Fuzzy pairwise (r, s)-irresolute mappings

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Abstract

In this paper, we introduce the concepts of fuzzy pairwise (r, s)-irresolute, fuzzy pairwise (r, s)-presemiopen and fuzzy pairwise (r, s)-presemiclosed mappings in smooth bitopological spaces and then we investigate some of their characteristic properties.

Key words : $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiopen sets, fuzzy pairwise (r, s)-irresolute mappings

1. Introduction

After the introduction of fuzzy sets by Zadeh [10], Chang [2] was the first to introduce the concept of a fuzzy topology on a set X by axiomatizing a collection T of fuzzy subsets of X, where he referred to each member of T as an open set. In his definition of fuzzy topology, fuzziness in the concept of openness of a fuzzy subset was absent. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [9], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [3], and by Ramadan [7]. Kandil [4] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. Lee [5] introduced the concept of smooth bitopological spaces as a generalization of smooth topological spaces and Kandil's fuzzy bitopological spaces.

In this paper, we introduce the concepts of fuzzy pairwise (r, s)-irresolute, fuzzy pairwise (r, s)-presemiopen and fuzzy pairwise (r, s)-presemiclosed mappings in smooth bitopological spaces and then we investigate some of their characteristic properties.

2. Preliminaries

Let I be the closed unit interval $[0, 1]$ of the real line and let I_0 be the half open interval $(0, 1]$ of the real line. For a set X, I^X denotes the collection of all mapping from X to I. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement $\tilde{1} - \mu$. All other notations are the standard notations of fuzzy set theory.

A *Chang's fuzzy topology* on X [2] is a family T of fuzzy sets in X which satisfies the following properties:

(1) $\tilde{0}, \tilde{1} \in T$.

(2) If $\mu_1, \mu_2 \in T$ then $\mu_1 \wedge \mu_2 \in T$.

(3) If $\mu_k \in T$ for all k, then $\bigvee \mu_k \in T$.

The pair (X, T) be called a *Chang's fuzzy topological space*. Members of T are called T-fuzzy open sets of X and their complements T-fuzzy closed sets of X.

A system (X, T_1, T_2) consisting of a set X with two Chang's fuzzy topologies T_1 and T_2 on X is called a *Kandil's fuzzy bitopological space*.

A *smooth topology* on X is a mapping $\mathcal{T} : I^X \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\tilde{0}) = \mathcal{T}(\tilde{1}) = 1$.

(2) $\mathcal{T}(\mu_1 \wedge \mu_2) \geq \mathcal{T}(\mu_1) \wedge \mathcal{T}(\mu_2)$.

(3) $\mathcal{T}(\bigvee \mu_i) \geq \bigwedge \mathcal{T}(\mu_i)$.

The pair $(X, \mathcal{T})$ is called a *smooth topological space*. For $r \in I_0$, we call μ a $\mathcal{T}$-fuzzy r-open set of X if $\mathcal{T}(\mu) \geq r$ and μ a $\mathcal{T}$-fuzzy r-closed set of X if $\mathcal{T}(\mu^c) \geq r$.

A system $(X, \mathcal{T}_1, \mathcal{T}_2)$ consisting of a set X with two smooth topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X is called a *smooth bitopological space*. Throughout this paper the indices i, j take values in $\{1, 2\}$ and $i \neq j$.

Let $(X, \mathcal{T})$ be a smooth topological space. Then it is easy to see that for each $r \in I_0$, an r-cut

$$\mathcal{T}_r = \{\mu \in I^X \mid \mathcal{T}(\mu) \geq r\}$$

is a Chang's fuzzy topology on X.

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Let (X, T) be a Chang's fuzzy topological space and $r \in I_0$. Then the mapping $T^r : I^X \to I$ is defined by

$$T^r(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ r & \text{if } \mu \in T - \{\tilde{0}, \tilde{1}\}, \\ 0 & \text{otherwise} \end{cases}$$

becomes a smooth topology.

Hence, we obtain that if $(X, \mathcal{T}_1, \mathcal{T}_2)$ is a smooth bitopological space and $r, s \in I_0$, then $(X, (\mathcal{T}_1)_r, (\mathcal{T}_2)_s)$ is a Kandil's fuzzy bitopological space. Also, if (X, T_1, T_2) is a Kandil's fuzzy bitopological space and $r, s \in I_0$, then $(X, (T_1)^r, (T_2)^s)$ is a smooth bitopological space.

Definition 2.1. [5] Let $(X, \mathcal{T})$ be a smooth topological space. For each $r \in I_0$ and for each $\mu \in I^X$, the *$\mathcal{T}$-fuzzy r-closure* is defined by

$$\mathcal{T}\text{-Cl}(\mu, r) = \bigwedge\{\rho \in I^X \mid \mu \leq \rho, \mathcal{T}(\rho^c) \geq r\}$$

and the *$\mathcal{T}$-fuzzy r-interior* is defined by

$$\mathcal{T}\text{-Int}(\mu, r) = \bigvee\{\rho \in I^X \mid \mu \geq \rho, \mathcal{T}(\rho) \geq r\}.$$

Lemma 2.2. [5] Let μ be a fuzzy set of a smooth topological space $(X, \mathcal{T})$ and let $r \in I_0$. Then we have:

(1) $\mathcal{T}\text{-Cl}(\mu, r)^c = \mathcal{T}\text{-Int}(\mu^c, r)$.

(2) $\mathcal{T}\text{-Int}(\mu, r)^c = \mathcal{T}\text{-Cl}(\mu^c, r)$.

Definition 2.3. [5] Let μ be a fuzzy set of a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then μ is said to be

(1) a *$(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiopen* set if there is a $\mathcal{T}_i$-fuzzy r-open set ρ in X such that $\rho \leq \mu \leq \mathcal{T}_j\text{-Cl}(\rho, s)$,

(2) a *$(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiclosed* set if there is a $\mathcal{T}_i$-fuzzy r-closed set ρ in X such that $\mathcal{T}_j\text{-Int}(\rho, s) \leq \mu \leq \rho$.

Definition 2.4. [5] Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a smooth bitopological space. For each $r, s \in I_0$ and for each $\mu \in I^X$, the *$(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiclosure* is defined by

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(\mu, r, s) = \bigwedge\{\rho \in I^X \mid \mu \leq \rho, \rho \text{ is } (\mathcal{T}_i, \mathcal{T}_j)\text{-fuzzy } (r, s)\text{-semiclosed}\}$$

and the *$(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiinterior* is defined by

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(\mu, r, s) = \bigvee\{\rho \in I^X \mid \mu \geq \rho, \rho \text{ is } (\mathcal{T}_i, \mathcal{T}_j)\text{-fuzzy } (r, s)\text{-semiopen}\}.$$

Lemma 2.5. [5] Let μ be a fuzzy set of a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and let $r, s \in I_0$. Then we have:

(1) $(\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(\mu, r, s)^c = (\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(\mu^c, r, s)$.

(2) $(\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(\mu, r, s)^c = (\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(\mu^c, r, s)$.

Definition 2.6. [5] Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a smooth bitopological space X to a smooth bitopological space Y and $r, s \in I_0$. Then f is said to be

(1) a *fuzzy pairwise (r, s)-continuous* mapping if the induced mapping $f : (X, \mathcal{T}_1) \to (Y, \mathcal{U}_1)$ is a fuzzy r-continuous mapping and the induced mapping $f : (X, \mathcal{T}_2) \to (Y, \mathcal{U}_2)$ is a fuzzy s-continuous mapping,

(2) a *fuzzy pairwise (r, s)-semicontinuous* mapping if $f^{-1}(\mu)$ is a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy (r, s)-semiopen set of X for each $\mathcal{U}_1$-fuzzy r-open set μ of Y and $f^{-1}(\nu)$ is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy (s, r)-semiopen set of X for each $\mathcal{U}_2$-fuzzy s-open set ν of Y,

(3) a *fuzzy pairwise (r, s)-precontinuous* mapping if $f^{-1}(\mu)$ is a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy (r, s)-preopen set of X for each $\mathcal{U}_1$-fuzzy r-open set μ of Y and $f^{-1}(\nu)$ is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy (s, r)-preopen set of X for each $\mathcal{U}_2$-fuzzy s-open set ν of Y.

3. Fuzzy pairwise (r, s)-irresolute, fuzzy pairwise (r, s)-presemiopen and fuzzy pairwise (r, s)-presemiclosed mappings

Definition 3.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a smooth bitopological space X to a smooth bitopological space Y and $r, s \in I_0$. Then f is called

(1) *fuzzy pairwise (r, s)-irresolute* if $f^{-1}(\mu)$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiopen set of X for each $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-semiopen set μ of Y,

(2) *fuzzy pairwise (r, s)-presemiopen* if $f(\rho)$ is a $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-semiopen set of Y for each $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiopen set ρ of X,

(3) *fuzzy pairwise (r, s)-presemiclosed* if $f(\rho)$ is a $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-semiclosed set of Y for each $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiclosed set ρ of X.

Theorem 3.2. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping and $r, s \in I_0$. Then the following statements are equivalent:

(1) f is a fuzzy pairwise (r, s)-irresolute mapping.

(2) $f^{-1}(\mu)$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiclosed set of X for each $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-semiclosed set μ of Y.

(3) For each fuzzy set ρ of X,

$$f((\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(\rho, r, s)) \leq (\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(f(\rho), r, s).$$

(4) For each fuzzy set μ of Y,

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(f^{-1}(\mu), r, s) \leq f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(\mu, r, s)).$$

(5) For each fuzzy set μ of Y,

$$f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(\mu, r, s)) \leq (\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(f^{-1}(\mu), r, s).$$

Proof. (1) $\Rightarrow$ (2) Let μ be any $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-semiclosed set of Y. Then μ^c is a $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-semiopen set of Y. Since f is a fuzzy pairwise (r, s)-irresolute mapping, $f^{-1}(\mu^c)$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiopen set of X. Thus $f^{-1}(\mu)$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiclosed set of X.

(2) $\Rightarrow$ (3) Let ρ be any fuzzy set of X. Then $(\mathcal{U}_i, \mathcal{U}_j)$-sCl$(f(\rho), r, s)$ is a $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-semiclosed set of Y. By (2), $f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(f(\rho), r, s))$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiclosed set of X. Since $f(\rho) \leq (\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(f(\rho), r, s)$, we have

$$\begin{aligned}
&(\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(\rho, r, s) \\
&\leq (\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(f^{-1}f(\rho), r, s) \\
&\leq (\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(f(\rho), r, s)), r, s) \\
&= f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(f(\rho), r, s)).
\end{aligned}$$

Hence

$$\begin{aligned}
&f((\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(\rho, r, s)) \\
&\quad \leq ff^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(f(\rho), r, s)) \\
&\quad \leq (\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(f(\rho), r, s).
\end{aligned}$$

(3) $\Rightarrow$ (4) Let μ be any fuzzy set of Y. By (3),

$$\begin{aligned}
&f((\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(f^{-1}(\mu), r, s)) \\
&\quad \leq (\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(ff^{-1}(\mu), r, s) \\
&\quad \leq (\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(\mu, r, s).
\end{aligned}$$

Thus

$$\begin{aligned}
&(\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(f^{-1}(\mu), r, s) \\
&\leq f^{-1}f((\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(f^{-1}(\mu), r, s)) \\
&\leq f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(\mu, r, s)).
\end{aligned}$$

(4) $\Rightarrow$ (5) Let μ be any fuzzy set of Y. Then μ^c is a fuzzy set of Y. By (4),

$$\begin{aligned}
&(\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(f^{-1}(\mu)^c, r, s) \\
&= (\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(f^{-1}(\mu^c), r, s) \\
&\leq f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(\mu^c, r, s)).
\end{aligned}$$

By Lemma 2.5,

$$\begin{aligned}
&f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(\mu, r, s)) \\
&\quad = f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sCl}(\mu^c, r, s))^c \\
&\quad \leq (\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(f^{-1}(\mu^c), r, s)^c \\
&\quad = (\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(f^{-1}(\mu), r, s).
\end{aligned}$$

(5) $\Rightarrow$ (1) Let μ be any $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-semiopen set of Y. Then $(\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(\mu, r, s) = \mu$. By (5),

$$\begin{aligned}
f^{-1}(\mu) &= f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(\mu, r, s)) \\
&\leq (\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(f^{-1}(\mu), r, s) \\
&\leq f^{-1}(\mu).
\end{aligned}$$

So $f^{-1}(\mu) = (\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(f^{-1}(\mu), r, s)$ and hence $f^{-1}(\mu)$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiopen set of X. Thus f is a fuzzy pairwise (r, s)-irresolute mapping. □

Theorem 3.3. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a bijection and $r, s \in I_0$. Then f is a fuzzy pairwise (r, s)-irresolute mapping if and only if $(\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(f(\rho), r, s) \leq f((\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(\rho, r, s))$ for each fuzzy set ρ of X.

Proof. Let f be a fuzzy pairwise (r, s)-irresolute mapping and ρ any fuzzy set of X. Since $(\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(f(\rho), r, s)$ is a $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-semiopen set of Y, we have $f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(f(\rho), r, s))$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiopen set of X. Since f is fuzzy pairwise (r, s)-irresolute and one-to-one, we have

$$\begin{aligned}
&f^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(f(\rho), r, s)) \\
&\quad \leq (\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(f^{-1}f(\rho), r, s) \\
&\quad = (\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(\rho, r, s).
\end{aligned}$$

Since f is onto,

$$\begin{aligned}
&(\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(f(\rho), r, s) \\
&= ff^{-1}((\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(f(\rho), r, s)) \\
&\leq f((\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(\rho, r, s)).
\end{aligned}$$

Conversely, let μ be any $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-semiopen set of Y. Then $(\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(\mu, r, s) = \mu$. Since f is onto,

$$\begin{aligned}
&f((\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(f^{-1}(\mu), r, s)) \\
&\quad \geq (\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(ff^{-1}(\mu), r, s) \\
&\quad = (\mathcal{U}_i, \mathcal{U}_j)\text{-sInt}(\mu, r, s) = \mu.
\end{aligned}$$

Since f is one-to-one, we have

$$\begin{aligned}
f^{-1}(\mu) &\leq f^{-1}f((\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(f^{-1}(\mu), r, s)) \\
&= (\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(f^{-1}(\mu), r, s) \\
&\leq f^{-1}(\mu).
\end{aligned}$$

Thus $f^{-1}(\mu) = (\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(f^{-1}(\mu), r, s)$ and hence $f^{-1}(\mu)$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiopen set of X. Therefore f is a fuzzy pairwise (r, s)-irresolute mapping. □

Theorem 3.4. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping and $r, s \in I_0$. Then the following statements are equivalent:

(1) f is a fuzzy pairwise (r,s)-presemiopen mapping.

(2) For each fuzzy set ρ of X,

$$\begin{aligned}&f((\mathcal{T}_i,\mathcal{T}_j)\text{-sInt}(\rho,r,s))\\&\quad\le(\mathcal{U}_i,\mathcal{U}_j)\text{-sInt}(f(\rho),r,s).\end{aligned}$$

(3) For each fuzzy set μ of Y,

$$\begin{aligned}&(\mathcal{T}_i,\mathcal{T}_j)\text{-sInt}(f^{-1}(\mu),r,s)\\&\quad\le f^{-1}((\mathcal{U}_i,\mathcal{U}_j)\text{-sInt}(\mu,r,s))\end{aligned}$$

Proof. (1) ⇒ (2) Let ρ be any fuzzy set of X. Clearly $(\mathcal{T}_i,\mathcal{T}_j)$-sInt$(\rho,r,s)$ is a $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy (r,s)-semiopen set of X. Since f is a fuzzy pairwise (r,s)-presemiopen mapping, $f((\mathcal{T}_i,\mathcal{T}_j)\text{-sInt}(\rho,r,s))$ is a $(\mathcal{U}_i,\mathcal{U}_j)$-fuzzy (r,s)-semiopen set of Y. Thus

$$\begin{aligned}&f((\mathcal{T}_i,\mathcal{T}_j)\text{-sInt}(\rho,r,s))\\&=(\mathcal{U}_i,\mathcal{U}_j)\text{-sInt}(f((\mathcal{T}_i,\mathcal{T}_j)\text{-sInt}(\rho,r,s)),r,s)\\&\le(\mathcal{U}_i,\mathcal{U}_j)\text{-sInt}(f(\rho),r,s).\end{aligned}$$

(2) ⇒ (3) Let μ be any fuzzy set of Y. Then $f^{-1}(\mu)$ is a fuzzy set of X. By (2),

$$\begin{aligned}&f((\mathcal{T}_i,\mathcal{T}_j)\text{-sInt}(f^{-1}(\mu),r,s))\\&\quad\le(\mathcal{U}_i,\mathcal{U}_j)\text{-sInt}(ff^{-1}(\mu),r,s)\\&\quad\le(\mathcal{U}_i,\mathcal{U}_j)\text{-sInt}(\mu,r,s).\end{aligned}$$

Thus we have

$$\begin{aligned}&(\mathcal{T}_i,\mathcal{T}_j)\text{-sInt}(f^{-1}(\mu),r,s)\\&\quad\le f^{-1}f((\mathcal{T}_i,\mathcal{T}_j)\text{-sInt}(f^{-1}(\mu),r,s))\\&\quad\le f^{-1}((\mathcal{U}_i,\mathcal{U}_j)\text{-sInt}(\mu,r,s)).\end{aligned}$$

(3) ⇒ (1) Let ρ be any $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy (r,s)-semiopen set of X. Then $(\mathcal{T}_i,\mathcal{T}_j)$-sInt$(\rho,r,s)=\rho$ and $f(\rho)$ is a fuzzy set of Y. By (3),

$$\begin{aligned}\rho&=(\mathcal{T}_i,\mathcal{T}_j)\text{-sInt}(\rho,r,s)\\&\le(\mathcal{T}_i,\mathcal{T}_j)\text{-sInt}(f^{-1}f(\rho),r,s)\\&\le f^{-1}((\mathcal{U}_i,\mathcal{U}_j)\text{-sInt}(f(\rho),r,s)).\end{aligned}$$

Hence we have

$$\begin{aligned}f(\rho)&\le ff^{-1}((\mathcal{U}_i,\mathcal{U}_j)\text{-sInt}(f(\rho),r,s))\\&\le(\mathcal{U}_i,\mathcal{U}_j)\text{-sInt}(f(\rho),r,s)\\&\le f(\rho).\end{aligned}$$

Thus $f(\rho)=(\mathcal{U}_i,\mathcal{U}_j)$-sInt$(f(\rho),r,s)$ and hence $f(\rho)$ is a $(\mathcal{U}_i,\mathcal{U}_j)$-fuzzy (r,s)-semiopen set of Y. Therefore f is a fuzzy pairwise (r,s)-presemiopen mapping. □

Theorem 3.5. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping and $r,s\in I_0$. Then the following statements are equivalent:

(1) f is a fuzzy pairwise (r,s)-presemiclosed mapping.

(2) For each fuzzy set ρ of X,

$$\begin{aligned}&(\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(f(\rho),r,s)\\&\quad\le f((\mathcal{T}_i,\mathcal{T}_j)\text{-sCl}(\rho,r,s))\end{aligned}$$

Proof. (1) ⇒ (2) Let ρ be any fuzzy set of X. Clearly $(\mathcal{T}_i,\mathcal{T}_j)$-sCl$(\rho,r,s)$ is a $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy (r,s)-semiclosed set of X. Since f is a fuzzy pairwise (r,s)-presemiclosed mapping, $f((\mathcal{T}_i,\mathcal{T}_j)\text{-sCl}(\rho,r,s))$ is a $(\mathcal{U}_i,\mathcal{U}_j)$-fuzzy (r,s)-semiclosed set of Y. Thus we have

$$\begin{aligned}&(\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(f(\rho),r,s)\\&\le(\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(f((\mathcal{T}_i,\mathcal{T}_j)\text{-sCl}(\rho,r,s)),r,s)\\&=f((\mathcal{T}_i,\mathcal{T}_j)\text{-sCl}(\rho,r,s)).\end{aligned}$$

(2) ⇒ (1) Let ρ be any $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy (r,s)-semiclosed set of X. Then $(\mathcal{T}_i,\mathcal{T}_j)$-sCl$(\rho,r,s)=\rho$. By (2),

$$\begin{aligned}(\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(f(\rho),r,s)&\le f((\mathcal{T}_i,\mathcal{T}_j)\text{-sCl}(\rho,r,s))\\&=f(\rho)\\&\le(\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(f(\rho),r,s).\end{aligned}$$

Thus $f(\rho)=(\mathcal{U}_i,\mathcal{U}_j)$-sCl$(f(\rho),r,s)$ and hence $f(\rho)$ is a $(\mathcal{U}_i,\mathcal{U}_j)$-fuzzy (r,s)-semiclosed set of Y. Therefore f is a fuzzy pairwise (r,s)-presemiclosed mapping. □

Theorem 3.6. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a bijection and $r,s\in I_0$. Then f is a fuzzy pairwise (r,s)-presemiclosed mapping if and only if $f^{-1}((\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(\mu,r,s))\le(\mathcal{T}_i,\mathcal{T}_j)\text{-sCl}(f^{-1}(\mu),r,s)$ for each fuzzy set μ of Y.

Proof. Let f be a fuzzy pairwise (r,s)-presemiclosed mapping and let μ be any fuzzy set of Y. Then $f^{-1}(\mu)$ is a fuzzy set of X. Since f is fuzzy pairwise (r,s)-presemiclosed and onto,

$$\begin{aligned}&(\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(\mu,r,s)\\&=(\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(ff^{-1}(\mu),r,s)\\&\le f((\mathcal{T}_i,\mathcal{T}_j)\text{-sCl}(f^{-1}(\mu),r,s)).\end{aligned}$$

Since f is one-to-one, we have

$$\begin{aligned}&f^{-1}((\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(\mu,r,s))\\&\quad\le f^{-1}f((\mathcal{T}_i,\mathcal{T}_j)\text{-sCl}(f^{-1}(\mu),r,s))\\&\quad=(\mathcal{T}_i,\mathcal{T}_j)\text{-sCl}(f^{-1}(\mu),r,s).\end{aligned}$$

Conversely, let ρ be any $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy (r,s)-semiclosed set of X. Then $(\mathcal{T}_i,\mathcal{T}_j)$-sCl$(\rho,r,s)=\rho$. Since f is one-to-one,

$$\begin{aligned}&f^{-1}((\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(f(\rho),r,s))\\&\quad\le(\mathcal{T}_i,\mathcal{T}_j)\text{-sCl}(f^{-1}f(\rho),r,s)\\&\quad=(\mathcal{T}_i,\mathcal{T}_j)\text{-sCl}(\rho,r,s)=\rho.\end{aligned}$$

Since f is onto, we have

$$\begin{aligned}&(\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(f(\rho),r,s)\\&=ff^{-1}((\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(f(\rho),r,s))\\&\leq f(\rho)\\&\leq(\mathcal{U}_i,\mathcal{U}_j)\text{-sCl}(f(\rho),r,s).\end{aligned}$$

Thus $f(\rho) = (\mathcal{U}_i,\mathcal{U}_j)$-sCl$(f(\rho),r,s)$ and hence $f(\rho)$ is a $(\mathcal{U}_i,\mathcal{U}_j)$-fuzzy (r,s)-semiclosed set of Y. Therefore f is a fuzzy pairwise (r,s)-presemiclosed mapping. □

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