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On Interval-Valued Fuzzy Weakly m^*-continuous Mappings on Interval-Valued Fuzzy Minimal Spaces

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Abstract

In this paper, we introduce the concept of IVF weakly m^*-continuous mappings on between IVF minimal spaces and investigate some characterizations for such mappings. Also we study the relationships IVF weakly m^*-continuous mappings and IVF M-compactness.

Key Words : IVF minimal space, IVF weakly m^*-continuous, IVF M-compact, almost IVF M-compact, nearly IVF M-compact

1. Introduction

Zadeh [5] introduced the concept of fuzzy set and investigated basic properties. Gorzalczany [1] introduced the concept of interval-valued fuzzy set which is a generalization of fuzzy sets. In [2], the author introduced and studied IVF minimal structures and IVF minimal spaces as a generalization of interval-valued fuzzy topology introduced by Mondal and Samanta [4]. The author [2] introduced the concepts of interval-valued fuzzy m-continuity and interval-valued fuzzy m-open mappings defined on between IVF minimal spaces. And we studied some characterizations and basic properties of such mappings. In this paper, we introduce the concept of IVF weakly m^*-continuous mappings and study some characterizations. Also we investigate the relationships IVF weakly m^*-continuous mappings and several types of IVF M-compactness.

2. Preliminaries

Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. The elements of $D[0,1]$ are generally denoted by capital letters $M, N, \cdots$ and note that $M = [M^L, M^U]$, where M^L and M^U are the lower and the upper end points respectively. Especially, we denote $\mathbf{0} = [0,0], \mathbf{1} = [1,1]$, and $\mathbf{a} = [a,a]$ for $a \in (0,1)$. We also note that

(1) For all $M, N \in D[0,1]$,

$$M = N \Leftrightarrow M^L = N^L, M^U = N^U.$$

(2) For all $M, N \in D[0,1]$,

$$M \leq N \Leftrightarrow M^L \leq N^L, M^U \leq N^U.$$

For every $M \in D[0,1]$, the complement of M, denoted by M^c, is defined by $M^c = \mathbf{1} - M = [1 - M^U, 1 - M^L]$.

Let X be a nonempty set. A mapping $A : X \to D[0,1]$ is called an interval-valued fuzzy set (simply, IVF set) in X. For each $x \in X$, $A(x)$ is a closed interval whose lower and upper end points are denoted by $A(x)^L$ and $A(x)^U$, respectively. For any $[a,b] \in D[0,1]$, the IVF set whose value is the interval $[a,b]$ for all $x \in X$ is denoted by $\widetilde{[a,b]}$. In particular, for any $a \in [a,b]$, the IVF set whose value is $\mathbf{a} = [a,a]$ for all x X is denoted by simply $\widetilde{a}$. For a point $p \in X$ and for $[a,b] \in D[0,1]$ with $b > 0$, the IVF set which takes the value $[a,b]$ at p and $\mathbf{0}$ elsewhere in X is called an interval-valued fuzzy point (simply, IVF point) and is denoted by $[a,b]_p$. In particular, if $b = a$, then it is also denoted by a_p. Denoted by $IVF(X)$ the set of all IVF sets in X.

For every $A, B \in IVF(X)$, we define

$$A = B \Leftrightarrow (\forall x \in X)([A(x)]^L = [B(x)]^L \text{ and } [A(x)]^U = [B(x)]^U),$$

$$A \subseteq B \Leftrightarrow (\forall x \in X)([A(x)]^L \subseteq [B(x)]^L \text{ and } [A(x)]^U \subseteq [B(x)]^U).$$

The complement A^c of A is defined by, for all $x \in X$,

$$[A^c(x)]^L = 1 - [A(x)]^U \text{ and } [A^c(x)]^U = 1 - [A(x)]^L.$$

For a family of IVF sets $\{A_i : i \in J\}$ where J is an index set, the union $G = \cup_{i \in J} A_i$ and $F = \cap_{i \in J} A_i$ are defined by

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$(\forall x \in X)$ $([G(x)]^L = \sup_{i\in J}[A_i(x)]^L,$
$[G(x)]^U = \sup_{i\in J}[A_i(x)]^U),$
$(\forall x \in X)$ $([F(x)]^L = \inf_{i\in J}[A_i(x)]^L,$
$[F(x)]^U = \inf_{i\in J}[A_i(x)]^U)$, respectively.

Let $f : X \to Y$ be a mapping and let A be an IVF set in X. Then the image of A under f, denoted by $f(A)$, is defined by

$$[f(A)(y)]^L = \begin{cases} \sup_{f(x)=y}[A(x)]^L, & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases}$$

$$[f(A)(y)]^U = \begin{cases} \sup_{f(x)=y}[A(x)]^U, & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases}$$

for all $y \in Y$.

Let B be an IVF set in Y. Then the inverse image of B under f, denoted by $f^{-1}(B)$, is defined by
$([f^{-1}(B)(x)]^L = [B(f(x))]^L$, $[f^{-1}(B)(x)]^U = [B(f(x))]^U)$ for all $x \in X$.

Definition 2.1 ([2]). A family $\mathcal{M}$ of interval-valued fuzzy sets in X is called an *interval-valued fuzzy minimal structure* on X if

$$\mathbf{0}, \mathbf{1} \in \mathcal{M}.$$

In this case, $(X, \mathcal{M})$ is called an *interval-valued fuzzy minimal space* (simply, *IVF minimal space*). Every member of $\mathcal{M}$ is called an IVF m-open set. An IVF set A is called an IVF m-closed set if the complement of A (simply, A^c) is an IVF m-open set.

Let $(X, \mathcal{M})$ be an IVF minimal space and A in IVF(X). The IVF minimal-closure of A [2], denoted by $mCl(A)$, is defined as
$mCl(A) = \cap\{B \in IVF(X) : B^c \in \mathcal{M}$ and $A \subseteq B\}$;
the IVF minimal-interior of A [2], denoted by $mInt(A)$, is defined as
$mInt(A) = \cup\{B \in IVF(X) : B \in \mathcal{M}$ and $B \subseteq A\}$.

Theorem 2.2 ([2]). Let $(X, \mathcal{M})$ be an IVF minimal space and A, B in IVF(X).
(1) $mInt(A) \subseteq A$ and if A is an IVF m-open set, then $mInt(A) = A$.
(2) $A \subseteq mCl(A)$ and if A is an IVF m-closed set, then $mCl(A) = A$.
(3) If $A \subseteq B$, then $mInt(A) \subseteq mInt(B)$ and $mCl(A) \subseteq mCl(B)$.
(4) $mInt(A) \cap mInt(B) \supseteq mInt(A \cap B)$ and $mCl(A) \cup mCl(B) \subseteq mCl(A \cup B)$.
(5) $mInt(mInt(A)) = mInt(A)$ and $mCl(mCl(A)) = mCl(A)$.
(6) $\mathbf{1} - mCl(A) = mInt(\mathbf{1} - A)$ and $\mathbf{1} - mInt(A) = mCl(\mathbf{1} - A)$.

Definition 2.3 ([2]). Let $(X, \mathcal{M}_X)$ be an IVF minimal space and let $(Y, \mathcal{M}_Y)$ be an IVF topological space. Then $f : X \to Y$ is said to be *interval-valued fuzzy m-continuous* (simply, IVF m-continuous) if for every $A \in \mathcal{M}_Y$, $f^{-1}(A)$ is in $\mathcal{M}_X$.

Definition 2.4 ([2]). Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be two IVF minimal spaces. Then $f : X \to Y$ is called *interval-valued fuzzy minimal open* (simply, IVF m-open) map if for every $A \in \mathcal{M}_X$, $f(A)$ is in $\mathcal{M}_Y$.

Theorem 2.5 ([2]). Let $f : X \to Y$ be a function on two IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$.
(1) f is IVF m-open.
(2) $f(mInt(A)) \subseteq mInt(f(A))$ for $A \in IVF(X)$.
(3) $mInt(f^{-1}(B)) \subseteq f^{-1}(mInt(B))$ for $B \in IVF(Y)$.
Then (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

3. IVF Weakly m^*-Continuous Mappings

Definition 3.1. Let $f : X \to Y$ be a mapping between IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then f is said to be *IVF weakly m^*-continuous* if for IVF point M_x in X and each IVF m-open set V containing $f(M_x)$, there is an IVF m-open set U containing M_x such that $f(U) \subseteq mCl(V)$.

Remark 3.2. Let $f : X \to Y$ be a mapping between IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then every IVF m-continuous mapping f is clearly IVF weakly m^*-continuous but the converse is not always true as shown in the next example.

Example 3.3. Let $X = \{a, b\}$. Let A, B and C be IVF sets defined as follows.

$$A(a) = [0.7, 0.9], A(b) = [0.3, 0.7],$$
$$B(a) = [0.6, 0.8], B(b) = [0.5, 0.6],$$
$$C(a) = [0.7, 0.9], C(b) = [0.5, 0.7].$$

Note $C = A \cup B$. Consider an IVF m-structure $\mathcal{M}_1 = \{\mathbf{0}, A, B, \mathbf{1}\}$ and an IVF topological space $\mathcal{M}_2 = \{\mathbf{0}, A, B, C, \mathbf{1}\}$. Let $f : (X, \mathcal{M}_1) \to (X, \mathcal{M}_2)$ be a function defined by $f(x) = x$ for each $x \in X$. Then f is IVF weakly m^*-continuous, but it is not IVF m-continuous because C is not in $\mathcal{M}_1$.

Theorem 3.4. Let $f : X \to Y$ be a mapping between IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following statements are equivalent:
(1) f is IVF weakly m^*-continuous.
(2) $f^{-1}(V) \subseteq mInt(f^{-1}(mCl(V)))$ for each IVF m-open set V in Y.
(3) $mCl(f^{-1}(mInt(B))) \subseteq f^{-1}(B)$ for each IVF m-closed set B in Y.
(4) $mCl(f^{-1}(V)) \subseteq f^{-1}(mCl(V))$ for each IVF m-open set V in Y.

Proof. (1) $\Rightarrow$ (2) Let V be an IVF m-open set in Y and IVF point $M_x \in f^{-1}(V)$. There exists an IVF m-open set U containing M_x such that $f(U) \subseteq mCl(V)$. From $M_x \in U \subseteq f^{-1}(mCl(V))$ it follows $M_x \in mInt(f^{-1}(mCl(V)))$. Hence $f^{-1}(V) \subseteq mInt(f^{-1}(mCl(V)))$.

(2) $\Rightarrow$ (3) Let B be an IVF m-closed set in Y. Then by (2) and Theorem 2.2,

$$\begin{aligned} f^{-1}(\mathbf{1}-B) &\subseteq mInt(f^{-1}(mCl(\mathbf{1}-B))) \\ &= mInt(f^{-1}(\mathbf{1}-mInt(B))) \\ &= mInt(\mathbf{1}-f^{-1}(mInt(B))) \\ &= \mathbf{1}-mCl(f^{-1}(mInt(B))). \end{aligned}$$

Thus $mCl(f^{-1}(mInt(B))) \subseteq f^{-1}(B)$. Similarly, we can prove that (3) $\Rightarrow$ (2).

(3) $\Rightarrow$ (4) Let V be IVF m-open Y. Suppose $M_x \notin f^{-1}(mCl(V))$. Then $f(M_x) \notin mCl(V)$ and so there exists an IVF m-open set U containing $f(M_x)$ such that $U \cap V = \emptyset$. It follows $mCl(U) \cap V = \emptyset$. By (2), $M_x \in f^{-1}(U) \subseteq mInt(f^{-1}(mCl(U)))$. Hence there exists an IVF m-open set G containing M_x such that $M_x \in G \subseteq f^{-1}(mCl(U))$. From $mCl(U) \cap V = \emptyset$ and $f(G) \subseteq mCl(U)$, it follows $G \cap f^{-1}(V) = \emptyset$. Hence $M_x \notin mCl(f^{-1}(V))$.

(4) $\Rightarrow$ (1) Let M_x be an IVF point in X and V an IVF m-open set in Y containing $f(M_x)$. Since $V = mInt(V) \subseteq mInt(mCl(V))$, by (4) and Theorem 2.2,

$$\begin{aligned} M_x \in f^{-1}(V) &\subseteq f^{-1}(mInt(mCl(V))) \\ &= \mathbf{1}-f^{-1}(mCl(\mathbf{1}-mCl(V))) \\ &\subseteq \mathbf{1}-mCl(f^{-1}(\mathbf{1}-mCl(V))) \\ &= mInt(f^{-1}(mCl(V))). \end{aligned}$$

It implies that there exists an IVF m-open U containing M_x such that $U \subseteq f^{-1}(mCl(V))$. Hence f is IVF weakly m^*-continuous. □

Definition 3.5 ([3]). Let $\mathcal{M}_X$ be an IVF minimal structure on X. Then $\mathcal{M}_X$ said to have property $(\mathcal{B})$ if the union of any family of IVF sets belong to $\mathcal{M}_X$ belongs to $\mathcal{M}_X$.

Lemma 3.6 ([3]). Let $\mathcal{M}_X$ be an IVF minimal structure on X. Then the following are equivalent.
(1) $\mathcal{M}_X$ has property $(\mathcal{B})$.
(2) If $mInt(B) = B$, then $B \in \mathcal{M}_X$.
(3) If $mCl(F) = F$, then $\mathbf{1} - F \in \mathcal{M}_X$.

From Lemma 3.6, we get the following:

Corollary 3.7. Let $f : X \to Y$ be a mapping between IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If $\mathcal{M}_X$ have property $(\mathcal{B})$, then the following statements are equivalent:
(1) f is IVF weakly m^*-continuous.
(2) $mCl(f^{-1}(mInt(F))) \subseteq f^{-1}(F)$ for each IVF m-closed set F in Y.
(3) $mCl(f^{-1}(mInt(mCl(B)))) \subseteq f^{-1}(mCl(B))$ for each $B \in IVF(Y)$.
(4) $f^{-1}(mInt(B)) \subseteq mInt(f^{-1}(mCl(mInt(B))))$ for each $B \in IVF(Y)$.
(5) $mCl(f^{-1}(V)) \subseteq f^{-1}(mCl(V))$ for an IVF m-open set V in Y.

Theorem 3.8. Let $f : X \to Y$ be a mapping between IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If f is IVF weakly m^*-continuous, then $f^{-1}(A) \subseteq mInt(f^{-1}(mCl(A)))$ for $A = mInt(A)$ in Y.

Proof. Let A be an IVF set in Y such that $A = mInt(A)$ and $M_x \in f^{-1}(A)$. Since $f(M_x) \in mInt(A)$, there exists an IVF m-open set V containing $f(M_x)$. From definition of IVF weakly m^*-continuity, there exists an IVF m-open set U containing M_x such that $f(U) \subseteq mCl(V)$, that is, $U \subseteq f^{-1}(mCl(V))$. So that we have $M_x \in mInt(f^{-1}(mCl(V))) \subseteq mInt(f^{-1}(mCl(A)))$. Hence we have $f^{-1}(A) \subseteq mInt(f^{-1}(mCl(A)))$. □

From Lemma 3.6 and Corollary 3.7, the following corollary is obtained:

Corollary 3.9. Let $f : X \to Y$ be a mapping between IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ and $\mathcal{M}_X$ have property $(\mathcal{B})$. Then f is IVF weakly m^*-continuous if and only if $f^{-1}(A) \subseteq mInt(f^{-1}(mCl(A)))$ for $A = mInt(A)$ in Y.

Definition 3.10. Let $(X, \mathcal{M}_X)$ be an IVF minimal space. A family of $\mathcal{C}$ is said to be an IVF cover of X if $\mathbf{1} = \cup_{A \in \mathcal{C}} A$. An IVF cover $\mathcal{C}$ of X is called an IVF M-cover of X if for each $A \in \mathcal{C}$, $A = mInt(A)$.

An IVF set A in X is said to be *IVF M-compact* if every IVF M-cover $\mathcal{A} = \{A_i : i \in J\}$ of A has a finite subcover. And an IVF set A in X is said to be *almost IVF M-compact* (resp., *nearly IVF M-compact*) if for every IVF M-cover $\mathcal{A} = \{A_i : i \in J\}$ of A, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} mCl(A_i)$ (resp., $A \subseteq \cup_{i \in J_0} mInt(mCl(A_i))$).

Remark 3.11. In $(X, \mathcal{M}_X)$ be an IVF minimal space, we have the following implications but the converses are not always true as in the next example.

IVF M-compact $\Rightarrow$ nearly IVF M-compact $\Rightarrow$ almost IVF M-compact

Example 3.12. Let $X = I$. Consider each IVF fuzzy set for $0 < n < 1$,

$$\sigma_n(x) = \begin{cases} [\frac{1}{n}x, \frac{1}{2n}x + \frac{1}{2}], & \text{if } 0 \le x \le n \\ [-\frac{x-1}{1-n}, -\frac{x-1}{2(1-n)} + \frac{1}{2}], & \text{if } n < x \le 1, \end{cases}$$

$$\alpha(x) = \begin{cases} [0, 0], & \text{if } 0 \le x < 1 \\ [1, 1], & \text{if } x = 1, \end{cases}$$

$$\beta(x) = \begin{cases} [1,1], & \text{if } 0 \le x < 1 \\ [0,0], & \text{if } x = 1. \end{cases}$$

Consider an IVF minimal structure

$$\mathcal{M}_1 = \{\mathbf{0}, \mathbf{1}, \alpha\} \cup \{\sigma_n : 0 < n < 1\}$$

and an IVF fuzzy set δ defined as follows.

$$\delta(x) = \begin{cases} [1,1], & \text{if } 0 < x < 1 \\ [0,0], & \text{if } x = 0, 1, \end{cases}$$

Let $\mathcal{C} = \{\sigma_n : 0 < n < 1\}$ be an IVF M-cover of δ. Then since β is an IVF m-closed set and $\sigma_n \subseteq \beta$ for $0 < n < 1$, the IVF fuzzy set δ is almost IVF M-compact but not nearly IVF M-compact.

Consider an IVF minimal structure

$$\mathcal{M}_2 = \{\mathbf{0}, \mathbf{1}, \alpha, \beta\} \cup \{\sigma_n : 0 < n < 1\}.$$

Then the IVF fuzzy set β is nearly IVF M-compact but not IVF M-compact.

Theorem 3.13. Let $f : X \to Y$ be an IVF weakly m^*-continuous mapping between IVF minimal spaces $(X, \mathcal{M}_X)$ and $(X, \mathcal{M}_Y)$. If A is an IVF M-compact set, then $f(A)$ is an almost IVF compact set.

Proof. Let $\{B_i \in IVF(Y) : i \in J\}$ be an IVF M-open cover of $f(A)$ in Y. Then by Theorem 3.4 (4), we have $f^{-1}(B_i) \subseteq mInt(f^{-1}(mCl(B_i)))$ for each $i \in J$. Thus $\{mInt(f^{-1}(mCl(B_i))) : i \in J\}$ is an IVF M-cover of A in X. By definition of IVF M-compactness, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} mInt(f^{-1}(mCl(B_i))) \subseteq f^{-1}(mCl(B_i))$. Hence $f(A) \subseteq \cup_{i \in J_0} mCl(B_i)$. □

Lemma 3.14. Let $f : X \to Y$ be an IVF weakly m^*-continuous mapping between IVF minimal spaces $(X, \mathcal{M}_X)$ and $(X, \mathcal{M}_Y)$. Then for $B = mCl(B)$ in Y, $mCl(f^{-1}(mInt(B))) \subseteq f^{-1}(B)$.

Proof. From Theorem 2.2 and Theorem 3.8, it is easily obtained. □

Theorem 3.15. Let $f : X \to Y$ be an IVF weakly m^*-continuous and IVF m-open mapping between an IVF minimal space $(X, \mathcal{M})$ and an IVF topological space (Y, τ). If A is an almost IVF M-compact set, then $f(A)$ is an almost IVF compact set.

Proof. Let $\{B_i \in IVF(Y) : i \in J\}$ be an IVF open cover of $f(A)$ in Y. Then by Theorem 3.4 (4), $\{mInt(f^{-1}(mCl(B_i))) : i \in J\}$ is an IVF M-cover of A in X. By definition of almost IVF M-compactness, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} mCl(mInt(f^{-1}(mCl(B_i))))$. Since $mCl(mCl(B_i)) = mCl(B_i)$, from Theorem 2.5 and Lemma 3.14, it follows

$$\begin{aligned} &\cup_{i \in J_0} mCl(mInt(f^{-1}(mCl(B_i)))) \\ &\subseteq \cup_{i \in J_0} mCl(f^{-1}(mInt(mCl(B_i)))) \\ &\subseteq \cup_{i \in J_0} f^{-1}(mCl(B_i)). \end{aligned}$$

Hence $f(A) \subseteq \cup_{i \in J_0} mCl(B_i)$. □

Theorem 3.16. Let $f : X \to Y$ be an IVF weakly m^*-continuous and IVF m-open mapping between an IVF minimal space $(X, \mathcal{M})$ and an IVF topological space (Y, τ). If A is a nearly IVF M-compact set, then $f(A)$ is a nearly IVF compact set.

Proof. Let $\{B_i \in IVF(Y) : i \in J\}$ be an IVF open cover of $f(A)$ in Y. Then $\{mInt(f^{-1}(mCl(B_i))) : i \in J\}$ is an IVF M-cover of A in X. By definition of nearly IVF M-compactness, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} mInt(mCl(mInt(f^{-1}(mCl(B_i)))))$. Since $mCl(mCl(B_i)) = mCl(B_i)$, from Theorem 2.5 and Lemma 3.14, it follows for $i \in J_0$,

$$\begin{aligned} &mInt(mCl(mInt(f^{-1}(mCl(B_i))))) \\ &\subseteq mInt(mCl(f^{-1}(mInt(mCl(B_i))))) \\ &\subseteq mInt(f^{-1}(mCl(B_i))) \\ &\subseteq f^{-1}(mInt(mCl(B_i))). \end{aligned}$$

Hence $f(A) \subseteq \cup_{i \in J_0} mInt(mCl(B_i))$. □

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