The parameter reference values are shown in Table 4.

Table 4 Parameter values

Parameter	Value
Window length	50 000 samples
Model order	80
Over determination factor	10

The window length is the signal length used for the correlation function, the model order is the number of modes estimated and the over determination factor is defined as the rank of the matrix divided by the number of values calculated in the pseudo inverse solution. An example of the result distribution for 200 iterations using the reference values is shown in Fig. 5.

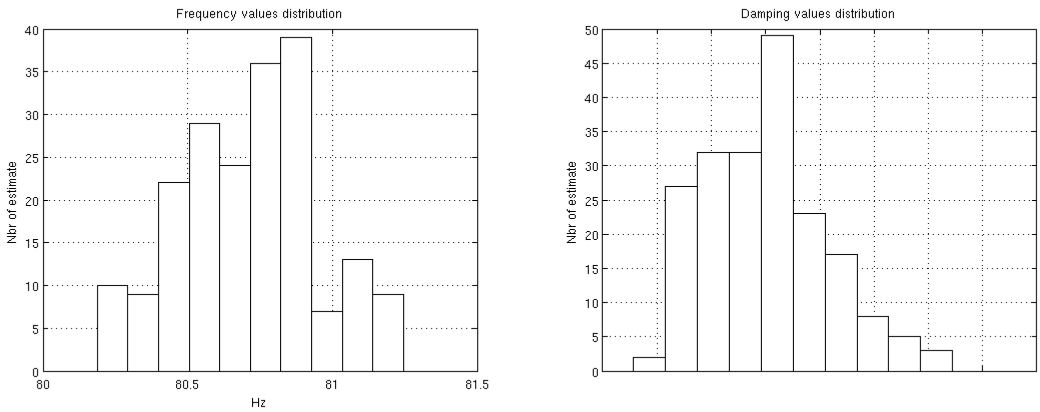

Fig. 5 Stay vane #2 – Modified LSCE – 39% BEP

Fig. 6 to Fig. 8 show the results of the sensitivity analysis. Only the results for stay vane #2 at 39% BEP are presented here. In each figure, the mean value +/- the standard deviation of 200 iterations is shown.

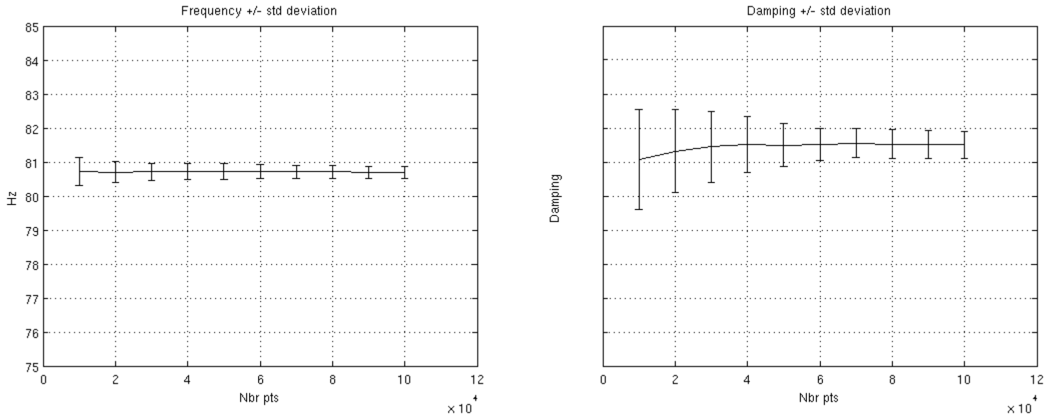

Fig. 6 Data length effect – Stay vane #2 – Modified LSCE – 39% BEP

As shown in Fig. 6, a reduction of standard deviation can be obtained with more data in each window. On the other hand the independence between these windows decreases with their length, especially at 24% BEP, where the available signal is only 150 seconds long.

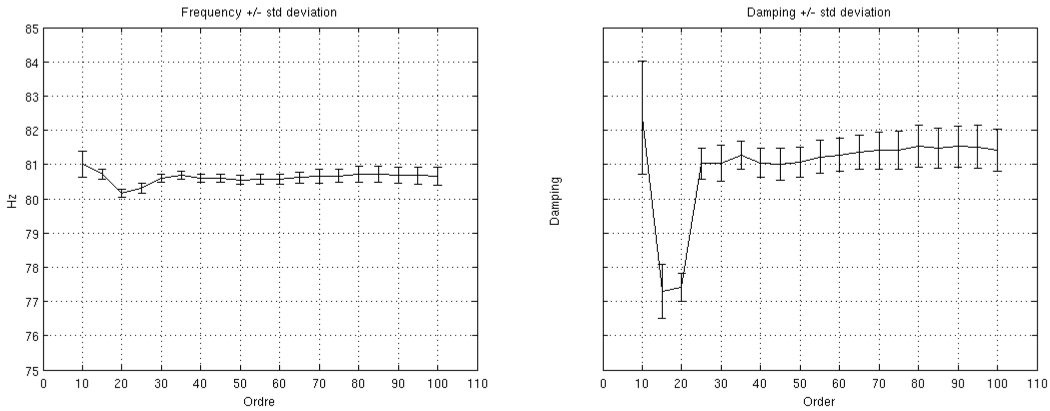

Fig. 7 Model order effect – Stay vane #2 – Modified LSCE – 39% BEP

Furthermore, it was observed that the over determination factor tends to increase the standard deviation above a certain level, and it can be observed in Fig. 8.

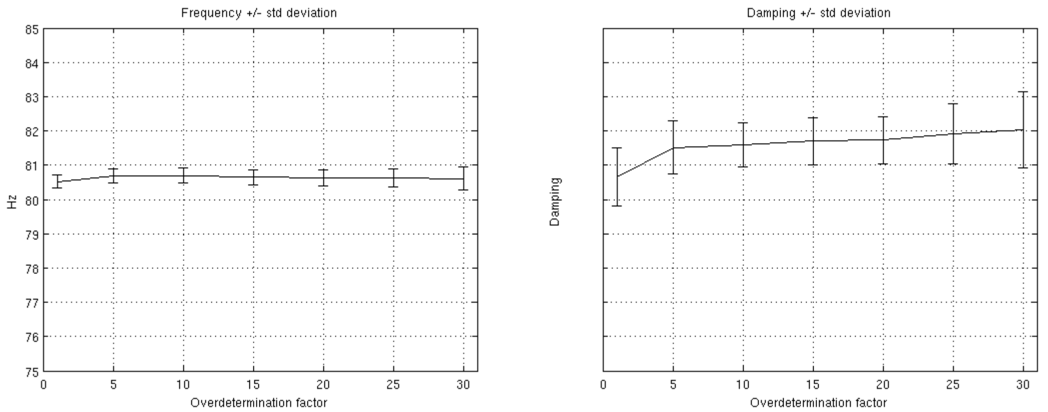

Fig. 8 Over determination effect – Stay vane #2 – Modified LSCE – 39% BEP

The sensitivity analysis results for all operating conditions on both stay vanes confirmed the reference values presented in Table 4 as an acceptable choice.

5. Results

A common set of parameters have been used for all operating conditions to avoid change in results bias from one condition to another. Thus, the comparison between results is done using the hypothesis of a constant bias. The added damping generated by flow-induced force changes with fluid flow characteristics. Therefore the a priori assumptions were that the natural frequencies will remain constant and the damping ratio will increase with load. The results obtained for the first mode on stay vane #2 are presented in Fig. 9 and on stay vane #18 presented in Fig. 10. For each load condition, the results mean value +/- the standard deviation for 200 iterations is shown. The results from stay vane #2 show a constant frequency and a significant increase in damping ratio at 106% of BEP which matches the a priori assumption. On the other hand, the results on stay vane #18 differ from stay vane #2 which was not expected. As for stay vane #2 the natural frequency is constant across the operating range but the damping ratio does not increase. Rather, the damping ratio shows an almost negligible drop at 57% of BEP then comes back to approximately the same value at 106% of BEP.

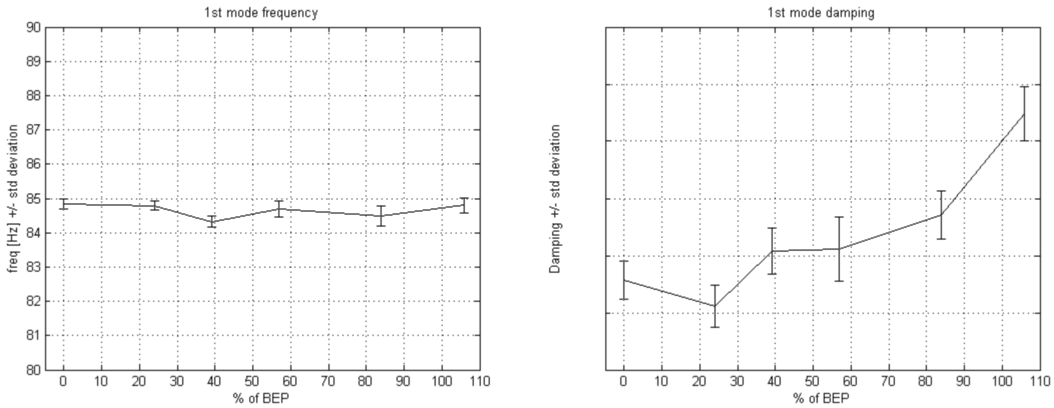

Fig. 9 Stay vane #2 – Modified LSCE

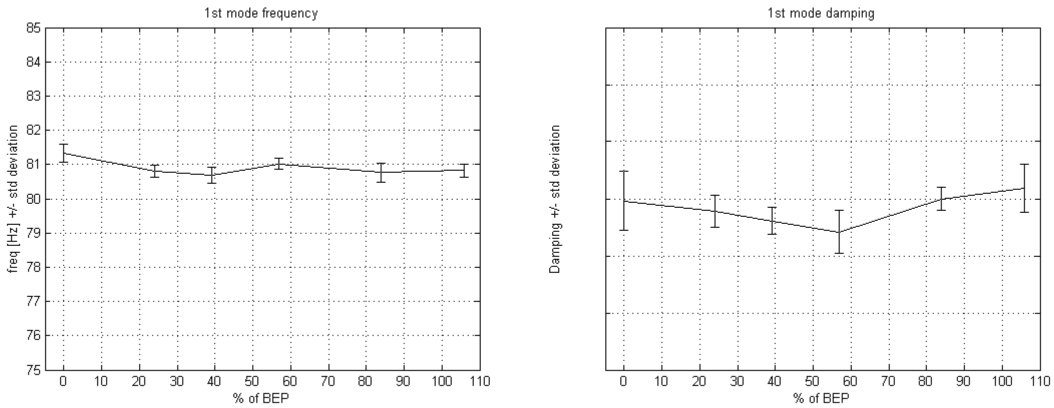

Fig. 10 Stay vane #18 – LSCE

The fluid flow characteristics are factors that can influence the added damping ratio [8, 9]. In this case, three main factors which can influence damping behavior have been identified: the position within the distributor, the profile geometry, and the presence of a harmonic excitation near the estimated frequency. Effects due to differences between the two identification algorithms (regular LSCE and modified LSCE) have been neglected because of the constant bias hypothesis.

6. Conclusion

In this paper, field test data have been used to estimate the modal parameters on two stay vanes from a Francis turbine. On stay vane #2, the modified LSCE method has been used because of the presence of a harmonic excitation near the first eigenvalue. On stay vane #18, the harmonic excitations were not close enough to influence the results, therefore allowing the use of the regular LSCE method. The variability of the results has been calculated using a Monte Carlos approach and the parameters used have been validated by a sensitivity analysis. The hypothesis of a constant bias across all data sets is being used in order to neglect the difference between the identification algorithms. Two different behaviors are observed: an increase in damping ratio with load is observed on stay vane #2 and a relatively constant damping ratio is observed on stay vane #18. Three main differences were identified as possible contributing factors: the position of the stay vane within the distributor, the stay vane profile geometry and the presence of a harmonic excitation near the estimated frequency on stay vane #2. It was not possible to validate the hypothesis or quantify the possible effect of each contributing factor with the data used here. More research needs to be done which might include field test data and controlled laboratory experiments. The use of field test data is necessary to identify the component behavior during operation but more controlled experiments will be needed to validate the assumptions and hypothesis.

Acknowledgments

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Nomenclature

A_{ri}	Constant associated to the j^{th} response signal	ϕ_{ri}	The i^{th} component of the eigenmode number r
C_{rj}	Constant associated with the r^{th} mode for the j^{th} response signal	ω_r^n	Eigenfrequency
m_r	Modal mass	ω_r^d	Damped eigenfrequency
$R_{ij}(t)$	Correlation function	ζ_r	Modal damping ratio
θ_r	Phase angle associated with the r^{th} modal response		

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