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Comparison of Proportional Hazards and Accelerated Failure Time Models in the Accelerated Life Tests

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Abstract. In the accelerated tests, the importance of correct failure analysis must be strongly emphasized. Understanding the failure mechanisms is requisite for designing and conducting successful accelerated life test. Under this presumption, a rational method must be identified to relate the results of accelerated tests quantitatively to the reliability or failure rates in use conditions, using a scientific acceleration transform. Most widely used models for relating the results of accelerated tests quantitatively to the reliability or failure rates in use conditions are an accelerated failure time model and a proportional hazards model. The purpose of this research is to compare the usability of the accelerated failure time model and proportional hazards model in the accelerated life tests.

Key Words : *accelerated failure time model, proportional hazards model, acceleration factor, Weibull*

1. INTRODUCTION

The accelerated test methods may be divided into two groups: qualitative accelerated testing and quantitative accelerated life testing. Qualitative accelerated tests are designed to identify weaknesses or potential design weaknesses and also weaknesses caused by manufacturing process that were not identified by any analytical methods during the product design period. Therefore engineer is mostly interested in identifying failures and failure modes without attempting to make predictions as to the product reliability under normal use conditions. In quantitative accelerated life testing, engineer is interested in predicting or quantifying the life characteristics under the normal use conditions from data obtained during a test where the test conditions are not identical to the use conditions. Typically, the stresses that contribute to product failure are increased to shorten test times.

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In the accelerated tests, the importance of correct failure analysis must be strongly emphasized. Understanding the failure mechanisms is essential for designing and conducting successful accelerated life or other test as advocated in physics-of-failure based reliability design and prediction methodologies using physics-of-failure approach. To achieve this, a rational method must be identified to relate the results of accelerated tests quantitatively to the reliability or failure rates in use conditions, using a scientific acceleration transform. The amount of test time compression achieved in an accelerated life test must be determined quantitatively, based on the physics of the relevant failure modes. Finally, accelerated life tests attempt to reduce the time it takes to observe failures.

2. ACCELERATED FAILURE TIME MODEL AND PROPORTIONAL HAZARDS MODEL

Most widely used models for relating the results of accelerated tests quantitatively to the reliability or failure rates in use conditions are an accelerated failure time model and a proportional hazards model.

2.1 Accelerated failure time model

Physical acceleration (sometimes called true acceleration or just acceleration) means that operating an item at high stress (i.e., higher temperature or voltage or humidity or duty cycle, etc.) produces the same failures that would occur at normal use stresses, except that they happen much quicker; the time scale is simply different.

When there is true acceleration, changing stress is equivalent to transforming the time scale used to record when failures occur. The transformations commonly used are linear, which means that time-to-fail at high stress just has to be multiplied by a constant (the acceleration factor) to obtain the equivalent time-to-fail at use stress. An acceleration factor is the constant multiplier between the two stress levels.

We use the following notation: let the subscripts *u* and *a* refer to use conditions and accelerated conditions, respectively.

t_a = time-to-fail at accelerated condition
t_u = corresponding time-to-fail at use condition
$F_a(t)$ = cumulative distribution function(CDF) at accelerated condition
$F_u(t)$ = cumulative distribution function(CDF) at use condition
$f_a(t)$ = probability density function(PDF) at accelerated condition
$f_u(t)$ = probability density function(PDF) at use condition
$h_a(t)$ = failure rate function at accelerated condition
$h_u(t)$ = failure rate function at use condition
μ_a = mean life at accelerated condition
μ_u = mean life function at use condition

Then, an acceleration factor *AF* between stress and use means the following relationships hold:

Time-to-Fail : $T_u = AF \times T_a$
CDF: $F_u(t) = Pr\,(T_u \le t)$
$= Pr\,(AF \cdot T_a \le t)$
$= Pr\,(T_a \le t/AF)$
$= F_a(t/AF)$
Reliability Function: $R_u(t) = R_a\,(t/AF)$
PDF: $f_u(t) = (1/AF) \cdot f_a(t/AF)$
Failure Rate function: $h_u(t) = (1/AF) \cdot h_a(t/AF)$
Mean Life: $\mu_u = AF \cdot \mu_a$

The AFT models are discussed in details in textbooks (Lawless, 1982; Elsayed, 1996).

2.2 Proportional hazards model

Introduced by Cox(1972), the proportional hazards model(PHM) was developed in order to estimate the effects of different covariates influencing the times-to-failure of a system. The model has been widely used in biomedical field and recently there has been an increasing interest in its application in reliability engineering. Argent *et al.* (1986), Marshall *et al.* (1990) studied the reliability analysis using PHM. Kumar and Westberg (1995), Kobbacy *et al.* (1997) studied the maintenance policies using PHM.

The PHM assumes that the failure rate (hazard rate) of a item is a product of

- An arbitrary and unspecified baseline rate, $h_0(t)$, which is a function of time only.
- A positive function $g(\underline{X}, \underline{A})$, independent of time, which incorporates the effects of a number of covariates such as humidity, temperature, pressure, voltage, etc.

The failure rate of a item is given by:

$$h(t, \underline{X}) = h_0(t) \cdot g(\underline{X}, \underline{A})$$

where $\underline{X} = (x_1, x_2, \cdots, x_m)$ is a row vector consisting the covariates, $\underline{A} = (\alpha_1, \alpha_2, \cdots, \alpha_m)^t$ is a column vector consisting of the unknown parameters (also called regression parameters) of the model and m is the number of stress related variables (time-independent).

Different forms of $g(\underline{X}, \underline{A})$ can be used. However, the exponential form is mostly used due to its simplicity and the failure rate of an item is given by:

$$h(t, \underline{X}) = h_0(t) \cdot \exp(\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_m x_m)$$

Therefore, in PHM
Reliability Function:

$$R(t, \underline{X}) = \exp\left[-\int_0^t h(u, \underline{X})\,du\right] = \exp\left[-g(\underline{X}, \underline{A}) \int_0^t h_0(u)\,du\right] = R_0(t)^{g(\underline{X}, \underline{A})},$$

where $R_0(t)$ is a baseline reliability function corresponding to the baseline rate, $h_0(t)$.

3. PARAMETRIC MODEL FORMULATION

Assuming that the life test is conducted at an accelerated condition for which the acceleration factor (*AF*) is known, and life time is fitted by the Weibull distribution, the accelerated failure time model can be expressed as follows. :
This is the case where the time to failure at an accelerated stress is distributed by the Weibull distribution with shape parameter β_a and scale parameter η_a. Thus,

$$F_a(t) = 1\text{-}exp\{-(t/\eta_a)^{\beta a}\}$$

and

$$F_u(t) = F_a(t/AF) = 1\text{-}exp[-\{t/(AF\cdot\eta_a)\}^{\beta a}].$$

Thus,

$$\beta_u = \beta_a,\ \eta_u = AF\cdot\eta_a, \tag{1}$$

where β_u is the shape parameter at use condition, β_a is the shape parameter at accelerated condition, η_u is the characteristic life at use condition and η_a is the characteristic life at accelerated condition.

From equation (1), we can see the shape parameter remains constant over all stress levels in the acceleration factor model. It is generally assumed a constant shape parameter across the different stress levels when analyzing data from an accelerated life test. This implies that the unit/component will fail in the same manner across different stress levels. If the shape parameters at different stress levels are significantly different, then either the assumption of true linear acceleration is invalid or the Weibull distribution is inappropriate to use for analysis of such data.

Now we will consider the Weibull distribution to formulate the parametric PHM. In this case, the baseline failure rate is given by:
$h_0(t) = \beta/\eta \cdot(t/\eta)^{\beta-1}$

The proportional hazards failure rate becomes

$$\begin{aligned} h(t, \underline{X}) &= \beta/\eta \cdot(t/\eta)^{\beta-1} \cdot g(\underline{X}, \underline{A}) \\ &= \beta/\eta \cdot(t/\eta)^{\beta-1} \cdot exp(\alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_m x_m) \end{aligned}$$

Therefore,
Reliability Function:

$$\begin{aligned} R(t, \underline{X}) &= exp\ [-\textstyle\int_0^t h(u, \underline{X})\ du\ ] \\ &= exp\ [-exp(\alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_m x_m)\textstyle\int_0^t h_0(u)\ du\ ] \\ &= exp\ [-\ (t/\eta)^{\beta}\cdot exp(\alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_m x_m)] \end{aligned} \tag{2}$$

After a transformation of the reliability function in (2), we can obtain

$$ln\{\ -\ ln\ R(t, \underline{X})\ \} = \beta\cdot ln(t) - \beta\cdot ln(\eta) + \alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_m x_m$$

The $ln\{\ -\ ln\ R(t, \underline{X})\ \}$ versus $ln(t)$ should give approximately a straight line if the Weibull distribution assumption is reasonable. The intercept of the line will be a rough estimate of

β. And if the two lines for two groups in this plot are essentially parallel, this means that the PHM is valid.

4. WHICH MODEL DO WE CHOOSE IN ACCELERATED LIFE TESTING?

In PHM, the effects of covariates alter the hazard function. On the other hand, under the accelerated failure time model, we measure the direct effect of the covariates on the failure time instead of hazard, as we do in PHM. Accelerated life tests attempt to reduce the time it takes to observe failures. In the accelerated failure time model, this works without actually changing the equation for the instantaneous failure rate. However, if the hazard function changes, it is termed a PHM. Mathematically, the differences between these two can be seen in the following two equations for a Weibull distribution in which $H_{AF}(t)$ is the cumulative hazard function for the accelerated failure time model, $H_{PH}(t)$ is the cumulative hazard function for the PHM ,AF is an acceleration factor due to some sort of stimulus and $H(t) = (t/\eta_u)^{\beta u}$ is the unmodified cumulative hazard for a Weibull distribution.

$$H_{AF}(t) = (AF \cdot t/\eta_u)^{\beta u} \tag{3}$$

$$H_{PH}(t) = [g(\underline{X}, \underline{A})^{1/\beta u} \cdot t/\eta_u]^{\beta u} \tag{4}$$

PHM is equivalent to the accelerated failure time model if we assume $AF = g(\underline{X}, \underline{A})^{1/\beta u}$. In this case, $H_{PH}(t)$ can be expressed as follows.

$$H_{PH}(t) = [AF \cdot t/\eta_u]^{\beta u}$$

In $H_{AF}(t)$, time is a linear function of the acceleration factor. In $H_{PH}(t)$, the hazard function itself is being modified. By rearranging the equation for $H_{PH}(t)$, it can be seen that time is a non-linear function of the $g(\underline{X}, \underline{A})$, the effects of covariates. That is, time is multiplied by $g(\underline{X}, \underline{A})^{1/\beta u}$. The difference between these two types of accelerated tests is that $H_{AF}(t)$ requires knowledge only of the ratio of the accelerated test time to non-accelerated time caused by the applied environmental stimulus whereas $H_{PH}(t)$ requires knowledge of the manner in which the $g(\underline{X}, \underline{A})$ changes as a function of the parameter β. For the Weibull distribution, the resultant distribution for either of these two conditions is still a Weibull distribution.

Equation (3) is usually applied when the acceleration is done with the increased repetition rate of the applied repetitious stress such as operational cycling. The equation (4) is preferred when the acceleration is applied to the physical states of the unit under test such as thermal acceleration (Brown's motion), where the acceleration factor itself depends on the distribution.

To summarize the above rationale it can be said that the stress acceleration provides reduction in time to failure by increasing the stress levels beyond those expected in the normal use of the item.

4. CONCLUSIONS

In the accelerated failure time model and the proportional hazards model, we assume that the stress levels applied at accelerated conditions are within a range of true acceleration-that is, if the failure time distribution at a high stress level is known and time-scale transformation to the normal conditions is also known, we can mathematically derive the failure time distributions at normal use conditions (or any other stress level). In the accelerated failure time model, we assume that the time-scale transformation is constant. Namely, in the accelerated failure time model, linear acceleration is assumed. It is important to emphasize that in PHM, it is assumed that the ratio of the hazards rates for two different stresses (such as two temperature, T_u and T_a); $h(t, T_u)/ h(t, T_a)$ does not vary with time. In other words, $h(t, T_u)$ is directly proportional to $h(t, T_a)$-hence the term proportional hazards model. The accelerated failure time model can be used for the analysis of failure-time even when hazards are not proportional.

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