Communications of the Korean Statistical Society
2009, Vol. 16, No. 3, 511–518

Parametric Tests and Estimation of Mean Change in Discrete Distributions

Jaehee Kim[1,a], Sooyoung Cheon[b]

[a]Department of Statistics, Duksung Women's University
[b]KU Industry-Academy Cooperation Group Team of Economics and Statistics, Korea University

Abstract

We consider the problem of testing for change and estimating the unknown change-point in a sequence of time-ordered observations from the binomial and Poisson distributions. Including the likelihood ratio test, Gombay and Horvath (1990) tests are studied and the proposed change-point estimator is derived from their test statistic. A power study of tests and a comparison study of change-point estimators are done via simulation.

Keywords: Binomial distribution, change-point, likelihood, mean change, Ornstein-Uhlenbeck process, Poisson distribution.

1. Introduction

Change-point problems have originally arisen in the context of quality control, where one typically observes the output of a production line and would wish to signal deviation from an acceptable level. The problem of abrupt changes in general arises in quite a variety of experimental and mathematical sciences. For instance, in epidemiology one may be interested in testing whether the incidence of a disease has remained constant over time, if not, in estimating the time of changes in order to suggest possible causes. Detection of possible change-points of the number of patients or the number of death due to a certain disease is also of interest according to time.

Gombay and Horvath (1990) considered the maximum likelihood type tests for change in the mean of independent random variables and proved the limit distribution as a double exponential distribution. Chen and Gupta (2000) considered the parametric change analysis including normal, exponential, Poisson and binomial distributions.

The change-point analysis has been done for the continuous probability models than for the discrete probability models. During the past several decades, a variety approaches have been developed to identify change-points in binomial and Poisson distributions. Hinkley and Hinkley (1970) studied the inference regarding the change-point for binomial distribution using maximum likelihood ratio method. Fitzgibbon *et al.* (2002) investigated the coding of change-points in the information-theoretic Minimum Message Length(MML) framework on a tractable binomial change-point problem. Worsley (1983) discussed the power of likelihood ratio and cumulative sum tests for the change-point problem incurred for a binomial probability model. Simpkin and Downham (2006) found that the change-point Poisson process, which is suitable for analyzing non-infectious disorders with low prevalence rates, does fit to the numbers of babies born with hypospadias in the Liverpool Congenital Malformations Registry. Recently, Hawkins (2001) and Perry and Pignatiello (2008) proposed a maximum likelihood

[1] Corresponding author: Professor, Department of Statistics, Duksung Women's University, Seoul 132-714, Korea.
E-mail: jaehee@duksung.ac.kr

change-point estimator for the natural location parameter of densities belonging to the exponential family including binomial and Poisson distributions.

We consider tests for the mean change and the change-point estimators in the discrete distributions. The remaining part of this paper is organized as follows. In Section 2, the mean change model in the binomial distribution is considered. In Section 3, we describe the mean change model in Poisson distribution. In Section 4, we compare the performance of several test statistics and change-point estimators. In Section 5, we conclude the paper with a brief discussion.

2. Binomial Model

The change-point occurring in a binomial model will be considered and studied. Let $X_1, X_2, \ldots, X_n$ be independent binomial random variables with success probability $p_1, p_2, \ldots, p_n$ respectively, say, $X_i \sim B(n_i, p_i)$ and $X_i = m_i$ = the number of successes for $i = 1, 2, \ldots, n$.

The hypothesis of interest is defined as

$$H_0 : p_1 = p_2 = \cdots = p_n = p \quad vs. \quad H_1 : p_1 = \cdots = p_k \neq p_{k+1} = \cdots = p_n, \tag{2.1}$$

where k is the unknown location of the single change-point. On these hypotheses, Pettit (1980) suggested a simple cumulative sum type statistic for the change-point with zero-one observations.

Under H_0, the log likelihood function is

$$\log L_0(p) = \sum_{i=1}^{n} f(X_i; p) = \sum_{i=1}^{n} \left[\log\binom{n_i}{m_i} + m_i \log p + (n_i - m_i)\log(1-p)\right]$$

and the maximum likelihood estimator(*mle*) of p is $\hat{p} = \sum_{i=1}^{n} m_i / \sum_{i=1}^{n} n_i$. Under H_1, the log-likelihood function is

$$\begin{aligned}\log L_1(p_1, p_n) &= \sum_{i=1}^{k} \left[\log\binom{n_i}{m_i} + m_i \log p_1 + (n_i - m_i)\log(1-p_1)\right] \\ &\quad + \sum_{j=k+1}^{n} \left[\log\binom{n_j}{m_j} + m_j \log p_n + (n_j - m_j)\log(1-p_n)\right]\end{aligned}$$

and the *mle*'s of $p_1 = M_k/N_k$ and $p_n = (M_n - M_k)/(N_n - N_k)$, where $M_k = \sum_{i=1}^{k} m_i$ and $N_k = \sum_{i=1}^{k} n_i$. Consider

$$\begin{aligned}\log\Lambda &= \log\frac{L_0(\hat{p})}{L_1(\hat{p}_1, \hat{p}_n)} = \log L_0(\hat{p}) - \log L_1(\hat{p}_1, \hat{p}_n) \\ &= M_n \log M_n + (N_n - M_n)\log(N_n - M_n) - N_n \log N_n \\ &\quad - M_k \log M_k - (N_k - M_k)\log(N_k - M_k) + N_k \log N_k \\ &\quad - M'_k \log M'_k - (N'_k - M'_k)\log(N'_k - M'_k) + N'_k \log N'_k\end{aligned} \tag{2.2}$$

where $M'_k = M_n - M_k$ and $N'_k = N_n - N_k$.

Define $l(n, m) = m\log m + (n-m)\log(n-m) - n\log n$. Then $-2\log$ maximum likelihood-ratio procedure statistic L_k is

$$L_k = -2\log\Lambda = 2\left[l(N_k, M_k) + l(N'_k, M'_k) - l(N_n, M_n)\right] \tag{2.3}$$

which has chi-square distribution as its asymptotic distribution. Therefore the likelihood based test statistic is then given by

$$U_{LRT} = \max_{1 \le k < n} L_k. \tag{2.4}$$

The likelihood ratio test(LRT) rejects H_0 if $U > c$. Worsley (1983) obtained the exact null and alternative distributions of likelihood ratio test statistic for the binomial distribution. Conditional on M_{k+1}, M_k has the hypergeometiric distribution with N_{k+1}, N_k, $N_{k+1} - N_k = n_{k+1}$, that is,

$$P(M_k = u | M_{k+1} = v) = \frac{\binom{N_k}{u}\binom{n_{k+1}}{v-u}}{\binom{N_{k+1}}{v}}.$$

Let the events be expressed and defined as

$$\{L_k < x\} = A_k = \{M_k : a_k \le M_k \le b_k\},$$

where $a_k = \inf\{M_k : L_k < x\}$ and $b_k = \sup\{M_k : L_k < x\}$. To derive the null distribution of U is to evaluate $P(\bigcap_{k=1}^{n} A_k)$ conditional on $M_n = m$. Let

$$F_k(v) = P\left(\bigcap_{i=1}^{k} A_i | M_k = v\right), \quad k = 1, 2, \ldots, n$$

be the conditional probability of $\{L_k < x\}$ given $M_k = v$, for example, $F_1(v) = 1$ if $a_1 \le u \le v \le b_1$. The general iterative procedure for evaluating $P(\bigcap_{k=1}^{n} A_k)$ gives the following probability equation shown in Chen and Gupta (2000):

Under H_0, if $p_i = p$, $i = 1, 2, \ldots, k+1$ for $k \le n-1$,

$$F_{k+1}(v) = \sum_{u=a_k}^{b_k} F_k(u) h_k(u, v), \quad a_{k+1} \le v \le b_{k+1},$$

where for $0 \le u \le N_k$, $0 \le v - u \le n_{k+1}$,

$$h_k(u, v) = \frac{\binom{N_k}{u}\binom{n_{k+1}}{v-u}}{\binom{N_{k+1}}{v}}.$$

3. Poisson Model

In this section we consider the change-point problems for Poisson model. Let $X_1, X_2, \ldots, X_n$ be independent Poisson random variables with parameters $\lambda_1, \lambda_2, \ldots, \lambda_n$ respectively.

The hypothesis of interest is defined as

$$H_0 : \lambda_1 = \lambda_2 = \cdots = \lambda_n = \lambda \quad vs. \quad H_1 : \lambda_1 = \cdots = \lambda_k \ne \lambda_{k+1} = \cdots = \lambda_n, \tag{3.1}$$

where k is the unknown location of the single change-point.

Under H_0, the likelihood function is

$$L_0(\lambda) = \prod_{i=1}^{n} f(X_i; \lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{X_i}}{X_i!} = \frac{e^{-n\lambda}\lambda^{\sum_{i=1}^{n} X_i}}{\prod_{i=1}^{n} X_i!}$$

and the maximum likelihood estimator(*mle*) of λ is $\hat{\lambda} = \bar{X} = \sum_{i=1}^{n} X_i/n$. Under H_1, the likelihood function is

$$L_1(\lambda) = \frac{e^{-k\lambda_1}\lambda_1^{\sum_{i=1}^{k} X_i}}{\prod_{i=1}^{k} X_i!} \cdot \frac{e^{-(n-k)\lambda_n}\lambda_n^{\sum_{i=k+1}^{n} X_i}}{\prod_{i=k+1}^{n} X_i!}$$

and the *mle*'s of λ_1 and λ_n are respectively,

$$\hat{\lambda}_1 = \bar{X}_k = \frac{\sum_{i=1}^{k} X_i}{k}, \qquad \hat{\lambda}_n = \bar{X}_{n-k} = \frac{\sum_{i=k+1}^{n} X_i}{n-k}. \tag{3.2}$$

Denote by $M_k = \sum_{i=1}^{k} X_i$ and $M_k' = M_n - M_k = \sum_{i=k+1}^{n} X_i$. Then under H_0, $\hat{\lambda} = M_n/n$ and under H_1, $\hat{\lambda}_1 = M_k/k$, and $\hat{\lambda}_n = M_k'/(n-k)$. Hence, the maximum likelihood-ratio procedure test statistic is

$$\log\Lambda = \log\frac{L_0\left(\hat{\lambda}\right)}{L_1\left(\hat{\lambda}_1, \hat{\lambda}_n\right)} = -M_k\log\frac{M_k}{k} - M_k'\log\frac{M_k'}{n-k} + M_n\log\frac{M_n}{n}.$$

With $L_k = -2\log\Lambda$, the likelihood based test statistic is then given by

$$U_{LRT} = \max_{1\le k<n} L_k. \tag{3.3}$$

The likelihood test rejects H_0 if $U > c$. Let the events be expressed and defined as

$$\{L_k < x\} = A_k = \{M_k : a_k \le M_k \le b_k\},$$

where $a_k = \inf\{M_k : L_k < x\}$ and $b_k = \sup\{M_k : L_k < x\}$. To derive the null distribution of U we evaluate $P(\bigcap_{k=1}^{n} A_k)$ conditional on $M = m$. Let

$$F_k(\nu) = P\left(\bigcap_{i=1}^{k} A_i | M_k = \nu\right), \quad k = 1, 2, \ldots, n$$

so that $F_1(\nu) = 1$ if $a_1 \le u \le \nu \le b_1$. The general iterative procedure for evaluating $P\left(\bigcap_{k=1}^{n} A_k\right)$ gives the following probability equation shown in Chen and Gupta (2000) :

If $\lambda_i = \lambda$, $i = 1, 2, \ldots, k+1$ for $k \ge 2$.

$$F_{k+1}(\nu) = \sum_{u=a_k}^{b_k} F_k(u)h_k^*(u,\nu), \quad a_{k+1} \le \nu \le b_{k+1},$$

where for $0 \le u \le \nu \le m$,

$$h_k^*(u,\nu) = \binom{\nu}{u}\frac{k^u}{(k+1)^\nu}.$$

Based on the likelihood, the change-point can be estimated as

$$\hat{k}_{LRT} = \arg\max_{1\le k<n} L_k. \tag{3.4}$$

in Chen and Gupta (2000).

4. Numerical Comparison

A simulation study is conducted to see the power of tests according to the amount of change, and the location of change. Gombay and Horvath (1990) proposed the test statistic as a function of *mle*'s based on

$$Z_k = 2\left\{kg\left(\bar{X}_k\right) + (n-k)g\left(\bar{X}_{n-k}\right) - ng\left(\bar{X}\right)\right\}, \tag{4.1}$$

where in the binomial case, $\hat{p}_1 = \bar{X}_k$ and $\hat{p}_n = \bar{X}_{n-k}$ in (2.2) where in the Poisson case, $\hat{\lambda}_1 = \bar{X}_k$ and $\hat{\lambda}_n = \bar{X}_{n-k}$ in (3.2) and g is a given function. For the hypotheses (2.1) and (3.1), their tests reject H_0 in favor of H_1 for large values of

$$Z(i,j) = \max_{i<m<j} \frac{|Z_m|}{g^{(2)}(\mu)}, \tag{4.2}$$

where $g^{(2)}$ is the second derivative of g and suitably chosen i and j. For testing of change, the LRT based test U_{LRT}, Gombay and Horvath (1990) tests T_{GH1} with $g_1(t) = t^2$ and T_{GH2} with $g_2(t) = \exp(t)$ are compared in power study. Let

$$T_{GH1} = \max_{i<k<j}\left|k\bar{X}_k^2 + (n-k)\bar{X}_{n-k}^2 - n\bar{X}^2\right|, \tag{4.3}$$

$$T_{GH2} = \max_{i<k<j} \frac{\left|k\exp\left(\bar{X}_k\right) + (n-k)\exp\left(\bar{X}_{n-k}\right) - n\exp\left(\bar{X}\right)\right|}{\exp\left(\bar{X}\right)}. \tag{4.4}$$

For the change-point estimation, the ability to detect the change-point is studied with calculation of the mean of change-point estimates and mean square error(MSE). For the power study, $\alpha = 0.10$ and $\alpha = 0.05$ level critical values were evaluated from the empirical distribution in 10,000 repetitions.

Note that the maximum of test for change occurs at the change-point. Therefore we consider the change-point estimation based on Gombay and Horvath (1990) test as follows:

$$\hat{k}_{GH} = \arg\max_{1\le i<n} Z(i,j), \tag{4.5}$$

where $\hat{k}_{GH1}$ with $g_1(t) = t^2$ and $\hat{k}_{GH2}$ with $g_2(t) = \exp(t)$ for $Z(i,j)$ in (4.2).

Gombay and Horvath (1990) showed that the limiting distribution of their test is

$$\frac{Z(m_1, m_2)}{\sigma^2} \to \sup_{0\le s\le \Lambda} |V(s)| \tag{4.6}$$

in distribution, where $0 < \lambda_1 \le 1 - \lambda_2 < 1$ as $n \to \infty$, $m_1 = n\lambda_1$, $m_2 = n(1-\lambda_2)$, $\Lambda = 1/2\{\log(1-\lambda_1)(1-\lambda_2)/\lambda_1\lambda_2\}$ and $\{V(s), -\infty < s < \infty\}$ is an Ornstein-Uhlenbeck process, *i.e.* a Gaussian process with mean zero and covariance $\exp(-|t-s|)$. Therefore the distribution of the change-point estimator can be shown as

$$\arg\max_{m_1,m_2} \frac{Z(m_1,m_2)}{\sigma^2} \to \arg\max\left\{s : \sup_{0\le s\le\Lambda} |V(s)|\right\} \quad \text{as } n\to\infty. \tag{4.7}$$

4.1. Binomial distribution case

A random sample $X_1, X_2, \ldots, X_n$ are generated from the binomial distribution say, $X_i \sim B(n_i, p_i)$ and $x_i = m_i$ for $i = 1, 2, \ldots, n$ where m_i = the number of successes. The success probability level change

Table 1: Power comparison study of Change-point tests in Binomial distribution with the sample size n=50 and the change-point $\tau = 15, \tau = 25, \tau = 40$ in 1,000 repetitions (start point = 5, end point = 45)

		$\tau = 15$		$\tau = 25$		$\tau = 40$	
	Test	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.05$
$p_1 = 0.5$ $p_2 = 0.3$	U_{LRT}	0.979	0.955	0.996	0.985	0.930	0.874
	T_{GH1}	0.972	0.948	0.994	0.980	0.928	0.871
	T_{GH2}	0.958	0.921	0.988	0.969	0.904	0.843
$p_1 = 0.5$ $p_2 = 0.4$	U_{LRT}	0.549	0.413	0.569	0.469	0.410	0.285
	T_{GH1}	0.541	0.409	0.563	0.469	0.409	0.285
	T_{GH2}	0.506	0.376	0.538	0.440	0.37	0.268
$p_1 = 0.5$ $p_2 = 0.55$	U_{LRT}	0.224	0.139	0.252	0.167	0.197	0.118
	T_{GH1}	0.222	0.145	0.252	0.174	0.198	0.120
	T_{GH2}	0.241	0.153	0.264	0.182	0.205	0.131
$p_1 = 0.5$ $p_2 = 0.6$	U_{LRT}	0.513	0.392	0.586	0.460	0.433	0.311
	T_{GH1}	0.499	0.392	0.583	0.459	0.427	0.312
	T_{GH2}	0.535	0.434	0.600	0.491	0.444	0.342

Table 2: Comparison of Change-point Estimators with $n = 50, \tau = 15, \tau = 25$ and $\tau = 40$ in 1,000 Repetitions in the Binomial distribution

		$\tau = 15$		$\tau = 25$		$\tau = 40$	
	Estimator	Mean	MSE	Mean	MSE	Mean	MSE
$p_1 = 0.5$ $p_2 = 0.3$	$\hat{k}_{LRT}$	15.533	19.027	25.028	15.150	38.308	40.950
	$\hat{k}_{GH1}$	15.197	15.001	24.826	15.142	38.157	42.217
	$\hat{k}_{GH2}$	14.994	14.382	24.636	15.124	37.863	47.347
$p_1 = 0.5$ $p_2 = 0.4$	$\hat{k}_{LRT}$	19.100	123.014	24.912	75.140	32.464	202.558
	$\hat{k}_{GH1}$	18.893	118.261	24.814	74.110	32.343	203.885
	$\hat{k}_{GH2}$	18.667	116.331	24.545	74.457	32.005	211.027
$p_1 = 0.5$ $p_2 = 0.55$	$\hat{k}_{LRT}$	23.702	254.568	24.273	144.883	27.431	335.781
	$\hat{k}_{GH1}$	23.705	254.649	24.224	144.246	27.447	334.899
	$\hat{k}_{GH2}$	23.814	256.754	24.474	143.672	27.639	330.745
$p_1 = 0.5$ $p_2 = 0.6$	$\hat{k}_{LRT}$	18.904	124.316	24.216	86.610	32.142	204.638
	$\hat{k}_{GH1}$	18.612	117.448	24.067	84.623	32.009	206.555
	$\hat{k}_{GH2}$	19.147	126.493	24.391	85.549	32.291	200.015

model with one change-point is as follows:

$$p_i = \begin{cases} p_1, & i = 1, \ldots, k, \\ p_2 = p_1 + \Delta, & i = k+1, \ldots, n, \end{cases} \tag{4.8}$$

where $p_1 = 0.5$ without loss of generality. The number of trials is fixed as $n_i = 10$. The amount of change $\Delta = -0.2, -0.1, 0.05, 0.1$, the sample size $n = 50$ and the location of change at $\tau/n = 0.3, 0.5, 0.8$ are considered. As Δ increases, the power of the tests and the ability of change-point estimation increase. Table 1 shows that the powers of U_{LRT}, T_{GH1} and T_{GH2} have the same pattern in which their powers are best when the change-point occurs in the middle of the data sequence since the sample sizes before and after the change-point are balanced. Therefore the location of the change-point affects the power. T_{GH1} has more power when the success probability decreases after the change-point while T_{GH2} has more power when the success probability increases after the change-point. Unlike LRT, Gombay and Horvath tests depend more on the change pattern. Table 2 gives that Gombay and Horvath type change-point estimators have slightly smaller MSE than LRT based when the success probability decreases after change-point. Overall the change-point of $\tau = 15$ is overestimated and the change-point of $\tau = 40$ is underestimated while the change-point of $\tau = 25$ is

Table 3: Power comparison study of Change-point tests in Poisson distribution with the sample size n=50 and the change-point $\tau = 15$, $\tau = 25$, $\tau = 40$ in 1,000 repetitions (start point = 5, end point = 45)

		$\tau = 15$		$\tau = 25$		$\tau = 40$	
	Test	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.05$
$\lambda_1 = 7$ $\lambda_2 = 5$	U_{LRT}	0.700	0.586	0.784	0.684	0.594	0.462
	T_{GH1}	0.587	0.453	0.701	0.574	0.508	0.361
	T_{GH2}	0.091	0.042	0.121	0.050	0.075	0.038
$\lambda_1 = 7$ $\lambda_2 = 6$	U_{LRT}	0.247	0.162	0.254	0.172	0.204	0.127
	T_{GH1}	0.217	0.132	0.220	0.147	0.177	0.101
	T_{GH2}	0.065	0.030	0.064	0.023	0.071	0.031
$\lambda_1 = 7$ $\lambda_2 = 8$	U_{LRT}	0.240	0.159	0.279	0.170	0.200	0.141
	T_{GH1}	0.271	0.189	0.315	0.207	0.226	0.158
	T_{GH2}	0.389	0.235	0.427	0.267	0.329	0.205
$\lambda_1 = 7$ $\lambda_2 = 9$	U_{LRT}	0.600	0.448	0.651	0.552	0.480	0.353
	T_{GH1}	0.673	0.553	0.711	0.622	0.535	0.422
	T_{GH2}	0.897	0.797	0.877	0.768	0.673	0.536

Table 4: Comparison of Change-point Estimators with $n = 50$, $\tau = 15$, $\tau = 25$ and $\tau = 40$ in 1,000 Repetitions in the Poisson distribution

		$\tau = 15$		$\tau = 25$		$\tau = 40$	
	Estimator	Mean	MSE	Mean	MSE	Mean	MSE
$\lambda_1 = 7$ $\lambda_2 = 5$	$\hat{k}_{LRT}$	17.184	69.868	24.902	48.780	34.851	136.615
	$\hat{k}_{GH1}$	1 6.513	61.021	24.258	50.240	32.646	165.168
	$\hat{k}_{GH2}$	13.511	49.451	19.910	92.894	27.024	349.618
$\lambda_1 = 7$ $\lambda_2 = 6$	$\hat{k}_{LRT}$	22.139	205.893	25.046	130.246	28.169	311.625
	$\hat{k}_{GH1}$	21.602	196.960	24.848	130.996	27.425	330.317
	$\hat{k}_{GH2}$	18.571	161.701	21.945	150.365	24.120	427.450
$\lambda_1 = 7$ $\lambda_2 = 8$	$\hat{k}_{LRT}$	22.363	212.035	25.253	141.107	28.670	314.032
	$\hat{k}_{GH1}$	22.825	220.455	26.080	137.364	29.206	298.132
	$\hat{k}_{GH2}$	26.243	295.163	28.753	156.247	31.613	249.589
$\lambda_1 = 7$ $\lambda_2 = 9$	$\hat{k}_{LRT}$	18.792	112.210	25.425	70.321	33.113	182.973
	$\hat{k}_{GH1}$	19.341	118.479	26.174	71.068	33.916	162.394
	$\hat{k}_{GH2}$	25.435	246.637	30.523	110.793	37.277	108.845

suitably estimated. All the estimators have the less MSE when the change-point occurs in the middle than when the change-point occurs in the former part or latter part of data.

4.2. Poisson distribution case

A random sample $X_1, X_2, \ldots, X_n$ are generated from the Poisson distribution with the parameter $\lambda_1, \lambda_2, \ldots, \lambda_n$. The mean level change model with one change-point is as follows:

$$\lambda_i = \begin{cases} \lambda_1, & i = 1, \ldots, k, \\ \lambda_2 = \lambda_1 + \Delta, & i = k+1, \ldots, n, \end{cases} \tag{4.9}$$

where $\lambda_1 = 7$ without loss of generality. The amount of change $\Delta = -2, -1, 1, 2$, the sample size $n = 50$ and the locations of change at $\tau/n = 0.3, 0.5, 0.8$ are considered.

Table 3 gives the powers of tests for the mean change in Poisson distributions. U_{LRT} has more power than T_{GH1}, T_{GH2} when the mean is decreasing after the change-point and U_{LRT} has less power than T_{GH1}, T_{GH2} in the increasing mean case. Unlike U_{LRT}, Gombay and Horvath tests, T_{GH1} and T_{GH2} heavily depend on the change pattern. Table 4 shows that the change-point estimator k_{LRT} based on LRT has smaller MSE than k_{GH1}, k_{GH2} when the change-point occurs in the middle of data sequence, which is the case of $\tau = 25$ from $n = 50$ samples. When the change-point occurs in the former part or the latter part of data, $\hat{k}_{GH1}$ and $\hat{k}_{GH2}$ have smaller MSE than $\hat{k}_{LRT}$. Overall the change-point of $\tau = 15$ is overestimated and the change-point of $\tau = 40$ is underestimated while the change-point of $\tau = 25$ is suitably estimated. Therefore the change-point estimation depends on the location of change-point.

5. Concluding Remarks

We considered testing for change and estimating of the change-point when the observations are from the discrete distributions such as the binomial and Poisson distributions. The numerical results lend support to the argument that the likelihood ratio test and change-point estimation are not always best even under the parametric distributional assumptions. But the function of the maximum likelihood estimator could play a role in change-point estimation. Also the numerical results show that testing and estimating depend on the location of change-point. Therefore one possible conclusion is that one should choose a test statistic and an estimator on a subjective basis depending on where one expects a change to take place.

References

Chen, J. and Gupta, A. K. (2000). *Parametric Statistical Change Point Analysis*, Birkhauser, Berlin.

Fitzgibbon, L. J., Dowe, D. L. and Allison, L. (2002). Change-point estimation using new minimum message length approximations, *Lecture Notes In Computer Science*, **2417**, 244–254.

Gombay, E. and Horvath, L. (1990). Asymptotic distributions of maximum likelihood tests for change in the mean, *Biometrika*, **77**, 411–414.

Hawkins, D. M. (2001). Fitting multiple change-point models to data, *Computational Statistics and Data Analysis*, **37**, 323–341.

Hinkley, D. V. and Hinkley, E. A. (1970). Inference about the change-point in a sequence of binomial random variables, *Biometrika*, **57**, 477–488.

Perry, M. B. and Pignatiello, J. R. (2008). A change point model for the location parameter of exponential family densities, *IEE Transactions*, **40**, 947–956.

Pettit, A. N. (1980). A Simple cumulative sum type statistics for the change-point problem with zero-one observations, *Biometrika*, **67**, 79–84.

Simpkin, J. M. and Downham, D. Y. (2006). Testing for a change-point in registry data with an example on hypospadias, *Statistics in Medicine*, **7**, 387–393.

Worsley, K. J. (1983). The power of likelihood ratio and cumulative sum tests for a change in a binomial probability, *Biometrika*, **70**, 455–464.

Received March 2009; Accepted March 2009