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AN EXPLICIT ITERATION METHOD FOR COMMON FIXED POINTS OF A FINITE FAMILY OF STRICTLY PSEUDOCONTRACTIVE MAPPINGS IN BANACH SPACES

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ABSTRACT. The aim of this paper is to present an explicit iteration method for finding a common fixed point of a finite family of strictly pseudocontractive mappings defined on q-uniformly smooth and uniformly convex Banach spaces.

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1. Introduction

Let $q > 1$, X be a q-uniformly smooth Banach space which is also uniformly convex and its dual space X^* be strictly convex. For the sake of simplicity, the norms of X and X^* are denoted by the symbol $\|.\|$. We write $\langle x, x^* \rangle$ instead of $x^*(x)$ for $x^* \in X^*$ and $x \in X$. Let $\{T_i\}_{i=1}^N$ be a family of strictly pseudocontractive mappings in X with the domain of definition $D(T_i) = X$.

Consider the following problem: find an element

$$x_* \in S := \cap_{i=1}^N F(T_i), \tag{1.1}$$

where $F(T_i)$ denotes the set of fixed points of the mapping T_i in X. In this paper we assume that $S \neq \emptyset$.

Recall that a mapping T_i in X is called strictly pseudocontractive in the terminology of Browder and Petryshyn [2] if for all $x, y \in D(T_i)$, there exists $\lambda_i > 0$ such that

$$\langle T_i(x) - T_i(y), j(x-y) \rangle \leq \|x-y\|^2 - \lambda_i \|x - y - (T(x) - T(y))\|^2, \tag{1.2}$$

where $j(x)$ denotes the normalized duality mapping of the space X. If I denotes the identity operator in X, then (1.2) can be written in the form

$$\langle (I-T_i)(x) - (I-T_i)(y), j(x-y) \rangle \geq \lambda_i \|(I-T_i)(x) - (I-T_i)(y)\|^2. \tag{1.3}$$

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In Hilbert space H, (1.2) (and hence (1.3)) is equivalent to the inequality

$$\|T_i(x) - T_i(y)\|^2 \leq \|x - y\|^2 + k_i\|(I - T_i)(x) - (I - T_i)(y)\|^2, k_i = 1 - \lambda_i.$$

Clearly, when $k_i = 0$, T_i is nonexpansive, i.e.,

$$\|T_i(x) - T_i(y)\| \leq \|x - y\|.$$

In [11] Wang proved the following result.

Theorem 1.1. *Let H be a Hilbert space, $T : H \to H$ a nonexpansive mapping with $F(T) \neq \emptyset$, and $F : H \to H$ an η-strongly monotone and k-Lipschitzian mapping. For any $x_0 \in H$, $\{x_n\}$ is defined by*

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)T^{\lambda_{n+1}}x_n, n \geq 0,$$

where $\{\alpha_n\}$ and $\{\lambda_n\} \subset [0, 1)$ satisfy the following conditions:
(1) $\alpha \leq \alpha_n \leq \beta$ *for some* $\alpha, \beta \in (0, 1)$;
(2) $\sum_{n=1}^{\infty} \lambda_n < +\infty$;
(3) $0 < \mu < 2\eta/k^2$.
Then,
(1) $\{x_n\}$ *converges weakly to a fixed point of T;*
(2) $\{x_n\}$ *converges strongly to a fixed point of T if and only if*

$$\lim_{n\to\infty} \inf d(x_n, F(T)) = 0.$$

Another methods are considered in [6] and [7] for the case $N = 1$.

In [14] Zeng and Yao proved the following results.

Theorem 1.2. *Let H be a Hilbert space, $F : H \to H$ be a mapping such that for some constants $k, \eta > 0$, F is k-Lipschitzian and η-strongly monotone. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of H such that $C = \cap_{i=1}^N F(T_i) \neq \emptyset$. Let $\mu \in (0, 2\eta/k^2)$, let $x_0 \in H, \{\lambda_n\}_{n=1}^{\infty} \subset [0, 1)$ and $\{\alpha_n\}_{n=1}^{\infty} \subset (0, 1)$ satisfying the conditions: $\sum_{n=1}^{\infty} \lambda_n < \infty$ and $\alpha \leq \alpha_n \leq \beta, n \geq 1$, for some $\alpha, \beta \in (0, 1)$. Then the sequence $\{x_n\}$ defined by*

$$\begin{aligned} x_n &= \alpha_{n-1}x_{n-1} + (1 - \alpha_n)T_n^{\lambda_n}x_n \\ &= \alpha_{n-1}x_{n-1} + (1 - \alpha_n)[T_n x_n - \lambda_n \mu F(T_n x_n)], n \geq 1, \end{aligned} \tag{1.4}$$

where $T_n = T_{n mod N}$, converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.

Theorem 1.3. *Let H be a Hilbert space, $F : H \to H$ be a mapping such that for some constants $k, \eta > 0$, F is k-Lipschitzian and η-strongly monotone. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of H such that $C = \cap_{i=1}^N F(T_i) \neq \emptyset$. Let $\mu \in (0, 2\eta/k^2)$, let $x_0 \in H, \{\lambda_n\}_{n=1}^{\infty} \subset [0, 1)$ and $\{\alpha_n\}_{n=1}^{\infty} \subset (0, 1)$ satisfying the conditions: $\sum_{n=1}^{\infty} \lambda_n < \infty$ and $\alpha \leq \alpha_n \leq \beta, n \geq 1$, for some $\alpha, \beta \in (0, 1)$. Then the sequence $\{x_n\}$ defined by (1.4) converges strongly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$ if and only if*

$$\lim_{n\to\infty} \inf d(x_n, \cap_{i=1}^N F(T_i)) = 0.$$

In [9, 12], Xu, Ori, and Osilike showed that if X is a Hilbert space, and the sequence $\{x_n\}$ defined by

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_n(x_n), x_0 \in C,$$

then $\{x_n\}$ converges weakly to a common fixed point of $\{T_i\}_{i=1}^N$. In [4] Chen, Lin, and Song extended the above result to a Banach spaces.

Theorem 1.4. *Let K be a nonempty closed convex subset of a q-uniformly smooth and p-uniformly convex Banach space E that has the Opial property. Let s be any element in $(0,1)$ and let $\{T_i\}_{i=1}^N$ be a finite family of strictly pseudocontractive self-maps of K such that $T_i, 1 \le i \le N$ have at least one common fixed point. For any point x_0 in K and any sequence $\{\alpha_n\}_{n=0}^\infty$, in $(0,s)$, the sequence*

$$x_n = \alpha_{n-1}x_{n-1} + (1-\alpha_n)T_n x_n,$$

converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.

Further, Gu in [5] introduced a new composite implicit iteration process as follows:

$$\begin{aligned} x_n &= (1-\alpha_n-\gamma_n)x_{n-1} + \alpha_n T_n(y_n) + \gamma_n u_n, n \ge 1, \\ y_n &= (1-\beta_n-\delta_n)x_n + \beta_n T_n(x_n) + \delta_n v_n, n \ge 1, \end{aligned} \tag{1.5}$$

where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}, \{\delta_n\}$ are four real sequences in $[0,1]$ satisfying $\alpha_n + \gamma_n \le 1$ and $\beta_n + \delta_n \le 1$ for all $n \ge 1$, $\{u_n\}$ and $\{v_n\}$ are two bounded sequences in C and x_0 is a given point. It is proved the following theorem.

Theorem 1.5. *Let X be a real Banach space and C be a nonempty closed convex subset of X. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive mappings of C into C with $S := \cap_{i=1}^N F(T_i) \ne \emptyset$. Let $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}, \{\delta_n\}$ are four real sequences in $[0,1]$ satisfying $\alpha_n + \gamma_n \le 1$ and $\beta_n + \delta_n \le 1$ for all $n \ge 1$, $\{u_n\}$ and $\{v_n\}$ are two bounded sequences in C satisfying the following conditions:*

(i) $\sum_{i=1}^\infty \alpha_n = \infty$;
(ii) $\sum_{i=1}^\infty \alpha_n^2 < \infty$;
(iii) $\sum_{i=1}^\infty \alpha_n\beta_n < \infty$;
(iv) $\sum_{i=1}^\infty \alpha_n\delta_n < \infty$;
(v) $\sum_{i=1}^\infty \gamma_n < \infty$;

Suppose further that and $x_0 \in C$ be a given point and $\{x_n\}$ is the implicit iteration sequence defined by (1.5), then the following conclusions hold:

(i) $\lim_{n\to\infty} \|x_n - p\|$ *exists for all* $p \in S$;
(ii) $\liminf_{n\to\infty} \|x_n - T_n(x_n)\| = 0$.

Set

$$A_i = I - T_i.$$

Obviously, $S_i := \{x \in X : A_i(x) = 0\} = F(T_i)$ and problem (1.1) is equivalent to one of finding a common solution of the following operator equations

$$A_i(x) = 0, \quad i = 1, ..., N,$$

where A_i are Lipschitz continuous and λ_i inverse strongly accretive, i.e., A_i satisfy (1.3).

In the following section, on the base of [3] and the implicit iterative method of the Tikhonov regularization type we present an explicit iterative method for solving (1.1) in Banach spaces. Later, the symbols $\to$ and $\rightharpoonup$ denote the strong and the weak convergence, respectively.

1. Main results

We formulate the following facts in [8], [13] which are necessary in the proof of our results.

Lemma 2.1. *Let $\{a_n\}, \{b_n\}, \{c_n\}$ be the sequences of positive numbers satisfying the conditions*

(i) $a_{n+1} \leq (1 - b_n)a_n + c_n, b_n < 1,$

(ii) $\sum_{n=0}^{\infty} b_n = +\infty, \quad \lim_{n\to+\infty} \frac{c_n}{b_n} = 0.$

Then, $\lim_{n\to+\infty} a_n = 0.$

Theorem 2.1. [13] *Let $q > 1$ and X be a real Banach space. Then the following are equivalent:*

(1) *X is q-uniformly smooth.*

(2) *There exists a constant $c_q > 0$ such that for all $x, y \in X$*

$$\|x + y\|^q \leq \|x\|^q + q\langle y, j_q(x)\rangle + c_q\|y\|^q,$$

where

$$j_q(x) = \{f \in X^* : \langle x, f\rangle = \|x\|^q \quad \text{and} \quad \|f\| = \|x\|^{q-1}\}.$$

Note that j_q is called the generalized duality mapping from X into 2^{X^*} for arbitrary Banach space X. When $q = 2$, j_2 is called the normalized duality mapping and it is usually denoted by j. It is well known [13] that $j_q(x) = \|x\|^{q-2}j(x)$ if $x \neq 0$, and that if X^* is strictly convex then j_q is single-valued. One important property of j used later is $j(-x) = -j(x)$. The modulus of smoothness of X is the function $\rho_X : [0, \infty) \to [0, \infty)$ defined by

$$\rho_X(\tau) = \sup\left\{\frac{1}{2}(\|x + y\| + \|x - y\|) - 1 : \|x\| \leq 1, \|y\| \leq \tau\right\}.$$

X is uniformly smooth if and only if $\lim_{\tau\to 0}(\rho_X(\tau)/\tau) = 0$. X is said to be q-unifomly smooth if there exists a constant $c > 0$ such that $\rho_X(\tau) \leq c\tau^q$. Hilbert spaces, L_p (or l_p) spaces, $1 < p < \infty$, and the Sobolev spaces, $W_m^p, 1 < p < \infty$, are q-uniformly smooth. Hilbert spaces are 2-uniformly smooth while

$$L_p(\text{or } l_p) \; or W_m^p \text{ is } \begin{cases} p - \text{uniformly smooth if} & 1 < p \leq 2 \\ 2 - \text{uniformly smooth if} & p \geq 2. \end{cases}$$

T is said to be *demiclosed at a point* p if whenever $\{x_n\}$ is a sequence in $D(T)$ such that $\{x_n\}$ converges weakly to $x \in D(T)$ and $\{T(x_n)\}$ converges strongly to p, then $T(x) = p$. Furthermore, T is said to be demicompact if whenever

$\{x_n\}$ is a bounded sequence in $D(T)$ such that $\{x_n - T(x_n)\}$ converges strongly, then $\{x_n\}$ has a subsequence which converges strongly.

Theorem 2.2. [8] *Let X be a q-uniformly smooth Banach space which is also uniformly convex. Let K be a nonempty closed convex subset of X and $T : K \to K$ a strictly pseudocontractive map. Then $(I - T)$ is demiclosed at zero.*

Consider the operator version of Tikhonov regularization method in the form

$$\sum_{i=1}^{N} A_i(x) + \alpha_n x = 0, \tag{2.1}$$

depending on the positive regularization parameter α_n that tens to zero as $n \to +\infty$.

We have the following results.

Theorem 2.3. *(i) For each $\alpha_n > 0$, problem (2.1) has a unique solution x_n.*
(ii) If one of the following conditions is satisfied:
(a) X possesses a weak sequential continuous duality mapping j,
(b) there exists a number $i_0 \in \{1, 2, ..., N\}$ such that T_{i_0} is demicompact,
then the sequence $\{x_n\}$ possesses a convergent subsequence, and each convergent subsequence of $\{x_n\}$ converges to a solution of (1.1).
(iii) If the sequence $\{\alpha_n\}$ is chosen such that

$$\lim_{n\to+\infty} \frac{|\alpha_n - \alpha_{n+p}|}{\alpha_n} = 0,$$

for any fixed positive natural number p, then

$$\lim_{n\to+\infty} x_n = x_* \in S.$$

Proof. (i) Since $\sum_{i=1}^{N} A_i$ are Lipschitz continuous and accretive, then it is m-accretive [1]. Hence, equation (2.1) has a unique solution denoted by x_n for each $\alpha_n > 0$.

(ii) From (2.1) it follows

$$\sum_{i=1}^{N} \langle A_i(x_n), j(x_n - y)\rangle + \alpha_n \langle x_n, j(x_n - y)\rangle = 0 \quad \forall y \in S. \tag{2.2}$$

Since $A_i(y) = 0, i = 1, ..., N$, then

$$\sum_{i=1}^{N} A_i(y) = 0.$$

The last equality, (2.2) and the accretive property of A_i give

$$\langle x_n, j(x_n - y)\rangle \le 0$$

or

$$\langle x_n - y, j(x_n - y)\rangle \le \langle -y, j(x_n - y)\rangle \quad \forall y \in S. \tag{2.3}$$

Consequently,

$$\|x_n - y\| \leq \|y\| \quad \text{and} \quad \|x_n\| \leq 2\|y\|, y \in S. \tag{2.4}$$

Hence, $\{x_n\}$ is bounded. Let $x_{n_k} \rightharpoonup \tilde{x} \in X$, as $k \to +\infty$. We shall prove that $\tilde{x} \in F(T_l), l = 1, ..., N$. For any $y \in S$ from (1.3), (2.1), (2.4) and the accretive property of A_l it implies that

$$\begin{aligned}\|A_l(x_{n_k})\|^2 &\leq \langle A_l(x_{n_k}), j(x_{n_k} - y)\rangle/\lambda_l \\ &\leq \sum_{i=1, i\neq l}^{N} \langle A_i(y) - A_i(x_{n_k}), j(x_{n_k} - y)\rangle/\lambda_l \\ &+ \alpha_{n_k}\langle -y, j(x_{n_k} - y)\rangle/\lambda_l\end{aligned}$$

or

$$\|A_l(x_{n_k})\| \leq \sqrt{\frac{\alpha_{n_k}}{\lambda_l}}\|y\|.$$

Therefore,

$$\lim_{k\to\infty} \|A_l(x_{n_k})\| = 0.$$

By virtue of the demiclosed property of A_i, we have $A_i(\tilde{x}) = 0$, i.e., $\tilde{x} \in F(T_l)$. It means that $\tilde{x} \in S$.

From the weak sequential continuous property of the duality mapping j and (2.3) with $y = \tilde{x}$ or the demicompact property of T_{i_0} it follows that $x_{n_k} \to \tilde{x} \in S$, as $k \to \infty$.

(iii) Let x_{n+p} be the solution of (2.1) when α_n is replaced by α_{n+p}. Then,

$$\sum_{i=1}^{N}\langle A_i(x_n) - A_i(x_{n+p}), j(x_n - x_{n+p})\rangle + \alpha_n\langle x_n, j(x_n - x_{n+p})\rangle + \alpha_{n+p}\langle x_{n+p}, j(x_{n+p} - x_n)\rangle = 0.$$

Hence,

$$\|x_n - x_{n+p}\| \leq \frac{|\alpha_n - \alpha_{n+p}|}{\alpha_n}2\|y\|.$$

Clearly, if

$$\lim_{n\to+\infty} \frac{|\alpha_n - \alpha_{n+p}|}{\alpha_n} = 0,$$

then $\{x_n\}$ is a Cauchy sequence in the Banach space X. Therefore,

$$\lim_{n\to+\infty} x_n = x_* \in S.$$

Theorem is proved now. □

Now, consider the iterative regularization method of zero order

$$z_{n+1} = z_n - \beta_n[\sum_{i=1}^{N} A_i(z_n) + \alpha_n z_n], z_0 \in X, n = 0, 1, ... \tag{2.5}$$

Set

$$L = \max\{\frac{1}{\lambda_i}, i = 1, ..., N\}.$$

Theorem 2.4. *Suppose that β_n, α_n satisfy the following conditions:*

$$0 < \beta_n < \beta_0, \alpha_n \searrow 0, \lim_{n\to\infty} \frac{\alpha_n - \alpha_{n+1}}{\alpha_n^2 \beta_n} = 0,$$

$$\sum_{n=0}^{\infty} \alpha_n \beta_n = \infty, \quad \overline{\lim}_{n\to\infty} \quad c_q \beta_n^{q-1} \frac{(LN + \alpha_n)^q}{\alpha_n} < 1.$$

Then $\lim_{n\to\infty} z_n = x_* \in S$, *where z_n is defined by (2.5).*

Proof. Let x_n be the solution of (2.1). Then, theorem 2.1 gives

$$\begin{aligned}\|z_{n+1} - x_n\|^q &= \|z_n - x_n - \beta_n[\sum_{i=1}^{N}(A_i(z_n) - A_i(x_n)) + \alpha_n(z_n - x_n)]\|^q \\ &\leq \|z_n - x_n\|^q \\ &- q\beta_n\langle\sum_{i=1}^{N}(A_i(z_n) - A_i(x_n) + \alpha_n(z_n - x_n), j_q(z_n - x_n)\rangle \\ &+ c_q\beta_n^q\|\sum_{i=1}^{N}(A_i(z_n) - A_i(x_n)) + \alpha_n(z_n - x_n)\|^q,\end{aligned}$$

where

$$\begin{aligned}\langle A_i(z_n) - A_i(x_n), j_q(z_n - x_n)\rangle &= \|z_n - x_n\|^{q-2} \times \\ &\quad \langle A_i(z_n) - A_i(x_n), j(z_n - x_n)\rangle \geq 0, \\ \langle z_n - x_n, j_q(z_n - x_n)\rangle &= \|z_n - x_n\|^q.\end{aligned}$$

Therefore,

$$\|z_{n+1} - x_n\|^q \leq \|z_n - x_n\|^q[1 - q\beta_n\alpha_n + c_q\beta_n^q(LN + \alpha_n)^q].$$

Thus,

$$\|z_{n+1} - x_n\| \leq \|z_n - x_n\|[1 - q\beta_n\alpha_n + c_q\beta_n^q(LN + \alpha_n)^q]^{1/q}.$$

Since $c_q\beta_n^q(LN + \alpha_n)^q < \beta_n\alpha_n$ and $(1-t)^\gamma \leq 1 - \gamma t$, for $0 < \gamma < 1$, then

$$\begin{aligned}\|z_{n+1} - x_n\| &\leq \|z_n - x_n\|[1 - (q-1)\beta_n\alpha_n]^{1/q} \\ &\leq \|z_n - x_n\|(1 - \frac{q-1}{q}\beta_n\alpha_n).\end{aligned}$$

Hence,

$$\begin{aligned}\|z_{n+1} - x_{n+1}\| &\leq \|z_{n+1} - x_n\| + \|x_{n+1} - x_n\| \\ &\leq \|z_n - x_n\|(1 - \frac{q-1}{q}\beta_n\alpha_n) + M\frac{\alpha_n - \alpha_{n+1}}{\alpha_n}.\end{aligned}$$

Applying lemma 2.1 with

$$a_n = \|z_n - x_n\|, \qquad b_n = \frac{q-1}{q}\beta_n\alpha_n$$

$$c_n = M\frac{\alpha_n - \alpha_{n+1}}{\alpha_n}$$

we obtain that $\lim_{n\to\infty}\|z_n - x_n\| = 0$. On the other hand, since

$$\lim_{n\to\infty}\frac{\alpha_n - \alpha_{n+1}}{\alpha_n^2\beta_n} = 0, \text{ then } \lim_{n\to\infty}\frac{\alpha_n - \alpha_{n+1}}{\alpha_n} = 0.$$

Consequently, for any fixed positive natural number p we have

$$\forall\varepsilon > 0\exists N(\varepsilon) > 0 : \forall n > N(\varepsilon) \Longrightarrow \frac{x_n - x_{n+1}}{x_n} < \frac{\varepsilon}{p}.$$

Thus,

$$0 < \frac{\alpha_n - \alpha_{n+p}}{\alpha_n} = \frac{\alpha_n - \alpha_{n+1}}{\alpha_n} + \frac{\alpha_{n+1} - \alpha_{n+2}}{\alpha_n} + ... + \frac{\alpha_{n+p-1} - \alpha_{n+p}}{\alpha_n}$$
$$\leq \frac{\alpha_n - \alpha_{n+1}}{\alpha_n} + \frac{\alpha_{n+1} - \alpha_{n+2}}{\alpha_{n+1}} + ... + \frac{\alpha_{n+p-1} - \alpha_{n+p}}{\alpha_{n+p-1}} < \varepsilon.$$

It means that $\lim_{n\to+\infty}\frac{\alpha_n - \alpha_{n+p}}{\alpha_n} = 0$ for any fixed positive natural number p. Theorem 2.3 permits us to conclude that $\lim_{n\to\infty} x_n = x_* \in S$. So,

$$\lim_{n\to\infty} z_n = x_* \in S.$$

Theorem is proved. □

Remark. The sequences $\alpha_n = (1+n)^{-p}, 0 < p < 1/2$, and $\beta_n = \gamma_0\alpha_n$ with

$$0 < \gamma_0 < \frac{1}{c_q^{1/(q-1)}(LN + \alpha_0)^{q/(q-1)}}$$

satisfy all the necessary conditions in theorem 2.4 for the case $q \geq 2$. For the case $1 < q < 2$, $\alpha_n = (1+n)^{-p}$ with $p < (q-1)/(2q)$ and $\beta_n = \gamma_0\alpha_n^{1/(q-1)}$ also satisfy all the necessary conditions in theorem 2.4.

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References

1. F.E. Browder, *Nonlinear mapping of nonexpansive and accretive type in Banach spaces,* Bull. Amer. Math. Soc. **73** (1967), 875-882.
2. F.E. Browder and W.V. Petryshyn, *Construction of fixed points of nonlinear mappings in Hilbert spaces,* J. Math. Anal. Appl. **20** (1967), 197-228.
3. Ng. Buong, *Regularization for unconstrained vector optimization of convex functionals in Banach spaces,* Zh. Vychisl. Mat. i Mat. Fiziki, **46**, (2006), n. 3, 372-378.
4. R. Chen, P.K. Lin, and Y. Song, *An approximation method for pseudocontractive mappings,* Nonlinear Analysis, **64** (2006), 2527-2535.

5. F. Gu, *The new composite iterative process with errors for common fixed points of a finite family of strictly pseudo contractive mappings,* J. Math. Anal. Appl. **329** (2007), n. 2, 766-776.
6. G. Marino and H.K. Xu, *Weak and strong convergence theorems for strict pseudo-contractions in Hilbert spaces,* J. Math. Anal. Appl. **329** (2007), n. 2, 336-346.
7. J.G. O'Hara, P. Pillay and H.K. Xu, *Iterative approachs to finding nearest common fixed points of nonexpansive mappings in Hilbert spaces,* Nonlinear Anal., **54** (2003), n. 8, 1417-1426.
8. M.O. Osilike and A. Udomene, *Demiclosedness principle and convergence theorems for strictly pseudocintractive mappings of Browder-Petryshyn type,* J. Math. Anal. Appl. **256** (2001), 431-445.
9. M.O. Osilike, *Implicit iteration process for common fixed points of a finite family of strictly pseudocontractive mappings,* J. Math. Anal. Appl. **294** (2004), 73-81.
10. G. Wang, J. Peng, and H.J. Lee, *Implicit iteration process with mean errors for common fixed point of a family of strictly pseudocontractive maps,* Int. Journal of Math. Ana.**1** (2007), n. 2, 89-99.
11. L. Wang, *An iteration method for nonexpansive mappings in Hilbert spaces,* Fixed Point Theory and Appl., Article ID 28619 Volume 2007, 8 pages.
12. H.K. Xu and G. Ori, *An implicit iteration process for nonexpansive mappings,* Numer. Funct. Anal. Optim. **22** (2001), 767-773.
13. H.K. Xu, *Inequalities in Banach spaces with applications,* Nonlinear Anal., **16** (1991), n. 12, 1127-1138.
14. L.C. Zeng, J.C. Yao, *Implicit iteration scheme with perturbed mapping for common fixed points of a finite family of nonexpansive mapping,* Nonlinear Anal., **64** (2006), n. 11, 2507-2515.

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