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INTERVAL OSCILLATION THEOREMS FOR SECOND-ORDER DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper,we are concerned with a class of nonlinear second-order differential equations with a nonlinear damping term and forcing term:

$$(r(t)k_1(x(t),x'(t)))' + p(t)k_2(x(t),x'(t))x'(t) + q(t)f(x(t)) = 0.$$

Passage to more general class of equations allows us to remove a restrictive condition usually imposed on the nonlinearity. And, as a consequence, our results apply to wider classes of nonlinear differential equations.Some illustrative examples are considered.

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1. Introduction

In this paper, we are concerned with the general nonlinear second-order differential equation

$$(r(t)k_1(x(t),x'(t)))' + p(t)k_2(x(t),x'(t))x'(t) + q(t)f(x(t)) = 0, \qquad (1)$$

where $t \geq t_0$, and the functions $r,\ k_1,\ p,\ k_2,\ q,\ f$ are to be specified in the following text.

As usual, a function $x : [t_0, t_1) \to (-\infty, \infty)$, $t_1 > t_0$ is called a solution of Eq.(1) if $x(t)$ satisfies Eq.(1) for all $t \in [t_0, t_1)$. In what follows, we always assume that solutions of this equation are continuable, that is, they exist for all $t \geq t_0$. A nonconstant continuable solution $x(t)$ of Eq.(1) is called proper if $\sup_{t\geq t_0} \mid x(t) \mid > 0$. A proper solution $x(t)$ is called oscillatory if it does not have the largest zero. otherwise it is called non-oscillatory. Finally, we call Eq.(1) oscillatory if all its proper solutions are oscillatory.

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Many papers on oscillation of differential equations are concerned with nonlinear differential equations with a linear damping term of the either form

$$(r(t)\psi((x(t))x'(t))' + p(t)x'(t) + q(t)f(x(t)) = 0$$

or

$$(r(t)x'(t))' + p(t)x'(t) + q(t)f(x(t)) = 0,$$

but only a few papers deal with equations with nonlinear damping term.The relevant examples are equations

$$x''(t) + g(t, x(t), x'(t))x'(t) + a(t)f(x(t)) = 0$$

and

$$x''(t) + q(t)\phi(x(t), x'(t))x'(t) + p(t)x^{\alpha}(t)g(x'(t)) = 0.$$

The general nonlinear differential equation with damping

$$(r(t)k(x(t), x'(t))x'(t))' + p(t)k(x(t), x'(t))x'(t) + q(t)f(x(t)) = 0$$

considered recently by Ayanlar and Tiryaki[4] and present authors[5].

However, from the Sturm Separation Theorem, we see that oscillation is only an interval property, i.e., if there exists a sequence of subinterval $[a_i, b_i]$ of $[t_0, \infty), a_i \to \infty$ such that for each i, there exists a solution of Eq.(1) that has at least two zeros in $[a_i, b_i]$, then every solution of Eq.(1) is oscillatory.

Using the thoughts mentioned above, Kwong and Zettle[6], El-sayed[7], Huang[8] and Kong[9] give some interval oscillation criteria for the linear differential equation $x''(t) + q(t)x(t) = 0, \quad t \geq t_0$. Then Li and Agarwal [10],Zhaowen Zheng [11] give some interval oscillation criteria for the nonlinear differential equation with damped term

$$x''(t) + p(t)x'(t) + q(t)f(x(t)) = 0, \quad t \geq t_0.$$

In this paper,we will give some interval oscillation criterions of Eq.(1). Hereafter, we assume that:

(i) the function $r : [t_0, \infty) \to (0, \infty)$ is continuously differential,$t_0 \geq 0$;

(ii) $p : [t_0, \infty) \to R$ is continuous, $p(t) \geq 0$ for all $t \geq t_0, \ t_0 \geq 0$;

(iii) $q : [t_0, \infty) \to R$ is continuous, $q(t) > 0$ for all $t \geq t_0, \ t_0 \geq 0$;

(iv) $f : R \to R$ is continuous and satisfies $f(x)/x \geq K$ for some positive constant K and for all $x \neq 0$;

(v) $k_1 : R^2 \to R^2$ is continuously differentiable and satisfies $k_1^2(u, v) \leq \alpha v k_1(u, v)$ for some positive constant α, all $v \in R/\{0\}$ and all $u \in R$;

(vi) $k_2 : R^2 \to R^2$ is continuous and has the sign of v for all $v \in R/\{0\}$ and all $u \in R$;

(vii) $f \in C^1[R, R]$ and $xf(x) > 0$ for $x \neq 0$, there exists $f'(x)$ for $x \in R$ and $f'(x) \geq \mu > 0$ for $x \neq 0$.

The following lemma is useful in our main results.

Lemma [1, Lemma 1]. *Suppose that the assumptions (i)-(vi) are satisfied.If $x(t)$ is a non-oscillatory solution of Eq.(1), then $x(t)x'(t) < 0$ for all large t.*

In the sequel we say that a function $H = H(t,s)$ belongs to a function class X, denoted by $H \in X$, if $H \in C(D, R_+)$ where $D = \{(t,s) : -\infty < s \le t < \infty\}$, which satisfies $H(t,t) = 0$, $H(t,s) > 0$, $t > s$, and has continuous partial derivatives $\partial H/\partial t$ and $\partial H/\partial s$ on D such that

$$\frac{\partial H}{\partial t} = h_1(t,s)H(t,s)^{1/2}, \quad \frac{\partial H}{\partial s} = -h_2(t,s)H(t,s)^{1/2}.$$

2. Interval oscillation results for $f(x)$ without monotonicity

Theorem 2.1. *Suppose that the main assumptions (i)-(vi) are satisfied, and assume that there exists a function $\rho \in C^1([t_0,\infty),(0,\infty))$ such that for some $H \in X$ and for each sufficiently large $T_0 \ge t_0$, there exist a, b, c with $T_0 \le a < c < b$ such that*

$$\frac{1}{H(c,a)}\int_a^c H(s,a)K\rho(s)q(s)ds + \frac{1}{H(b,c)}\int_c^b H(b,s)K\rho(s)q(s)ds$$

$$> \frac{\alpha}{4H(c,a)}\int_a^c r(s)\rho(s)Q_1^2(s,a)ds + \frac{\alpha}{4H(b,c)}\int_c^b r(s)\rho(s)Q_2^2(b,s)ds, \quad (2)$$

where $Q_1(s,t) = h_1(s,t)+(\rho(s))^{-1}\rho'(s)(H(s,t))^{1/2}$, $Q_2(t,s) = h_2(t,s)-(\rho(s))^{-1}\rho'(s)(H(t,s))^{1/2}$. Then Eq.(1) is oscillatory.

Proof. Suppose to the contrary. Let $x(t)$ be a non-oscillatory solution of Eq.(1), and suppose that there exists a $T_* \ge t_0$ such that $x(t) \ne 0$ for all $t \ge T_*$. Without loss of generality, we may assume that $x(t) > 0$ for all $t \ge T_*$. Then, by Lemma, this non-oscillatory solution $x(t)$ is eventually monotonic and there exists a $T_0 \ge T_*$ such that $x'(t) < 0$ for all $t \ge T_0 \ge T_*$. Define

$$w(t) = \rho(t)\frac{r(t)k_1(x(t),x'(t))}{x(t)}, \quad (3)$$

differentiating (3) and using Eq.(1), we have

$$w'(t) \le \frac{\rho'(t)}{\rho(t)}w(t) - \rho(t)p(t)\frac{k_2(x(t),x'(t))x'(t)}{x(t)}$$

$$-\rho(t)q(t)\frac{f(x(t))}{x(t)} - \frac{w^2(t)}{\alpha\rho(t)r(t)}. \quad (4)$$

Since by the assumptions of the theorem, the second summand in (4) is non-positive and by Lemma, $x'(t) < 0$, we obtain

$$w'(t) \le -K\rho(t)q(t) + \frac{\rho'(t)}{\rho(t)}w(t) - \frac{w^2(t)}{\alpha r(t)\rho(t)}, \quad (5)$$

for all $t \ge T_0$. Multiplying both sides of the inequality (5) by $H(t,s)$ and integrating it with respect to s from c to t for $t \in [c,b)$, we have

$$\int_c^t H(t,s)K\rho(s)q(s)ds$$

$$\le -\int_c^t H(t,s)w'(s)ds + \int_c^t H(t,s)\frac{\rho'(s)}{\rho(s)}w(s)ds - \int_c^t H(t,s)\frac{w^2(s)}{\alpha\rho(s)r(s)}ds$$

$$= H(t,c)w(c) - \int_c^t \left[\left(\frac{H(t,s)}{\alpha\rho(s)r(s)}\right)^{1/2} w(s) + \frac{1}{2}(\alpha\rho(s)r(s))^{1/2}Q_2(t,s)\right]^2 ds$$

$$+ \int_c^t \frac{\alpha\rho(s)r(s)}{4}Q_2^2(t,s)ds \leq H(t,c)w(c) + \int_c^t \frac{\alpha\rho(s)r(s)}{4}Q_2^2(t,s)ds. \quad (6)$$

Dividing both sides of the inequality (6) by $H(t,c)$ and let $t \to b^-$, we obtain

$$\frac{1}{H(b,c)}\int_c^b H(b,s)K\rho(s)q(s)ds \leq w(c) + \frac{1}{H(b,c)}\int_c^b \frac{\alpha\rho(s)r(s)}{4}Q_2^2(b,s)ds. \quad (7)$$

Next, we go back to (5). We multiply (5) by $H(s,t)$, integrate it with respect to s from t to c for $t \in (a,c]$, then we get that

$$\int_t^c H(s,t)K\rho(s)q(s)ds$$

$$\leq -\int_t^c H(s,t)w'(s)ds + \int_t^c H(s,t)\frac{\rho'(s)}{\rho(s)}w(s)ds - \int_t^c H(s,t)\frac{w^2(s)}{\alpha\rho(s)r(s)}ds$$

$$= -H(c,t)w(c) + \int_t^c \left[\left(\frac{H(s,t)}{\alpha\rho(s)r(s)}\right)^{1/2} w(s) + \frac{1}{2}(\alpha\rho(s)r(s))^{1/2}Q_1(s,t)\right]^2 ds$$

$$+ \int_t^c \frac{\alpha\rho(s)r(s)}{4}Q_1^2(s,t)ds$$

$$\leq -H(c,t)w(c) + \int_t^c \frac{\alpha\rho(s)r(s)}{4}Q_1^2(s,t)ds. \quad (8)$$

Dividing both sides of the inequality (8) by $H(c,t)$ and let $t \to a^+$, we obtain

$$\frac{1}{H(c,a)}\int_a^c H(s,a)K\rho(s)q(s)ds \leq -w(c) + \frac{1}{H(c,a)}\int_a^c \frac{\alpha\rho(s)r(s)}{4}Q_1^2(s,a)ds. \quad (9)$$

Adding (7) and (9), we have

$$\frac{1}{H(c,a)}\int_a^c H(s,a)K\rho(s)q(s)ds + \frac{1}{H(b,c)}\int_c^b H(b,s)K\rho(s)q(s)ds$$

$$\leq \frac{\alpha}{4H(c,a)}\int_a^c r(s)\rho(s)Q_1^2(s,a)ds + \frac{\alpha}{4H(b,c)}\int_c^b r(s)\rho(s)Q_2^2(b,s)ds,$$

which contradicts the assumption (2). Therefore, every solution of Eq.(1) is oscillatory. The proof is complete. □

Theorem 2.2. *Suppose that the main assumptions (i)-(vi) are satisfied, and assume that there exists a function $\rho \in C^1([t_0,\infty),(0,\infty))$ such that*

$$\limsup_{t\to\infty}\int_l^t \left[H(s,l)K\rho(s)q(s) - \frac{\alpha r(s)\rho(s)}{4}Q_1^2(s,l)\right]ds > 0 \quad (10)$$

and

$$\limsup_{t\to\infty}\int_l^t \left[H(t,s)K\rho(s)q(s) - \frac{\alpha r(s)\rho(s)}{4}Q_2^2(t,s)\right] ds > 0 \tag{11}$$

for some $H \in X$, and for each $l \geq t_0$, then Eq.(1) is oscillatory.

Proof. For any $T \geq t_0$, let $a = T$. In (10) we choose $l = a$. Then there exists $c > a$ such that

$$\int_a^c \left[H(s,a)K\rho(s)q(s) - \frac{\alpha r(s)\rho(s)}{4}Q_1^2(s,a)\right] ds > 0. \tag{12}$$

In (11) we choose $l = c$. Then there exists $b > c$ such that

$$\int_c^b \left[H(b,s)K\rho(s)q(s) - \frac{\alpha r(s)\rho(s)}{4}Q_2^2(b,s)\right] ds > 0. \tag{13}$$

Combining (12) and (13) we obtain (2), the conclusion thus comes from Theorem 1. The proof is complete. □

With the standard yet choice of the $H(t,s)$

$$H(t,s) = (t-s)^\lambda, \quad t \geq s \geq t_0,$$

where $\lambda > 1$ is a constant, we obtain the following corollary.

Corollary 2.1. *Suppose that the main assumptions (i)-(vi) are satisfied. Then every solution of Eq.(1) is oscillatory provided that there exists a function $\rho \in C^1([t_0,\infty),(0,\infty))$ such that for each $l \geq t_0$ and for $\lambda > 1$, the following two inequalities hold:*

$$\limsup_{t\to\infty}\frac{1}{t^{\lambda-1}}\int_l^t \left[(s-l)^\lambda K\rho(s)q(s) - \frac{\alpha r(s)\rho(s)}{4}(s-l)^{\lambda-2} \times \left(\lambda + \frac{\rho'(s)}{\rho(s)}(s-l)\right)^2\right] ds > 0$$

and

$$\limsup_{t\to\infty}\frac{1}{t^{\lambda-1}}\int_l^t \left[(t-s)^\lambda K\rho(s)q(s) - \frac{\alpha r(s)\rho(s)}{4}(t-s)^{\lambda-2} \times \left(\lambda + \frac{\rho'(s)}{\rho(s)}(t-s)\right)^2\right] ds > 0.$$

Theorem 2.3. *Suppose that the main assumptions (i)-(vi) are satisfied, and assume that there exists a function $\rho \in C^1([t_0,\infty),(0,\infty))$ such that for any $u \in C[a,b]$ satisfying $u'(t) \in L^2[a,b]$ and $u(a) = u(b) = 0$, we have*

$$\int_a^b \left[u^2(s)Kq(s)\rho(s) - \alpha r(s)\rho(s)\left(u'(s) + \frac{1}{2}u(s)\frac{\rho'(s)}{\rho(s)}\right)^2\right] ds > 0,$$

then Eq.(1) is oscillatory.

Proof. Suppose the contrary. Let $x(t)$ be a non-oscillatory solution of Eq.(1) and suppose that there exists a $T_* \geq t_0$ such that $x(t) \neq 0$ for all $t \geq T_*$. Without loss of generality, we may assume that $x(t) > 0$ for all $t \geq T_*$. Similar to the proof of Theorem 2.1, we multiply (5) by $u^2(t)$, integrate it with respect to s from a to b and use $u(a) = u(b) = 0$, then we get

$$\begin{aligned}
&\int_a^b u^2(s)Kq(s)\rho(s)ds \\
&\leq -\int_a^b u^2(s)w'(s)ds - \int_a^b u^2(s)\frac{w^2(s)}{\alpha r(s)\rho(s)}ds + \int_a^b u^2(s)w(s)\frac{\rho'(s)}{\rho(s)}ds \\
&= 2\int_a^b u(s)u'(s)w(s)ds - \int_a^b u^2(s)\frac{w^2(s)}{\alpha r(s)\rho(s)}ds + \int_a^b u^2(s)w(s)\frac{\rho'(s)}{\rho(s)}ds \\
&= -\int_a^b \Bigg\{ \left[\sqrt{\frac{1}{\alpha r(s)\rho(s)}}u(s)w(s) - \sqrt{\alpha r(s)\rho(s)}\left(u'(s) + \frac{1}{2}u(s)\frac{\rho'(s)}{\rho(s)}\right)\right]^2 \\
&\quad + \alpha r(s)\rho(s)\left(u'(s) + \frac{1}{2}u(s)\frac{\rho'(s)}{\rho(s)}\right)^2 \Bigg\} ds.
\end{aligned}$$

So

$$\int_a^b \left[u^2(s)Kq(s)\rho(s) - \alpha r(s)\rho(s)\left(u'(s) + \frac{1}{2}u(s)\frac{\rho'(s)}{\rho(s)}\right)^2\right] ds \leq 0,$$

which contradicts the assumption, so every solution of Eq.(1) is oscillatory. The proof is complete. □

3. Interval oscillation results for $f(x)$ with monotonicity

Theorem 3.1. *Suppose that the main assumptions (i)-(v) and (vii) are satisfied, and assume that there exists a function $\rho \in C^1([t_0,\infty),(0,\infty))$ such that for some $H \in X$ and for each sufficiently large $T_0 \geq t_0$, there exist a, b, c with $T_0 \leq a < c < b$ such that*

$$\begin{aligned}
&\frac{1}{H(c,a)}\int_a^c H(s,a)\rho(s)q(s)ds + \frac{1}{H(b,c)}\int_c^b H(b,s)\rho(s)q(s)ds \\
&> \frac{\alpha}{4\mu H(c,a)}\int_a^c r(s)\rho(s)Q_1^2(s,a)ds + \frac{\alpha}{4\mu H(b,c)}\int_c^b r(s)\rho(s)Q_2^2(b,s)ds,
\end{aligned}$$

where $Q_1(s,t), Q_2(t,s)$ are defined as Theorem 2.1. Then Eq.(1) is oscillatory.

Proof. Suppose the contrary. Let $x(t)$ be a non-oscillatory solution of Eq.(1), and suppose that there exists a $T_* \geq t_0$ such that $x(t) \neq 0$ for all $t \geq T_*$. Without loss of generality, we may assume that $x(t) > 0$ for all $t \geq T_*$. Then, by Lemma, this non-oscillatory solution $x(t)$ is eventually monotonic and there exists a $T_0 \geq T_*$ such that $x'(t) < 0$ for all $t \geq T_0 \geq T_*$. Define

$$w(t) = \rho(t)\frac{r(t)k_1(x(t),x'(t))}{f(x(t))}. \tag{14}$$

Differentiating (14) and using Eq.(1), we have

$$w'(t) \le \frac{\rho'(t)}{\rho(t)}w(t) - \rho(t)p(t)\frac{k_2(x(t),x'(t))x'(t)}{f(x(t))} - \rho(t)q(t) - \frac{\mu w^2(t)}{\alpha\rho(t)r(t)}. \quad (15)$$

Since by the assumptions of the theorem, the second summand in (15) is non-positive and by Lemma, $x'(t) < 0$, we obtain

$$w'(t) \le \frac{\rho'(t)}{\rho(t)}w(t) - \rho(t)q(t) - \frac{\mu w^2(t)}{\alpha\rho(t)r(t)}.$$

The rest of the proof is similar to that of Theorem 2.1. □

Theorem 3.2. *Suppose that the main assumptions (i)-(v) and (vii) are satisfied, and assume that there exists a function* $\rho \in C^1([t_0,\infty),(0,\infty))$ *such that*

$$\limsup_{t\to\infty}\int_l^t \left[H(s,l)\rho(s)q(s) - \frac{\alpha r(s)\rho(s)}{4\mu}Q_1^2(s,l)\right] ds > 0$$

and

$$\limsup_{t\to\infty}\int_l^t \left[H(t,s)\rho(s)q(s) - \frac{\alpha r(s)\rho(s)}{4\mu}Q_2^2(t,s)\right] ds > 0$$

for some $H \in X$, *and for each* $l \ge t_0$, *then Eq.(1) is oscillatory.*

Corollary 3.1. *Suppose that the main assumptions (i)-(v) and (vii) are satisfied. Then every solution of Eq.(1) is oscillatory provided that there exists a function* $\rho \in C^1([t_0,\infty),(0,\infty))$ *such that for each* $l \ge t_0$ *and for* $\lambda > 1$, *the following two inequalities hold:*

$$\limsup_{t\to\infty}\frac{1}{t^{\lambda-1}}\int_l^t \Bigg[(s-l)^\lambda\rho(s)q(s) - \frac{\alpha r(s)\rho(s)}{4\mu}(s-l)^{\lambda-2} \times\left(\lambda + \frac{\rho'(s)}{\rho(s)}(s-l)\right)^2\Bigg]ds > 0$$

and

$$\limsup_{t\to\infty}\frac{1}{t^{\lambda-1}}\int_l^t \Bigg[(t-s)^\lambda\rho(s)q(s) - \frac{\alpha r(s)\rho(s)}{4\mu}(t-s)^{\lambda-2} \times\left(\lambda + \frac{\rho'(s)}{\rho(s)}(t-s)\right)^2\Bigg]ds > 0.$$

Theorem 3.3. *Suppose that the main assumptions (i)-(v) and (vii) are satisfied, and assume that there exists a function* $\rho \in C^1([t_0,\infty),(0,\infty))$ *such that for any* $u \in C[a,b]$ *satisfying* $u'(t) \in L^2[a,b]$ *and* $u(a) = u(b) = 0$, *we have*

$$\int_a^b \left[u^2(s)q(s)\rho(s) - \frac{\alpha r(s)\rho(s)}{\mu}\left(u'(s) + \frac{1}{2}u(s)\frac{\rho'(s)}{\rho(s)}\right)^2\right] ds > 0,$$

then Eq.(1) is oscillatory.

Remark. We have required in this paper that the function $p(t)$ is nonnegative and $q(t)$ is strictly positive. Although positive damping is commonly encountered in applications, it would be desirable to consider also the cases where the damping term is non-positive and of variable sign. Careful examination of the proof of Lemma reveals that nonnegativity of $p(t)$ can be replaced with non-positivity of this function provided that the assumption

$k_2(u,v)$ has the sign opposite to that of v for all $v \in R/\{0\}$ and $u \in R$ is considered in place of condition (vi) for the function $k_2(u,v)$.

4. Examples

Example 1. Consider the nonlinear differential equation
$\left(\frac{1+\sin^2 t}{2+\sin^2 t}\frac{x^4(t)}{1+x^4(t)}x'(t)\frac{1+(x'(t))^2}{2+(x'(t))^2}\right)' + \exp\,(t+\sin t)x^4(t)(x'(t))^2 + x(t)(1+x^2(t)) = 0,$
where $t \geq 0$. $f'(x) = 1+3x^2 \geq 1$. Let $\rho(t) \equiv 1,\ a = 2k\pi,\ b = 2k\pi+\pi,\ u(t) = \sin t$,

$$\int_{2k\pi}^{2k\pi+\pi}\left(\sin^2 t - \frac{1+\sin^2 t}{2+\sin^2 t}\cos^2 t\right)dt$$
$$= \int_{2k\pi}^{2k\pi+\pi}(\sin^2 t - \cos^2 t)dt + \int_{2k\pi}^{2k\pi+\pi}\cos^2 t(\frac{1}{2+sin^2 t})dt$$
$$= \int_{2k\pi}^{2k\pi+\pi}\cos^2 t(\frac{1}{2+sin^2 t})dt > 0.$$

Hence, the equation is oscillatory by Theorem 3.3.

Example 2. Consider the nonlinear differential equation

$$\left((1-\sin t)\frac{x^2(t)}{1+x^2(t)}x'(t)\frac{1+(x'(t))^2}{2+(x'(t))^2}\right)' + (1+\sin^2 t)x^4(t)(x'(t))^2$$
$$+\frac{x(t)(2+x^2(t))}{1+x^2(t)} = 0,$$

where $t \geq 0$. Note that $f'(x)$ is complicated, so Theorem 3.3 can not apply to this equation, so we can apply Theorem 2.3 because $f(x)/x \geq 1$. Thus let

$\rho(t) \equiv 1,\ a = 2k\pi,\ b = 2k\pi+\pi,\ u(t) = \sin t$,

$$\int_{2k\pi}^{2k\pi+\pi}[\sin^2 t - (1-\sin t)\cos^2 t]dt = 2/3 > 0.$$

Hence, the equation is oscillatory by Theorem 2.3.

Remark. We stress that the oscillatory character of this two examples are not deducible from previously known oscillation criteria.

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