

# Vibration Analysis of a Rotor considering Nonlinear Reaction of Hydrodynamic Bearing

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## Abstract

In this paper it was attempted to treat the hydrodynamic journal bearing as a time-based nonlinear reaction source in each step of rotor rotation in order to observe the bearing effect more realistically and accurately in stead of the conventional method of simple linearized stiffness and damping. Lubrication analysis based on finite element method is employed to calculate the hydrodynamic reaction of bearing and Newmark's method was used to calculate the rotor dynamics in the time domain. Simulation for an industrial electrical motor showed remarkable results with differences compared to those by the conventional method in the dynamic behavior of the rotor.

**Keywords:** Rotating Machinery, Lateral Vibration Analysis, Journal Bearing, Nonlinear Hydrodynamic Reaction, Unbalance Response Analysis, Electric Motor

## 1. Introduction

High performance rotating machinery is now extremely important elements of worldwide industry. The electric power, petrochemical, marine and aircraft industries are prime examples for which rotating machinery is crucial to business success.

In the design of modern rotating machinery, there are increasing requirements for high efficiency, reliability, operability and maintainability. These considerations usually lead to the use of more flexible and more complex rotor systems. The trend towards greater flexibility results in critical speeds in or near the operational speed, which may cause severe vibration problems [1]. The increasing complexity of the system makes both the system simulation and the design much more complicated due to the large number of parameters under considerations [2]. Among these quantities, critical speeds, unbalance response, the deflection of the shafts and transmitted loads through bearings are the most important ones to be taken into account in the design process [3].

The vibration behavior of a rotor system supported by hydrodynamic bearings is significantly influenced by the dynamic characteristics (i.e. stiffness and damping coefficients) of the bearings [4]. The exact values of stiffness and damping coefficients are difficult to predict. Conventional approach to handle the journal bearing in the rotor vibration problem has been to model it as a set of linear coupled stiffness and damping coefficients equivalent at a certain fixed point of the clearance orbit [5]. But more often the shaft in the bearing moves off the assumed orbital path and hence this results in discrepancies in the reactions of the bearing and rotor shaft responses.

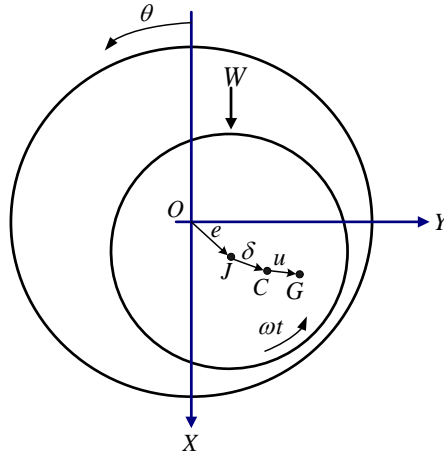
In this paper, it was attempted to simulate this phenomena using more realistic bearing model which takes into account the nonlinear bearing reaction at every step of the rotating cycle of the rotor by the modified time-integration method. As the case result of coupled analysis the behavior of 2-pole motor under the unbalance are simulated using the developed procedure and reviewed with conventional method.

## 2. Method of Analysis

The coordinate system of journal bearing used in this paper is shown in Fig. 1, where O is the bearing center, J is the journal center, C is the geometric center of the shafting system, and G is its center of gravity, respectively.

The system equations describing the rotor dynamics have the following form

$$M\ddot{x} + C\dot{x} + Kx = F(t) \quad (1)$$



**Fig. 1** Coordinate system

where,  $F(t) = F_{unbalance}(t) + F_{bearing}(t, x, y, \dot{x}, \dot{y}) + W$

Unbalance forces are given as following harmonic forms.

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix}_{unbalance} = mu\omega^2 \begin{bmatrix} \cos(\omega t + \beta) \\ \sin(\omega t + \beta) \end{bmatrix} \quad (2)$$

One of the most widely used methods to solve the nonlinear eq. (1) is the constant average acceleration Newmark method [1]. The time interval is split in steps of  $h$ , and the state at time  $t_{i+1}$  is obtained by Newmark schemes using velocities and displacements of the time  $t_i$ .

$$M\ddot{x}_{i+1} + C\dot{x}_{i+1} + Kx_{i+1} = F_{i+1} \quad (3)$$

where

$$\dot{x}_{i+1} = -\dot{x}_i + \frac{2}{h}(x_{i+1} - x_i), \quad \ddot{x}_{i+1} = -\ddot{x}_i + \frac{4}{h^2}(x_{i+1} - x_i - h\dot{x}_i) \quad (4)$$

Substituting eq. (4) in eq. (3) for time  $t_{i+1}$ , we get

$$\left[ \frac{4M}{h^2} + \frac{2C}{h} + K \right] x_{i+1} = M \left[ \ddot{x}_i + \frac{4}{h}\dot{x}_i + \frac{4}{h^2}x_i \right] + C \left[ \dot{x}_i + \frac{2}{h}x_i \right] + F_{i+1} \quad (5)$$

If  $F_{i+1}$  is defined, we can easily get  $x_{i+1}$  using eq. (5). But  $F_{i+1}$  is not determinable at time  $t_{i+1}$ , because  $F_{i+1}$  is dependent on journal velocity and displacement at time  $t_{i+1}$ . This problem can be solved with recursive method. At first iteration  $x_{i+1}$ ,  $\dot{x}_{i+1}$  are calculated with  $F_i$  instead of  $F_{i+1}$ .  $F_{i+1}$  can be calculated using journal displacement and velocity at time  $t_{i+1}$  (i.e.,  $x_{i+1}$  and  $\dot{x}_{i+1}$ ). This is more accurate value than the first iteration. We repeat this process until the difference between  $F_{i+1}^{n \text{ iter}}$  and  $F_{i+1}^{n+1 \text{ iter}}$  is small enough. This method is relatively practical and seems to give most accurate results. But this algorithm allows performing large time steps and sometimes brings about numerical instability problem. One of the approaches to solve the problem can be following method. Dynamic bearing stiffness and damping are considered linear at small time interval  $h$ .  $\vec{F}_{i+1} = \vec{F}_i + \Delta\vec{F}$  and  $\Delta\vec{F}$  can be written as following forms.

$$\begin{aligned} \Delta\vec{F} = \begin{Bmatrix} \Delta\vec{F}_x \\ \Delta\vec{F}_y \end{Bmatrix} &= \begin{bmatrix} \frac{\Delta F_x}{\Delta x} & \frac{\Delta F_x}{\Delta y} \\ \frac{\Delta F_y}{\Delta x} & \frac{\Delta F_y}{\Delta y} \end{bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} + \begin{bmatrix} \frac{\Delta F_x}{\Delta \dot{x}} & \frac{\Delta F_x}{\Delta \dot{y}} \\ \frac{\Delta F_y}{\Delta \dot{x}} & \frac{\Delta F_y}{\Delta \dot{y}} \end{bmatrix} \begin{Bmatrix} \Delta \dot{x} \\ \Delta \dot{y} \end{Bmatrix} \\ &= \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} + \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{Bmatrix} \Delta \dot{x} \\ \Delta \dot{y} \end{Bmatrix} = [K_B] \{ \vec{x}_{i+1} - \vec{x}_i \} + [C_B] \{ \vec{\dot{x}}_{i+1} - \vec{\dot{x}}_i \} \end{aligned} \quad (6)$$

From eq. (4)  $\dot{x}_{i+1} = -\dot{x}_i + \frac{2}{h}(x_{i+1} - x_i)$ , and substituting this in eq. (6), we get

$$\Delta \vec{F} = K_B \{ \bar{x}_{i+1} - \bar{x}_i \} + C_B \left\{ -2\dot{\bar{x}}_i + \frac{2}{h}(\bar{x}_{i+1} - \bar{x}_i) \right\} \quad (7)$$

Substituting this in eq.  $\vec{F}_{i+1} = \vec{F}_i + \Delta \vec{F}$ , then eq. (5) is rewritten as follows.

$$\left[ \frac{4M}{h^2} + (K - K_B) + \frac{2}{h}(C - C_B) \right] x_{i+1} = M \left[ \ddot{x}_i + \frac{4}{h}\dot{x}_i + \frac{4}{h^2}x_i \right] + C \left[ \dot{x}_i + \frac{2}{h}x_i \right] + F_i - K_B x_i - 2C_B \left[ \dot{x}_i + \frac{1}{h}x_i \right] \quad (8)$$

With this equation it is possible to simulate rotor unbalance response more realistically. Moreover this model accounts for the nonlinear bearing reaction at every step of the rotating cycle of the rotor without iterative procedure.

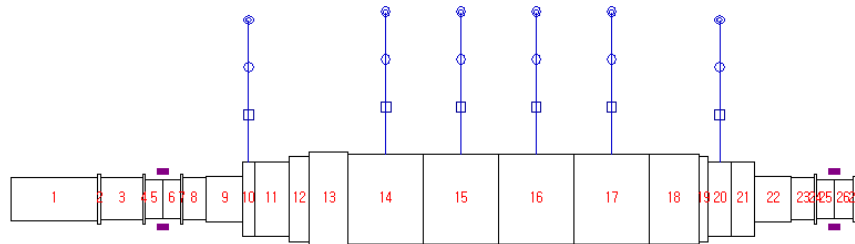
### 3. Analysis and Results

Rotor dynamic analysis program based on new algorithm was developed using MATLAB to determine the transient response of a rotor under the static loads and unbalance forces. Bearing film is modeled as the finite element based on the Reynolds equation for lubrication analysis. Dynamic characteristics of these bearings are then obtained with a perturbation method proposed by Klit and Lund [7]. The shaft is modeled as a series of mass inertia and beam elements where the gyroscopic effect of rotor disc and shear stiffness of Timoshenko beam are included.

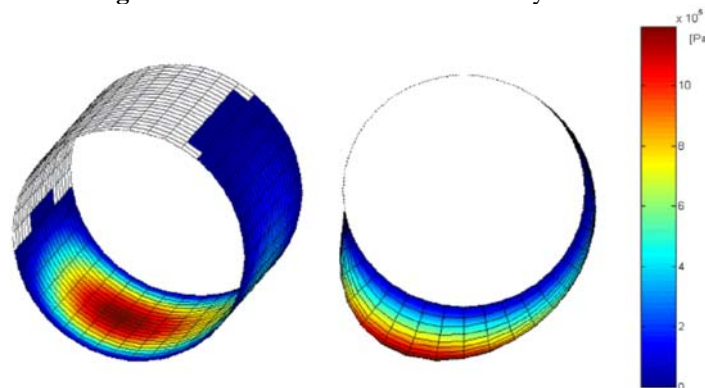
As the case result of the analysis, the behaviors of a 2-pole motor under the unbalance are simulated. The general specifications of the motor are shown in Table 1. As shown in Fig. 2, the analysis model for the motor consists of 27 beam elements, 6 concentrated mass inertia, and 2 sleeve bearings. Rotor core is modeled with equivalent concentrated mass and inertia. Fig. 3 shows oil film pressure distribution of the motor journal bearing obtained from the lubrication analysis. The film pressure distribution was integrated to be the reaction forces against the shaft motion where the relative displacement and the velocity of the shaft are both involved. In Fig. 4, the nonlinear unbalance responses and the journal locus at 3600 rpm are shown as an example.

**Table 1** General specification of the analyzed motor

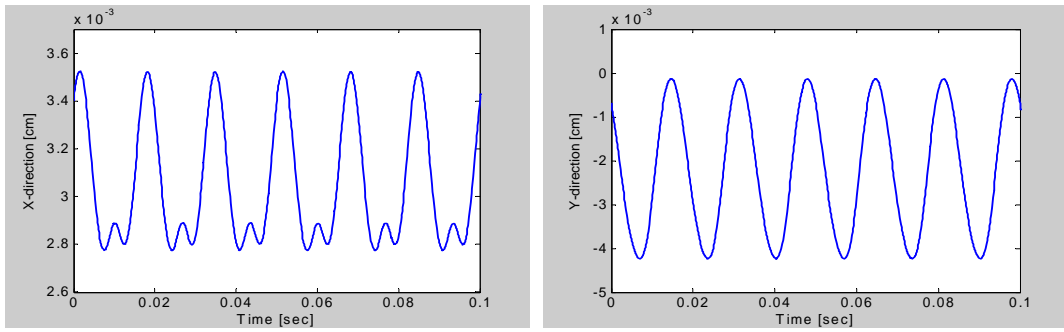
Rated power	1,250 HP
Rated voltage	3,300 V / 3 Phase
Frequency	60 Hz
Rotor	161 cm / 550 kg
Bearing	Sleeve bearing (L/D = 50/75, C/R = 1.9×10 <sup>-3</sup> )
Lubrication oil	ISO VG32



**Fig. 2** FE model for lateral vibration analysis

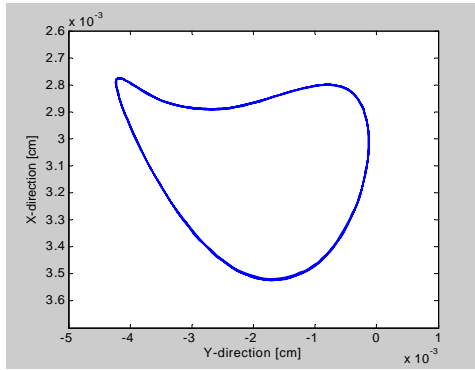


**Fig. 3** Oil film pressure distributions of hydrodynamic bearing



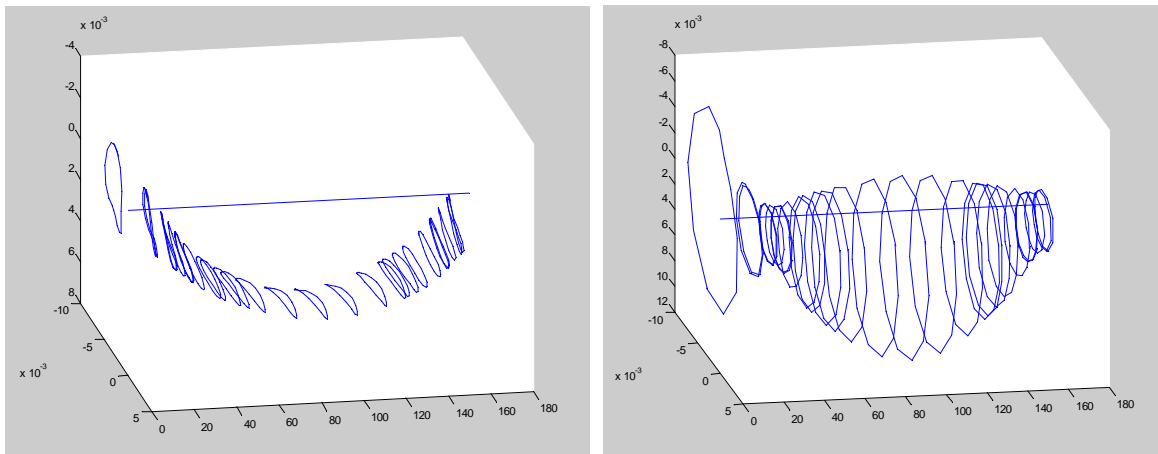
(a) X direction

(b) Y direction



(c) Locus of the journal center

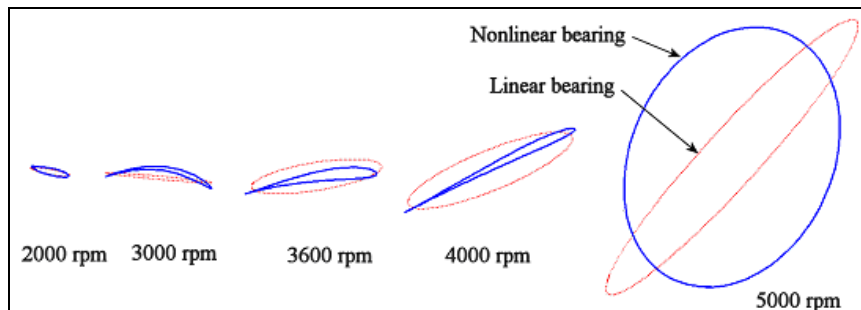
**Fig. 4** Time response of journal center at 3,600 rpm



(a) 3,600 rpm

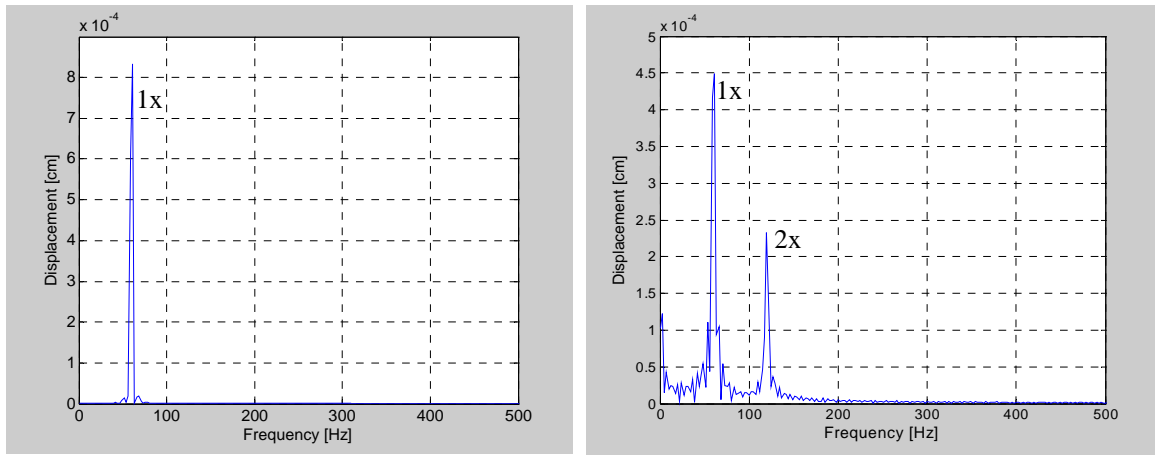
(b) 5,000 rpm

**Fig. 5** Rotor whirling orbits

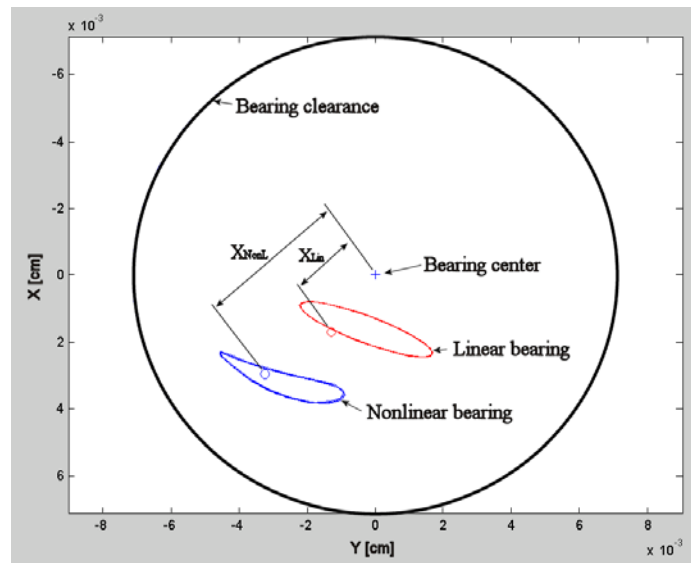


**Fig. 6** Whirling orbits at the rotor mid position w.r.t shaft rpm

Fig. 5 shows whirling orbits of the rotor system with respect to shaft revolution speeds. Both static load and unbalance force effects can be observed from the results. Whirling orbits from linear and nonlinear analyses in the middle of rotor are shown with respect to shaft revolution speeds shown in Fig. 6. It shows that orbital shapes are quite different from each other. These discrepancies are more evident as shaft speed increase.



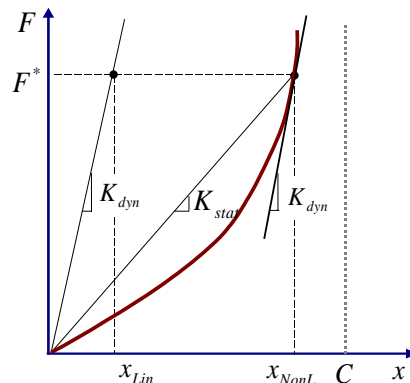
(a) Linear bearing (b) Nonlinear bearing  
**Fig. 7** Frequency spectrum of unbalance response at 3,600 rpm in X direction



**Fig. 8** Whirling orbit and static equilibrium position of the journal at 3,600 rpm

Fig. 7 shows frequency spectrum of the unbalance response at 3,600 rpm from linear and nonlinear analyses. Even though excitation force has only 1x component, the nonlinear analysis results show not only vibration of 1x component but also 2x component in the frequency spectra, which is nonlinear effect resulting from hydrodynamic bearings. Fig. 8 shows unbalance responses of the journal at 3,600 rpm obtained from two different methods. Even though their magnitudes are similar each other, the shape by nonlinear analysis is more complicated with the 2x component as previously shown.

There is another remarkable difference between the two analysis results. Small circle in Fig. 8 means static equilibrium point of the journal center obtained from two different analysis methods. There is great discrepancy between the two results. If static load  $F^*$  is acting on a bearing, journal will converges to the equilibrium position  $x_{NonL}$  as shown in Fig. 9. The positions are determined by static bearing coefficient  $K_{st}$ . At this position dynamic coefficient of the bearing is  $K_{dyn}$ . If we use linear dynamic coefficient  $K_{dyn}$  to find out equilibrium position on loading  $F^*$ , there is a possibility to misunderstand  $x_{Lin}$  as static equilibrium position of the journal center. Nonlinear bearing model approach is free from this problem.



**Fig. 9** Static equilibrium positions from the linear and nonlinear bearing models

## 4. Conclusions

Direct nonlinear bearing model based on finite element method is used to consider the hydrodynamic bearing reaction in the rotor dynamic analysis. Consequence of the analysis for an industrial motor showed quite remarkable results about the behavior of rotor. Basically the orbits from the nonlinear bearing model showed non-elliptic shapes in that only the first order component by the eccentricity was considered in the excitation but higher harmonic components including the second order showed up in the response as the typical nonlinear phenomenon is. The equilibrium position of the rotor moved far from that of the linear bearing model on account of difference in static stiffness and dynamic stiffness of the journal bearing. This sophisticated approach to the hydrodynamic bearing model is expected to contribute to the analysis of some delicate problems in the rotor vibration.

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