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On Interval-Valued Fuzzy Weakly M-continuous Mappings

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Abstract

In this paper, we introduce the concept of IVF weakly M-continuity and investigate some characterizations for IVF weakly M-continuous mappings between an IVF minimal space and an IVF topological space.

Key words : IVF minimal space, IVF weakly M-continuous, M^*-open mappings, IVF M-compact, almost IVF M-compact, nearly IVF M-compact

1. Introduction

Zadeh [7] introduced the concept of fuzzy set and investigated basic properties. Gorzalczany [1] introduced the concept of interval-valued fuzzy set which is a generalization of fuzzy sets. In [3], the author introduced and studied an IVF minimal structure as a generalization of interval-valued fuzzy topology introduced by Mondal and Samanta [6]. The author and Kim [5] introduced the concepts of interval-valued fuzzy M-continuity and interval-valued fuzzy M^*-open mappings defined between an IVF minimal space and an IVF topological space. And we studied some characterizations and basic properties of such mappings. In this paper, we introduce the concept of IVF weakly M-continuous mappings and study some characterizations.

2. Preliminaries

Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. The elements of $D[0,1]$ are generally denoted by capital letters $M, N, \cdots$ and note that $M = [M^L, M^U]$, where M^L and M^U are the lower and the upper end points, respectively. Especially, we denote $\mathbf{0} = [0,0], \mathbf{1} = [1,1]$, and $\mathbf{a} = [a,a]$ for $a \in (0,1)$. We also note that

(1) For all $M, N \in D[0,1]$,

$$M = N \Leftrightarrow M^L = N^L, M^U = N^U.$$

(2) For all $M, N \in D[0,1]$,

$$M \le N \Leftrightarrow M^L \le N^L, M^U \le N^U.$$

For every $M \in D[0,1]$, the complement of M, denoted by M^c, is defined by $M^c = \mathbf{1} - M = [1 - M^U, 1 - M^L]$.

Let X be a nonempty set. A mapping $A : X \to D[0,1]$ is called an interval-valued fuzzy set (simply, IVF set) in X. For each $x \in X$, $A(x)$ is a closed interval whose lower and upper end points are denoted by $A(x)^L$ and $A(x)^U$, respectively. For any $[a,b] \in D[0,1]$, the IVF set whose value is the interval $[a,b]$ for all $x \in X$ is denoted by $\widetilde{[a,b]}$. In particular, for any $a \in [a,b]$, the IVF set whose value is $\mathbf{a} = [a,a]$ for all x X is denoted by simply $\widetilde{a}$. For a point $p \in X$ and for $[a,b] \in D[0,1]$ with $b > 0$, the IVF set which takes the value $[a,b]$ at p and $\mathbf{0}$ elsewhere in X is called an interval-valued fuzzy point (simply, IVF point) and is denoted by $[a,b]_p$. In particular, if $b = a$, then it is also denoted by a_p. We denote the set of all IVF sets in X by $IVF(X)$.

For every $A, B \in IVF(X)$, we define

$$A = B \Leftrightarrow (\forall x \in X)([A(x)]^L = [B(x)]^L \text{ and } [A(x)]^U = [B(x)]^U),$$

$$A \subseteq B \Leftrightarrow (\forall x \in X)([A(x)]^L \subseteq [B(x)]^L \text{ and } [A(x)]^U \subseteq [B(x)]^U).$$

The complement A^c of A is defined by, for all $x \in X$,

$$[A^c(x)]^L = 1 - [A(x)]^U \text{ and } [A^c(x)]^U = 1 - [A(x)]^L.$$

For a family of IVF sets $\{A_i : i \in J\}$ where J is an index set, the union $G = \cup_{i \in J} A_i$ and $F = \cap_{i \in J} A_i$ are defined by

$$(\forall x \in X)\ ([G(x)]^L = \sup_{i \in J}[A_i(x)]^L, [G(x)]^U = \sup_{i \in J}[A_i(x)]^U),$$
$$(\forall x \in X)\ ([F(x)]^L = \inf_{i \in J}[A_i(x)]^L, [F(x)]^U = \inf_{i \in J}[A_i(x)]^U),$$

respectively.

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Let $f : X \to Y$ be a mapping and let A be an IVF set in X. Then the image of A under f, denoted by $f(A)$, is defined by

$$[f(A)(y)]^L = \begin{cases} \sup_{f(x)=y}[A(x)]^L, & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise ,} \end{cases}$$

$$[f(A)(y)]^U = \begin{cases} \sup_{f(x)=y}[A(x)]^U, & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise ,} \end{cases}$$

for all $y \in Y$.

Let B be an IVF set in Y. Then the inverse image of B under f, denoted by $f^{-1}(B)$, is defined by

$([f^{-1}(B)(x)]^L = [B(f(x))]^L$, $[f^{-1}(B)(x)]^U = [B(f(x))]^U)$ for all $x \in X$.

Definition 2.1 ([6]). A family τ of IVF sets in X is called an *interval-valued fuzzy topology* on X if it satisfies:

(1) $\mathbf{0}, \mathbf{1} \in \tau$.

(2) $A, B \in \tau \Rightarrow A \cap B \in \tau$.

(3) For $i \in J$, $A_i \in \tau \Rightarrow \cup_{i \in J} A_i \in \tau$.

Every member of τ is called an IVF open set. An IVF set A is called an IVF closed set if the complement of A is an IVF open set. And (X, τ) is called an *interval-valued fuzzy topological space*.

In an IVF topological space (X, τ), for an IVF set A in X, the IVF closure and the IVF interior of A [6], denoted by $cl(A)$ and $int(A)$, respectively, are defined as

$cl(A) = \cap\{B \in IVF(X) : B^c \in \tau$ and $A \subseteq B\}$,

$int(A) = \cup\{B \in IVF(X) : B \in \tau$ and $B \subseteq A\}$.

Theorem 2.2 ([6]). Let A be an IVF set in an IVF topological space (X, τ). Then $\mathbf{1} - cl(\mathbf{1} - A) = int(A)$

An IVF set A in an IVF topological space X is said to be *IVF compact* [6] if every IVF open cover $\mathcal{A} = \{A_i : i \in J\}$ of A has a finite IVF subcover. And an IVF set A in X is said to be *almost IVF compact* (resp., *nearly IVF compact*) [4] if for every IVF open cover $\mathcal{A} = \{A_i : i \in J\}$ of A, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} cl(A_i)$ (resp., $A \subseteq \cup_{i \in J_0} int(cl(A_i))$).

Definition 2.3 ([3]). A family $\mathcal{M}$ of interval-valued fuzzy sets in X is called an *interval-valued fuzzy minimal structure* on X if

$$\mathbf{0}, \mathbf{1} \in \mathcal{M}.$$

In this case, $(X, \mathcal{M})$ is called an *interval-valued fuzzy minimal space* (simply, *IVF minimal space*). Every member of $\mathcal{M}$ is called an IVF m-open set. An IVF set A is called an IVF m-closed set if the complement of A (simply, A^c) is an IVF m-open set.

Let $(X, \mathcal{M})$ be an IVF minimal space and A in IVF(X). The IVF minimal-closure of A [3], denoted by $mCl(A)$, is defined as

$mCl(A) = \cap\{B \in IVF(X) : B^c \in \mathcal{M}$ and $A \subseteq B\}$;

the IVF minimal-interior of A [2], denoted by $mInt(A)$, is defined as

$mInt(A) = \cup\{B \in IVF(X) : B \in \mathcal{M}$ and $B \subseteq A\}$.

Theorem 2.4 ([3]). Let $(X, \mathcal{M})$ be an IVF minimal space and A, B in IVF(X).

(1) $mInt(A) \subseteq A$ and if A is an IVF m-open set, then $mInt(A) = A$.

(2) $A \subseteq mCl(A)$ and if A is an IVF m-closed set, then $mCl(A) = A$.

(3) If $A \subseteq B$, then $mInt(A) \subseteq mInt(B)$ and $mCl(A) \subseteq mCl(B)$.

(4) $mInt(A) \cap mInt(B) \supseteq mInt(A \cap B)$ and $mCl(A) \cup mCl(B) \subseteq mCl(A \cup B)$.

(5) $mInt(mInt(A)) = mInt(A)$ and $mCl(mCl(A)) = mCl(A)$.

(6) $\mathbf{1} - mCl(A) = mInt(\mathbf{1} - A)$ and $\mathbf{1} - mInt(A) = mCl(\mathbf{1} - A)$.

Definition 2.5 ([5]). Let $(X, \mathcal{M}_X)$ be an IVF minimal space and let (Y, τ) be an IVF topological space. Then $f : X \to Y$ is said to be *interval-valued fuzzy M-continuous* (simply, IVF M-continuous) if for every $A \in \tau$, $f^{-1}(A)$ is in $\mathcal{M}_X$.

Definition 2.6 ([5]). Let $(X, \mathcal{M}_X)$ be an IVF minimal space and (Y, τ) be an IVF topological space. Then $f : X \to Y$ is called an *interval-valued fuzzy M^*-open* (simply, IVF M^*-open) mapping if for every IVF m-open set A in X, $f(A)$ is IVF open in Y.

Theorem 2.7 ([5]). Let $f : X \to Y$ be a mapping on an IVF minimal space $(X, \mathcal{M}_X)$ and an IVF topological space (Y, τ). Then the following are equivalent:

(1) f is IVF M^*-open.

(2) $f(mInt(A)) \subseteq Int(f(A))$ for $A \in IVF(X)$.

(3) $mInt(f^{-1}(B)) \subseteq f^{-1}(Int(B))$ for $B \in IVF(Y)$.

3. IVF Weakly M-continuous Mappings

Definition 3.1. Let $f : X \to Y$ be a mapping between an IVF minimal space $(X, \mathcal{M})$ and an IVF topological space (Y, τ). Then f is said to be IVF weakly M-continuous if for every IVF point M_x and each IVF open set V of $f(M_x)$, there exists IVF m-open set U of M_x such that $f(U) \subseteq cl(V)$.

Remark 3.2. Let $f : X \to Y$ be a mapping between an IVF minimal space $(X, \mathcal{M})$ and an IVF topological space (Y, τ). Then every IVF M-continuous mapping f is clearly IVF weakly M-continuous but the converse is not always true as seen by the next example.

Example 3.3. Let $X = \{a, b\}$. Let A, B and C be IVF sets defined as follows

$$A(a) = [0.8, 0.9], A(b) = [0.5, 0.7],$$

$$B(a) = [0.7, 0.9], B(b) = [0.6, 0.7],$$

$$C(a) = [0.8, 0.9], C(b) = [0.6, 0.7].$$

Let us consider an IVF m-structure $\mathcal{M}_X = \{\mathbf{0}, A, B, \mathbf{1}\}$ and an IVF topological space $\tau = \{\mathbf{0}, A, B, A \cap B, C, \mathbf{1}\}$. Let $f : (X, \mathcal{M}_X) \to (X, \tau)$ be a function defined as follows $f(x) = x$ for each $x \in X$. Then f is IVF weakly M-continuous but it is not IVF M-continuous.

Theorem 3.4. Let $f : X \to Y$ be a mapping between an IVF minimal space $(X, \mathcal{M})$ and an IVF topological space (Y, τ). Then the following statements are equivalent:

(1) f is IVF weakly M-continuous.

(2) $f^{-1}(B) \subseteq mInt(f^{-1}(cl(B)))$ for each IVF open set B of Y.

(3) $mCl(f^{-1}(int(F))) \subseteq f^{-1}(F)$ for each IVF closed set F in Y.

(4) $mCl(f^{-1}(int(cl(B)))) \subseteq f^{-1}(cl(B))$ for each $B \in IVF(Y)$.

(5) $f^{-1}(int(B)) \subseteq mInt(f^{-1}(cl(int(B))))$ for each $B \in IVF(Y)$.

(6) $mCl(f^{-1}(V)) \subseteq f^{-1}(cl(V))$ for an IVF open set V in Y.

Proof. (1) $\Rightarrow$ (2) Let B be an IVF open set in Y. Since f is IVF weakly M-continuous, for each $M_x \in f^{-1}(B)$, there exists an IVF m-open set U_{M_x} of M_x such that $f(U_{M_x}) \subseteq cl(B)$. Now we can say for each $M_x \in f^{-1}(B)$, there exists an IVF m-open set U_{M_x} such that

$$M_x \in U_{M_x} \subseteq f^{-1}(f(U_{M_x})) \subseteq f^{-1}(cl(B)).$$

This implies $M_x \in mInt(f^{-1}(cl(B)))$. Hence $f^{-1}(B) \subseteq mInt(f^{-1}(cl(B)))$.

(2) $\Rightarrow$ (1) Let M_x be an IVF point in X and V an IVF open set containing $f(M_x)$. Then since $M_x \in f^{-1}(V) \subseteq mInt(f^{-1}(cl(V)))$, there exists an IVF m-open set U containing M_x such that $M_x \in U \subseteq f^{-1}(cl(V))$. This implies $f(U) \subseteq f(f^{-1}(cl(V))) \subseteq cl(V)$. Hence f is IVF weakly M-continuous.

(1) $\Rightarrow$ (3) Let F be any IVF closed set of Y. Then $\mathbf{1} - F$ is an IVF open set in Y, from Theorem 2.2 and Theorem 2.4, it follows

$$\begin{aligned} f^{-1}(\mathbf{1} - F) &\subseteq mInt(f^{-1}(cl(\mathbf{1} - F))) \\ &= mInt(f^{-1}(\mathbf{1} - int(F))) \\ &= mInt(\mathbf{1} - f^{-1}(int(F))) \\ &= \mathbf{1} - mCl(f^{-1}(int(F))). \end{aligned}$$

Hence we have $mCl(f^{-1}(int(F))) \subseteq f^{-1}(F)$.

(3) $\Rightarrow$ (4) Let B be any IVF set in Y. Since $cl(B)$ is an IVF closed set in Y, by (3),

$$mCl(f^{-1}(int(cl(B)))) \subseteq f^{-1}(cl(B)).$$

(4) $\Rightarrow$ (5) Let B be any IVF set of Y. Then,

$$\begin{aligned} f^{-1}(int(B)) &= \mathbf{1} - (f^{-1}(cl(\mathbf{1} - B))) \\ &\subseteq \mathbf{1} - mCl(f^{-1}(int(cl(\mathbf{1} - B)))) \\ &= mInt(f^{-1}(cl(int(B)))). \end{aligned}$$

Hence,

$$f^{-1}(int(B)) \subseteq mInt(f^{-1}(cl(int(B)))).$$

(5) $\Rightarrow$ (6) Let V be any IVF open set of Y. Then by (5),

$$\begin{aligned} \mathbf{1} - f^{-1}(cl(V)) &= f^{-1}(int(\mathbf{1} - V)) \\ &\subseteq mInt(f^{-1}(cl(int(\mathbf{1} - V)))) \\ &= mInt(\mathbf{1} - (f^{-1}(int(cl(V))))) \\ &= \mathbf{1} - mCl(f^{-1}(int(cl(V)))) \\ &\subseteq \mathbf{1} - mCl(f^{-1}(V)). \end{aligned}$$

Hence we have

$$mCl(f^{-1}(V)) \subseteq f^{-1}(cl(V)).$$

(6) $\Rightarrow$ (1) Let V be an IVF open set containing $f(M_x)$. By (6),

$$\begin{aligned} M_x \in f^{-1}(V) &\subseteq f^{-1}(int(cl(V))) \\ &= \mathbf{1} - f^{-1}(cl(\mathbf{1} - cl(V))) \\ &\subseteq \mathbf{1} - mCl(f^{-1}(\mathbf{1} - cl(V))) \\ &= mInt(f^{-1}(cl(V))). \end{aligned}$$

It implies $M_x \in mInt(f^{-1}(cl(V)))$. Thus there exists an IVF m-open set U such that $M_x \in U \subseteq f^{-1}(cl(V))$. Hence $f(U) \subseteq cl(V)$.

□

Definition 3.5. Let A be an IVF set in an IVF topological space (X, τ). Then A is said to be

(1) *IVF semiopen* [2] if there is an IVF -open set B in X such that $B \subseteq A \subseteq cl(B)$,

(2) *IVF preopen* [2] if $A \subseteq int(cl(A))$,

(3) *IVF regular open* (resp., *IVF regular closed*) [4] if $A = int(cl(A))$ (resp., $A = cl(int(A))$),

(4) *IVF β-open* [4] if $A \subseteq cl(int(cl(A)))$.

Theorem 3.6. Let $f : X \to Y$ be a mapping between an IVF minimal space $(X, \mathcal{M})$ and an IVF topological space (Y, τ). Then the following statements are equivalent:

(1) f is IVF weakly M-continuous.

(2) $mC(f^{-1}(int(cl(G)))) \subseteq f^{-1}(cl(G))$ for each IVF open set G in Y.

(3) $mCl(f^{-1}(int(cl(V)))) \subseteq f^{-1}(cl(V))$ for each IVF preopen set V in Y.

(4) $mCl(f^{-1}(int(K))) \subseteq f^{-1}(K)$ for each IVF regular closed set K in Y.

(5) $mCl(f^{-1}(int(cl(G)))) \subseteq f^{-1}(cl(G))$ for each IVF β-open set G in Y.

(6) $mCl(f^{-1}(int(cl(G)))) \subseteq f^{-1}(cl(G))$ for each IVF semiopen set G in Y.

Proof. (1) $\Rightarrow$ (2) Let G be an IVF open set in Y. Then by Theorem 3.4 (3), we have $mCl(f^{-1}(int(cl(G)))) \subseteq f^{-1}(cl(G))$.

(2) $\Rightarrow$ (3) Let V be an IVF preopen of Y. Then $V \subseteq int(cl(V))$. Put $A = int(cl(V))$. Since A is an IVF open set, from (2), it follows

$$mCl(f^{-1}(int(cl(A)))) \subseteq f^{-1}(cl(A)).$$

From $cl(A) = cl(V)$, it follows

$$mCl(f^{-1}(int(cl(V)))) \subseteq f^{-1}(cl(V)).$$

(3) $\Rightarrow$ (4) Let K be an IVF regular closed set of Y. Then since $int(K)$ is an IVF preopen set, by (3),

$$mCl(f^{-1}(int(cl(int(K))))) \subseteq f^{-1}(cl(int(K))).$$

From $int(K) = int(cl(int(K)))$ and IVF regular closedness of K, we have

$$mCl(f^{-1}(int(K))) \subseteq f^{-1}(K).$$

(4) $\Rightarrow$ (5) Let G be an IVF β-open set. Then $G \subseteq cl(int(cl(G)))$ and $cl(G) = cl(int(cl(G)))$, and so $cl(G)$ is an IVF regular closed set. Hence by (4), we have

$$mCl(f^{-1}(int(cl(G)))) \subseteq f^{-1}(cl(G)).$$

(5) $\Rightarrow$ (6) It is obvious.

(6) $\Rightarrow$ (1) Let V be an IVF open set; then since V is an IVF semiopen set, by (6) and $V \subseteq int(cl(V))$, we have

$$mCl(f^{-1}(V)) \subseteq mCl(f^{-1}(int(cl(V)))) \subseteq f^{-1}(cl(V)).$$

Hence, by Theorem 3.4 (6), we find that f is IVF weakly M-continuous. □

Definition 3.7. Let $(X, \mathcal{M}_X)$ be an IVF minimal space. A family of $\mathcal{C}$ is said to be an IVF cover of X if $\mathbf{1} = \cup_{A\in\mathcal{C}} A$. An IVF cover $\mathcal{C}$ of X is called an IVF M-cover of X if for each $A \in \mathcal{C}$, $A = mInt(A)$.

An IVF set A in X is said to be *IVF M-compact* if every IVF M-cover $\mathcal{A} = \{A_i : i \in J\}$ of A has a finite subcover. And an IVF set A in X is said to be *almost IVF M-compact* (resp., *nearly IVF M-compact*) if for every IVF M-cover $\mathcal{A} = \{A_i : i \in J\}$ of A, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i\in J_0} mCl(A_i)$ (resp., $A \subseteq \cup_{i\in J_0} mInt(mCl(A_i))$).

Example 3.8. Let $X = \{a, b\}$. For each $n \in N$ let A_n be an IVF set defined as follows

$$A_n(a) = [\frac{n}{1+n}, 1], A_n(b) = [1, 1].$$

Consider $\tau = \{\mathbf{0}, A_n, \mathbf{1}\}$ as an IVF minimal space on X. Let $\mathcal{C} = \{A_n : n \in N\}$ be an IVF M-cover of X. Then there does not exist a finite subcover of $\mathcal{C}$. Thus X is not IVF M-compact but it is almost IVF M-compact.

Theorem 3.9. Let $f : X \to Y$ be an IVF weakly M-continuous mapping between an IVF minimal space $(X, \mathcal{M})$ and an IVF topological space (Y, τ). If A is an IVF M-compact set, then $f(A)$ is an almost IVF compact set.

Proof. Let $\{B_i \in IVF(Y) : i \in J\}$ be an IVF open cover of $f(A)$ in Y. Then by Theorem 3.4 (2), $\{mInt(f^{-1}(cl(B_i))) : i \in J\}$ is an IVF M-cover of A in X. By definition of IVF M-compactness, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i\in J_0} mInt(f^{-1}(cl(B_i))) \subseteq f^{-1}(cl(B_i))$. Hence $f(A) \subseteq \cup_{i\in J_0} cl(B_i)$. □

Theorem 3.10. Let $f : X \to Y$ be an IVF weakly M-continuous and IVF M^*-open mapping between an IVF minimal space $(X, \mathcal{M})$ and an IVF topological space (Y, τ). If A is an almost IVF M-compact set, then $f(A)$ is an almost IVF compact set.

Proof. Let $\{B_i \in IVF(Y) : i \in J\}$ be an IVF open cover of $f(A)$ in Y. Then by Theorem 3.4 (2), $\{mInt(f^{-1}(cl(B_i))) : i \in J\}$ is an IVF M-cover of A in X. By definition of almost IVF M-compactness, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i\in J_0} mCl(mInt(f^{-1}(cl(B_i))))$. Since $int(cl(B_i))$ is IVF open in in Y, from Theorem 2.7 and Theorem 3.4, it follows

$$\begin{aligned}\cup_{i\in J_0} \quad & mCl(mInt(f^{-1}(cl(B_i)))) \\ & \subseteq \cup_{i\in J_0} mCl(f^{-1}(int(cl(B_i)))) \\ & \subseteq \cup_{i\in J_0} f^{-1}(cl(int(cl(B_i)))) \\ & \subseteq \cup_{i\in J_0} f^{-1}(cl(B_i)).\end{aligned}$$

Hence $f(A) \subseteq \cup_{i\in J_0} cl(B_i)$. □

Theorem 3.11. Let $f : X \to Y$ be an IVF weakly M-continuous and IVF M^*-open mapping between an IVF minimal space $(X, \mathcal{M})$ and an IVF topological space (Y, τ). If A is a nearly IVF M-compact set, then $f(A)$ is a nearly IVF compact set.

Proof. Let $\{B_i \in IVF(Y) : i \in J\}$ be an IVF open cover of $f(A)$ in Y. Then $\{mInt(f^{-1}(cl(B_i))) : i \in J\}$ is an IVF M-cover of A in X. By definition of nearly IVF M-compactness, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} mInt(mCl(mInt(f^{-1}(cl(B_i)))))$. Since $int(cl(B_i))$ is IVF open, from Theorem 2.7 and Theorem 3.4, it follows

$$\begin{aligned}\cup_{i \in J_0} \quad & mInt(mCl(mInt(f^{-1}(cl(B_i)))))\\ & \subseteq \cup_{i \in J_0} mInt(mCl(f^{-1}(int(cl(B_i)))))\\ & \subseteq \cup_{i \in J_0} mInt(f^{-1}(cl(int(cl(B_i)))))\\ & \subseteq \cup_{i \in J_0} mInt(f^{-1}(cl(B_i)))\\ & \subseteq \cup_{i \in J_0} f^{-1}(int(cl(B_i)))).\end{aligned}$$

Hence $f(A) \subseteq \cup_{i \in J_0} int(cl(B_i))$. □

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