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Original Paper

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# Design Optimization of a High Specific Speed Francis Turbine Using Multi-Objective Genetic Algorithm

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## Abstract

A design optimization system for Francis turbine was developed. The system consists of design program and CFD solver. Flow passage shapes are optimized automatically by using the system with Multi-Objective Genetic Algorithm (MOGA). In this study, the system was applied to a high specific speed Francis turbine ( $n_{SP} = 250\text{m-kW}$ ). The runner profile and the draft tube shape were optimized to decrease hydraulic losses. As the results, it was shown that the turbine efficiency was improved in wide operating range, furthermore, the height of draft tube was reduced with the hydraulic performance kept.

**Keywords:** Francis Turbine, Multi-Objective Genetic Algorithm, Optimization Design, Performance.

## 1. Introduction

The hydraulic turbines with a reaction type, such as Francis turbine, have been widely used as an electrical power generate machine of natural resource energy. In order to improve the performances of such machines, many designers have been used numerical flow simulation techniques. However, design optimization of a Francis turbine runner and draft tube is a time-consuming task even for an experienced designer because of the complex shape. Therefore, it is desired to develop the automatic optimization system for these flow-passages. Recently many optimization methods are developed. Genetic Algorithm (GA) is one of the optimization methods. This method is based on the theory of evolution, where a biological population evolves over generations in order to adapt to an environment by selection, crossover and mutation. And this method is known to be effective to seek optimum solution in wide space of objective function compared with gradient method. Although the only fault of GA is that the exploration efficiency of optimum solution is not good, it has become possible to apply for an optimization design of fluid machine since the processing speed of a computer became quick in recent years. So GA begins to be applied for simple optimization design problems, such as single component and single-objective function (Reference [1-4]).

However, the interaction effect between runner and draft tube grows strong with the increase of the specific speed. Therefore, when a high specific Francis turbine runner is optimized, it is desirable to optimize, considering the trade-off relationship between runner loss and draft tube loss. In addition, as for the optimization of the draft tube hydraulic performance, it is also needed to minimize the height size from the viewpoint of the cost down. And the height size and the performance are in the relation of the trade-off.

In this paper, an optimization system using Multi-Objective Genetic Algorithm (MOGA) is presented and the results applied for a Francis turbine runner and draft tube, whose specific speed is 250 m-kW, are shown.

## 2. Optimization Methods

The schematic of the optimization process is shown in Figure 1. The process is described by following steps.

### 2.1 Initial Population

Initial population with design candidates is formed. Each of the design candidates is defined as a design parameter vector  $X = (x_1, \dots, x_n)$  and generated by random sequence method in the constraint of  $x_{\min} < x < x_{\max}$ .

### 2.2 Geometry Definition

In order to carry out GA, flow-passage shape must be defined by parameter values. So geometry definition programs were used to design runner shape and draft tube shape. As for the runner shape definition program, it is 3D shape is defined by design parameters, and runner meridian passage and blade shape can be generated variously by changing the parameters. In this study, approximately 30 design parameters were used in optimization variables such as number of blade, blade inlet diameter, blade inlet and outlet angle, and

so on. Runner inlet height and runner outlet diameter were fixed in order to confirm the optimized runner performance in model test using same stationary parts.

On the other hand, as for the draft tube definition program, its 3D shape is defined by design parameters related to meridian flow-passage geometry and several cross section shapes. In this study, approximately 50 design parameters were used in optimization variables. And also, the cross section area of draft tube outlet was fixed to constrain the pressure recovery factor in the draft tube outlet.

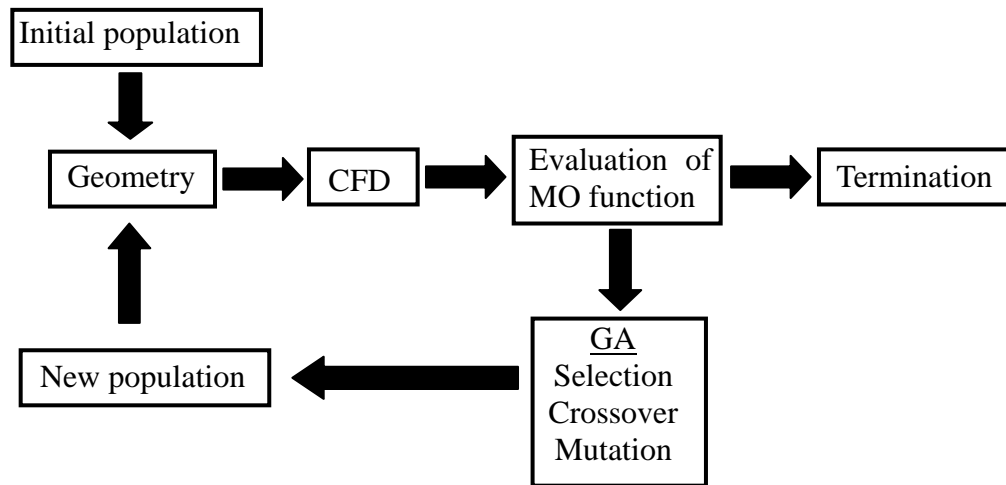


Fig. 1 Schematic of the optimization process

### 2.3 Flow Analysis

To evaluate hydraulic performance, CFD was performed in 100% flow rate. The governing equation of flow analysis is three-dimensional Reynolds-averaged Navier-Stokes equation. And the discretization of the governing equation is done by finite volume method. Convective terms are approximated by Self Filtered Central Differencing scheme. Reynolds stress is determined to use RNG  $k-\varepsilon$  turbulence model.

#### 2.3.1 Finite Volume Modeling

##### (1) Runner optimization

Computational model for flow analysis is shown in Figure 2. Computational domain was the flow passage from the runner to the draft tube to consider the interaction effect of these parts. About the runner part of the computational model, one pitch of flow passage between blades was modelled with periodic condition on the boundary surface in the circumferential direction except of rigid vanes. And also, due to apply mixing plane condition, the relative motion between the rotating part and the stationary part is simulated. The number of runner domain grid points and draft tube domain grid points are 106,000 and 137,000, respectively.

CFD around stay vanes and guide vanes was performed prior to CFD for runners, and circumferentially averaged velocity distribution at the guide vane pitch circle was adapted to inlet condition for runners. As for the outlet boundary condition, the average pressure was set to fix. Furthermore, about the surface of walls, the wall function representation of the boundary layer behavior with the high Reynolds number was introduced. It was assumed to be hydro-dynamically smooth walls and the dimensionless normal distance from the walls ( $y^+$ ) was arranged within the range from 200 to 10 by controlling the height of the grid cell adjacent to the walls.

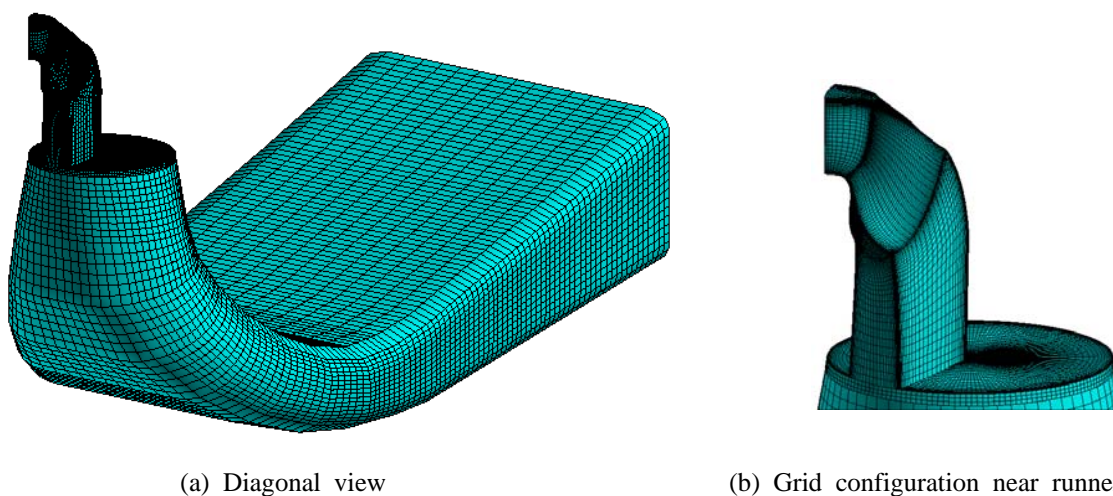


Fig. 2 Computational domain and grids for runner optimization

## (2) Draft tube optimization

Computational model for flow analysis is shown in Figure 3. Computational domain was limited within the draft tube. About the inlet boundary condition, it was assumed the circumferentially averaged velocity distribution calculated by the flow analysis of the runner flow passage. As for the outlet boundary condition and the wall boundary condition, they assumed to the conditions similar to the above-mentioned

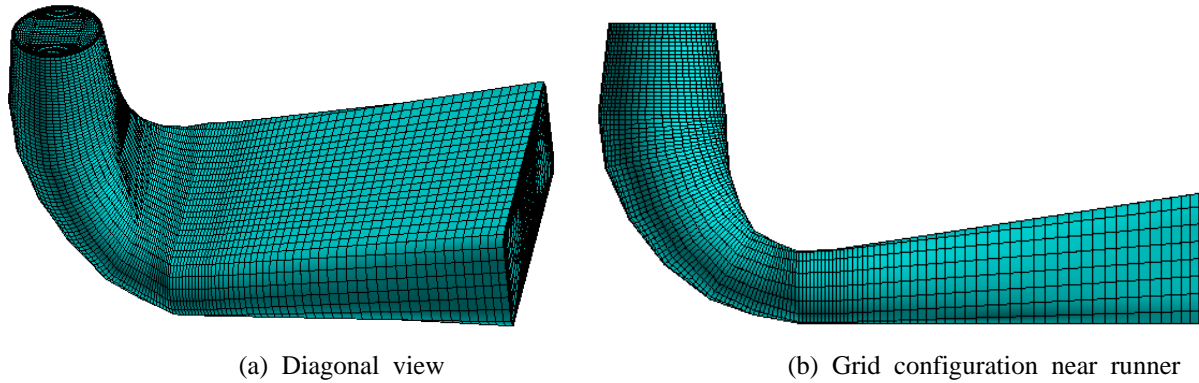


Fig. 3 Computational domain and grids for draft tube optimization

## 2.4 Evaluation of Objective Functions

### (1) Runner optimization

In order to optimize turbine performance, runner design parameters are chosen to minimize total pressure loss at runner and draft tube. The total pressure loss of runner ( $P_{tloss-r}$ ) and draft tube ( $P_{tloss-d}$ ) are defined by next equations.

$$P_{tloss-r} = \Delta P_{t-r} - H_{th} \quad (1)$$

$\Delta P_{t-r}$  : Differential total pressure between runner inlet and runner outlet

$H_{th}$  : Theoretical head between runner inlet and runner outlet

$$P_{tloss-d} = \Delta P_{t-d} \quad (2)$$

$\Delta P_{t-d}$  : Differential total pressure between draft tube inlet and draft tube outlet

$P_{tloss-r}$  and the value which is added  $P_{tloss-r}$  and  $P_{tloss-d}$  are selected as objective functions. These objective functions are evaluated in parallel without specifying weights between the objective functions.

### (2) Draft tube optimization

As for optimization design of draft tube, it is necessary to minimize total pressure loss and height size. Therefore, the values, which are expressed by Equations (3) and (4), are selected as objective functions.

$$P_{tloss-dl} = \Delta P_{t-dl} \quad (3)$$

$\Delta P_{t-dl}$  : Differential total pressure between inlet boundary and outlet boundary

$$H-d = \Delta H_{level-d} \quad (4)$$

$\Delta H_{level-d}$  : Differential distance between turbine center level and draft tube bottom level

## 2.5 New Population generating by Genetic Algorithm (GA)

GA is based on the theory of evolution, where a biological population evolves over generations in order to adapt to an environment by selection, crossover and mutation. These processes of GA are described as follows.

### (1) Selection

According to evaluation, a several percent of the best individuals (geometry) are chosen. Parents are selected in these individuals.

### (2) Crossover

In this process, design parameters are regarded as gene. The selected parents' genes are replaced each other and new child (geometry), which has parents' gene, is born. This new child remains next generation. In this stage, the number of replaced gene is important. Because if it is too much or too less than desirable one, superior children cannot be remained next generation.

### (3) Mutation

The mutation is used to obtain new children, which cannot be obtained only the crossover. The genes are changed at random and the children remain next generation. To do this operation, the solution can be search in more widely region.

The number of these processes reputation, the population of individuals and the probability of these processes at each generation are shown in Table 1. In this study, different and constant probability of cross over and mutation were used. Constant probability was effective in hydraulic turbine optimization because of our past GA study result (Reference [4]). Moreover, optimization of draft tube was the first time for us. In general, it is known that draft tube depth becomes shallower, hydraulic loss is larger. Therefore, the probability of mutation was set a little larger than runner optimization and the probability of crossover was set a little smaller in order to get innovative shallow shape of draft tube.

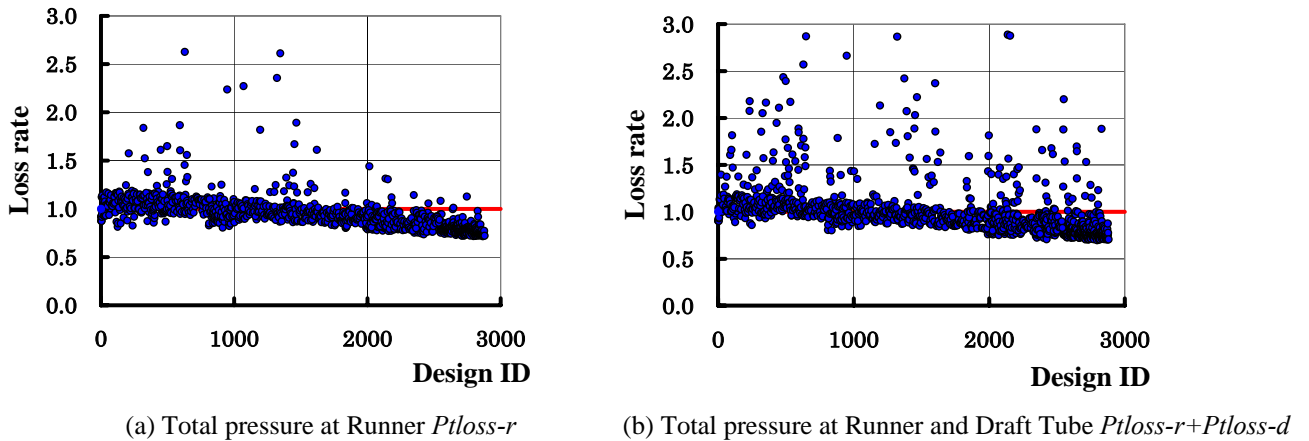
**Table 1** Specification of GA optimization

	Runner Optimization	Draft Tube Optimization
The Number of generation	50	40
Population of individual	58	100
Probability of crossover	0.70	0.65
Probability of mutation	0.05	0.10

## 3. Optimization Results and Discussion

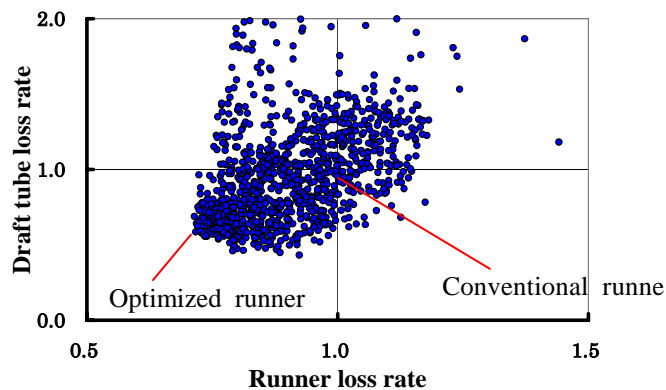
### 3.1 Runner

Figure 4 shows the optimization history. The vertical axis shows objective function, which is expressed relatively to the value of the conventional design. And horizontal axis shows design ID. Design ID means each runner shape which was designed and considered by CFD analysis. From these figure, it can be seen that the objective functions decreased and converged successfully.

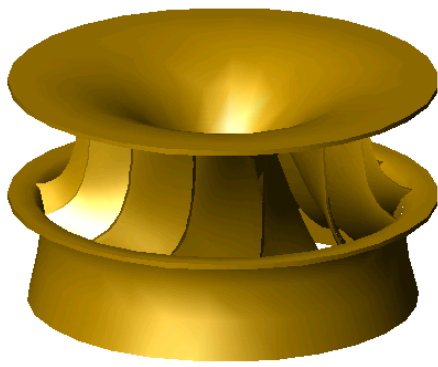


**Fig. 4** Convergence history of the runner optimization

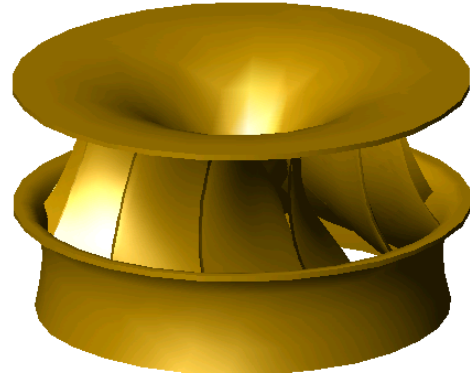
Figure 5 shows the resulting Pareto solutions on the function plane in the total pressure loss of runner and draft tube. From this figure, it can be recognized that the optimization calculation is proceeded stably by MOGA in constrain of the tradeoffs relation for runner loss and draft tube loss although exploration range is concentrated near the region whose runner loss is small. For example, in vicinity of Pareto front, in proportional as runner loss is decrease, draft tube loss is increase. Figure 6 shows typical runner shape, whose loss is plotted on Figure 5. There is obvious difference at blade shape.



**Fig. 5** Pareto solutions on the function plane in runner loss and draft tube loss



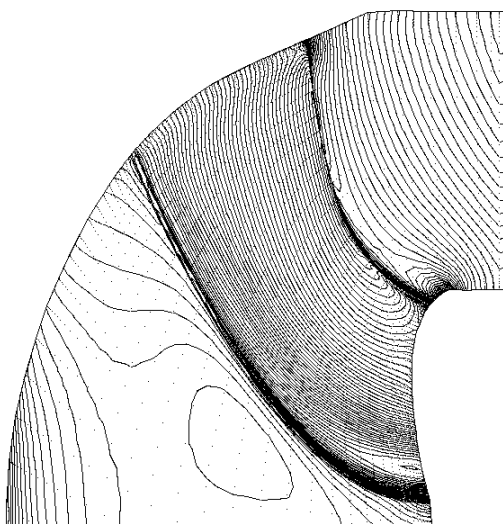
(a) Conventional runner



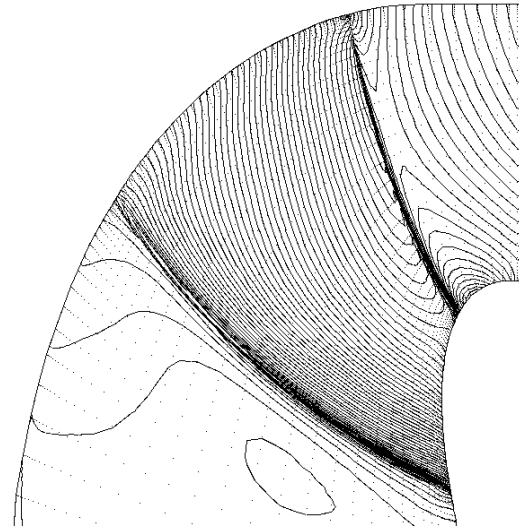
(b) Optimized runner

**Fig. 6** Comparison with runner shapes

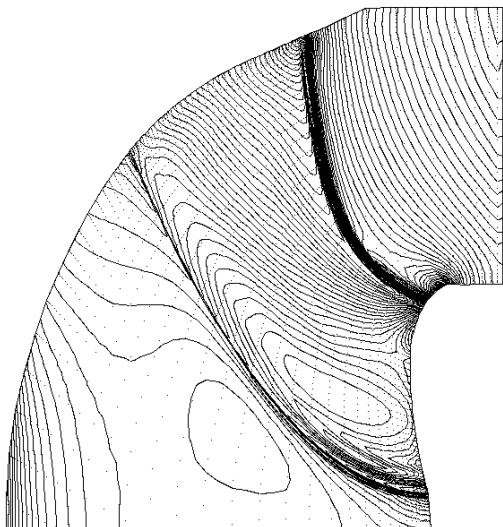
Figure 7 shows the static pressure distribution of runner blade surface on meridian plane. From these figures, it can be found that the optimized runner's pressure is changed smoothly from inlet to outlet compared with the conventional runner's one. And also, as for the conventional runner, pressure contours distortion is recognized near the inlet region on the suction surface, however, this pressure contours distortion is greatly improved in the optimized runner. These improvements of flow field are caused by the blade inlet lean shape and the meridional flow passage geometry that was explored by MOGA.



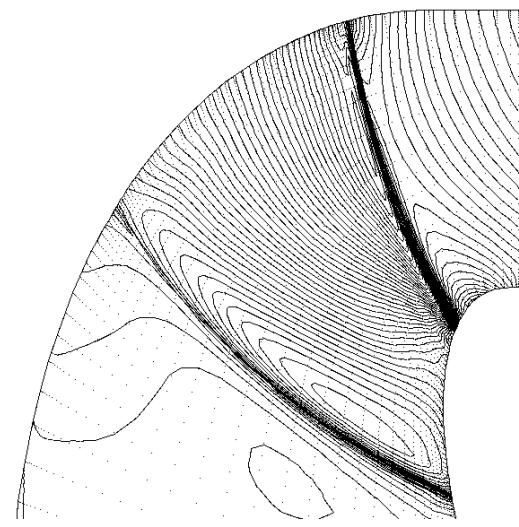
(a) Conventional runner (pressure side)



(b) Optimized runner (pressure side)



(c) Conventional runner (suction side)



(d) Optimized runner (suction side)

**Fig. 7** Static pressure contours on runner blade surface

Figure 8 shows the turbine efficiency characteristics calculated by the hybrid turbine performance prediction method (Reference [5]). In this figure, the turbine efficiency is normalized by the maximum efficiency of the conventional runner and the experiment value of the conventional runner is written together for reference. It can be found that the optimized runner has great improvement in turbine efficiency in wide operating range. Therefore, it is thought that the presented optimization system enables to obtain a superior high specific Francis turbine runner.

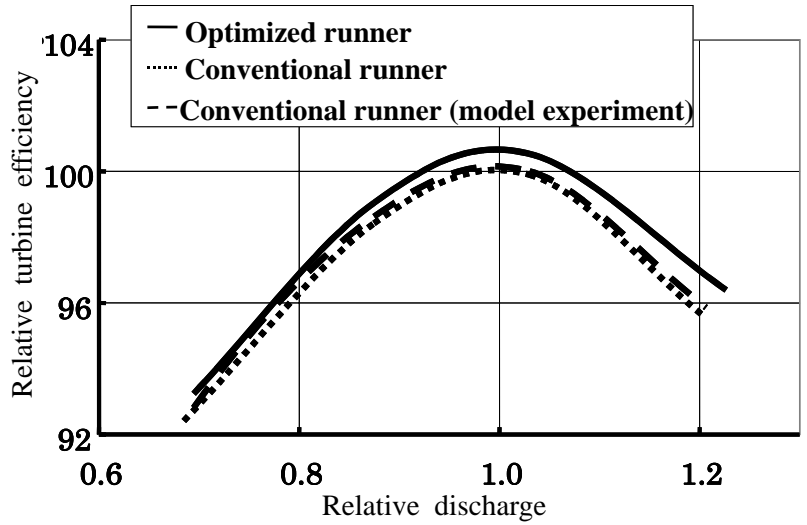


Fig. 8 Comparison with turbine efficiency characteristics

### 3.2 Draft Tube

The optimization history is shown in Figure 9. The vertical axis shows objective function, which is expressed relatively to the value of the conventional design, and horizontal axis shows design ID in the same way of the runner optimization results. As for the height size rate, it can be seen that the function value is changed discretely since it is also used as design parameter. However, the exploration range is shifted to the lower value region of the height size rate with the increase of design ID and as for the total pressure loss rate, the function value is decreased and converged successfully.

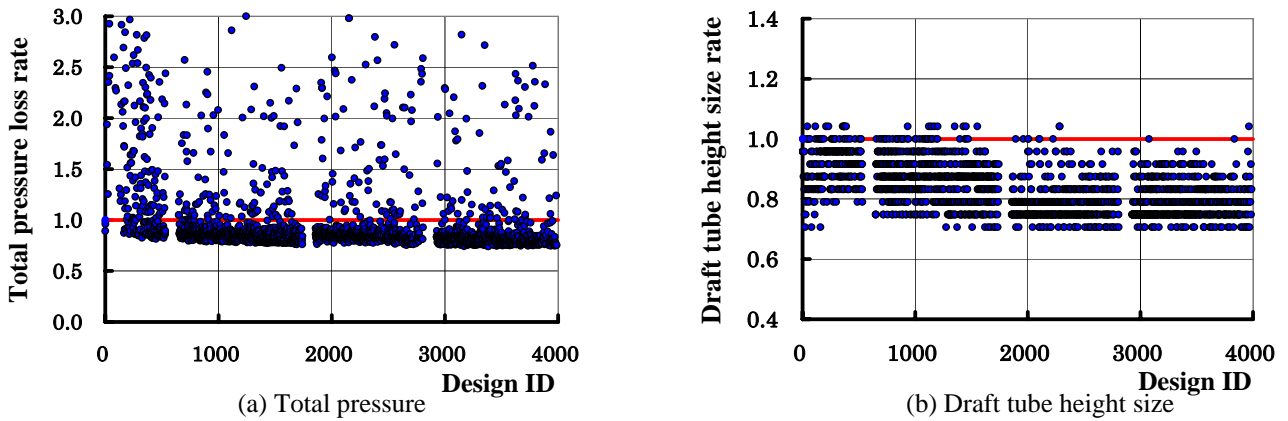


Fig. 9 Convergence history of the draft tube optimization

Figure 10 shows the resulting Pareto solutions on the function plane in the height size and the total pressure loss. From this figure, it can be recognized that the optimization calculation is proceeded stably by MOGA in constrain of the tradeoffs relation for height size and total pressure loss although these objective functions are quite different qualitatively. Figure 11 shows typical draft tube shape, whose objective function values are plotted on Figure 11. There is obvious difference at the curvature of the elbow part.

The velocity magnitude distribution on the meridian plane is shown in Figure 12. From these figures, it can be found that the optimized draft tube's velocity distortion in the elbow part is smaller than the conventional draft tube's one. As a result, the velocity deceleration near the elbow outlet region is improved in the optimized draft tube. And also, as for the flow distortion near the left-hand side of the draft tube outlet, it is improved a little in the optimized draft tube. These improvements of flow field are caused by the elbow part shape that was explored by MOGA.

Therefore, it is thought that the presented optimization system enables to obtain an optimized flow passage geometry using the objective functions which are quite different qualitatively.

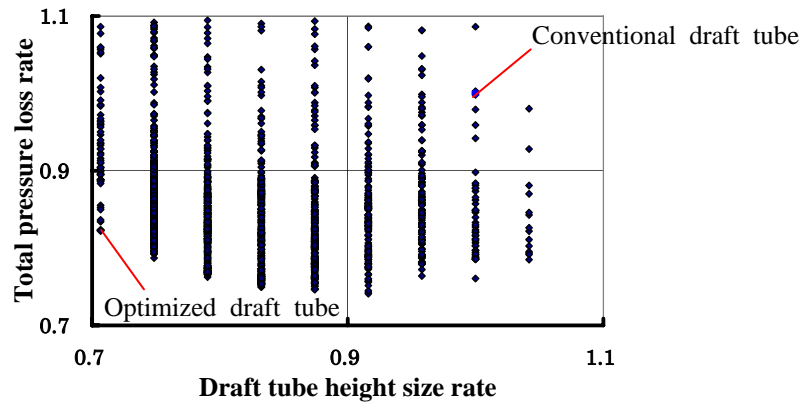


Fig. 10 Pareto solutions on the draft tube height size rate and total pressure loss

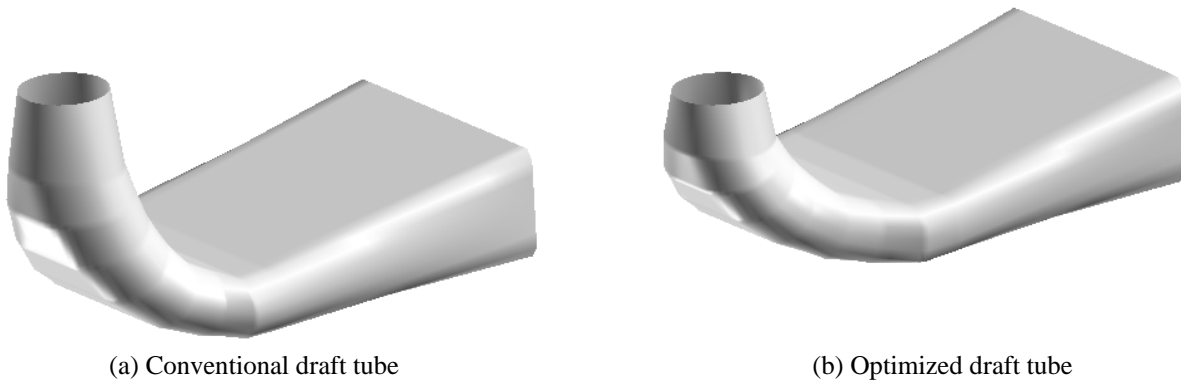


Fig. 11 Comparison with draft tube shapes

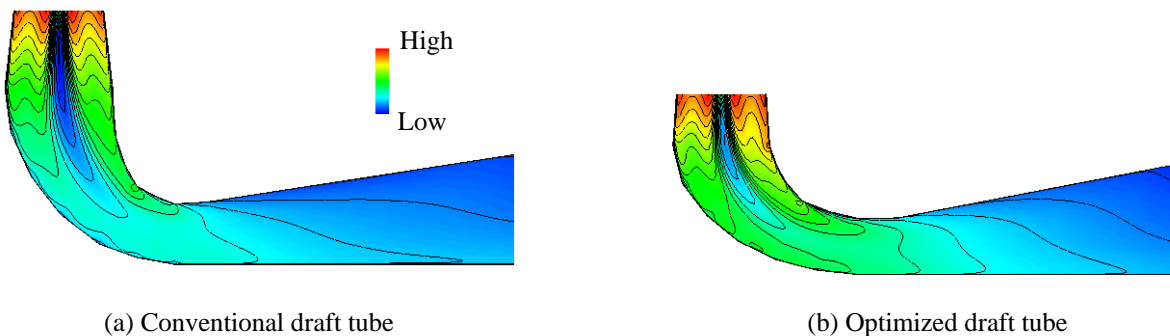


Fig. 12 Velocity distributions on meridian plane

#### 4. Conclusion

The new optimization system coupled CFD and design tool of Francis turbine shape to Multi-Objective Genetic Algorithm (MOGA) has been described and the results applied for Francis turbine runner and draft tube, whose specific speed is 250 m-kW, were shown. The obtained results are as follows:

- (1) The presented optimization system enables to optimize runner shape automatically on the constraint of the trade-off relationship between the runner loss and the draft tube loss.
- (2) The optimized runner has great improvement in the turbine efficiency in the wide operating range.
- (3) The presented optimization system enables to optimize a draft tube shape automatically on the constraint of the trade-off relationship between the height size and the total pressure loss which are quite different qualitatively.
- (4) Therefore, the presented optimization system can be used as an engineering tool of a hydraulic turbine development.

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