FREE VIBRATION ANALYSIS OF CIRCULAR PLATE WITH ECCENTRIC HOLE SUBMERGED IN FLUID

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Circular plates with holes are extensively used in mechanical components. The existence of a hole in a circular plate results in a significant change in the natural frequencies and mode shapes of the structure. Especially if the hole is located eccentrically, the vibration behavior of these structures is expected to deviate significantly from that of a plate with a concentric hole. In addition, if the plate is in contact with or submerged in fluid, the situation is more complex. Therefore, in this study, an analytical method to determine the modal characteristics of a plate submerged in fluid is developed based on the finite Fourier-Bessel series expansion and Rayleigh-Ritz method and is verified by the finite element analysis using a commercial program. Also, the relationship between parameter variations and vibration modes is investigated. These results can be used as guidance for the modal analysis and damage detection of a circular plate with a hole.

KEYWORDS : Circular Plate, Eccentricity, Hole, Mode Shape, Natural Frequency

1. INTRODUCTION

Circular plates submerged in fluid are used extensively in mechanical engineering for nuclear reactor internal components. There have been many studies of the vibration of such plates. Vibration analysis of circular plates has attracted extensive interest when exploring the structural responses to various excitations, such as seismic and pump pulsation excitations. This kind of plate may have many holes in it to provide the smooth passage of flow; the holes are sometimes located eccentrically. In this case, modal characteristics of a plate with an eccentric hole will be significantly different from that of a plate with a concentric hole. These characteristics are important to analyze in order to determine structural parameter identification, damage detection, and vibration control of the structure.

In general, most research has focused analytically or experimentally on vibration analysis of circular plate with a concentric hole [1-3]. However, for engineering applications, many mechanical components can be modeled as circular plate with an eccentric hole. Usually, the existence of an eccentric hole in a circular plate will result in the splitting of doublet frequencies and the distortion of mode shapes; consequently, the vibration characteristics of the structure are affected due to the influence of asymmetric location of the hole. In such cases, the influence of eccentricity and hole size on frequencies or mode shapes should be considered carefully. Especially if the plate is in contact with or submerged in fluid, the situation is more complex, necessitating a detailed investigation because most of such submerged plates are used in the nuclear industry.

Therefore, in this study, an analytical method of obtaining the modal characteristics of the plate submerged in fluid is developed based on the finite Fourier-Bessel series expansion and the Rayleigh-Ritz method and is verified by finite element analysis using a commercial program. The relationship between parameter variations (fluid, hole, eccentricity, hole size) and vibration modes is investigated, and the results can be used as guidance for the modal analysis and damage detection of a circular plate with a hole.

2. THEORETICAL DEVELOPMENT

For a single circular solid plate submerged in a fluid-filled rigid cylinder as shown in Figs. 1 and 2, where R_1, h, d_1 and d_2 represent the radius and thickness of the plate, and height of upper and lower fluid, respectively, the equation of motion for transverse displacement, w, is:

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = p \tag{1}$$

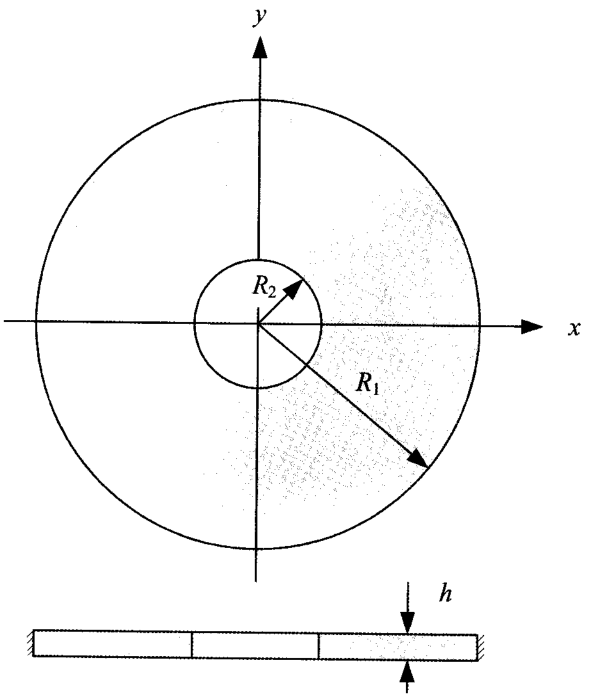

Fig. 1. Circular Plate with a Hole

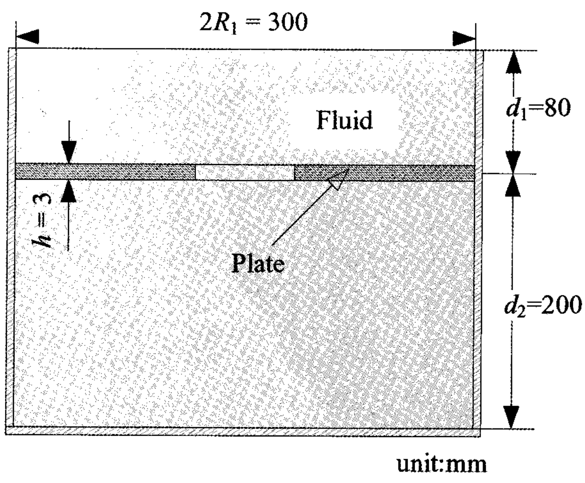

Fig. 2. Plate Submerged in Fluid

where $D=Eh^3/12(1-\mu^2)$ is the flexural rigidity of the plate; ρ, μ, p and E are density, Poisson's ratio, hydrodynamic pressure on the plate and Young's modulus of the plate, respectively [4]. The upper fluid is referred to with a subscript "1" while the lower fluid is denoted by a subscript "2". The solution of Eq. (1) takes the following form of combinations for plate deformation with respect to polar coordinates (r,θ) :

$$w(r,\theta,t)=\cos(n\theta)\sum_{m=1}^{M} q_m W_{nm}(r)\exp(\mathrm{i}\omega t) \tag{2}$$

where q_m is an unknown coefficient and n and m are the numbers of the nodal diameters and circles of the plate, respectively. For the plate with clamped boundary condition, the displacement along the edge of the plate, $r = R_1$, must be zero and therefore dynamic displacement of Eq. (2) will be reduced to:

$$W_{nm}(r)=\mathrm{J}_n(\lambda_{nm} r)-\mathrm{J}_n(\lambda_{nm} R_1)\frac{\mathrm{I}_n(\lambda_{nm} r)}{\mathrm{I}_n(\lambda_{nm} R_1)} \tag{3}$$

where λ_{nm} is the frequency parameter for the plate in air, which is also determined by the boundary conditions and is related to the circular frequency of the plate in air ω. J_n and I_n are the Bessel function and the modified Bessel function of the first kind, respectively. For the fixed boundary condition, the eigenvalues λ_{nm} for the plate in a vacuum can be obtained from the zero slope and zero moment boundary conditions as follows [5]:

$$\frac{\mathrm{J}_n'(\lambda_{nm} R_1)}{\mathrm{J}_n(\lambda_{nm} R_1)}=\frac{\mathrm{I}_n'(\lambda_{nm} R_1)}{\mathrm{I}_n(\lambda_{nm} R_1)} \tag{4}$$

The fluid region contained in a cylindrical rigid vessel is bisected into two parts, an upper fluid and a lower fluid, by the circular plate. The three dimensional oscillatory fluid flow in the cylindrical coordinates can be described by the velocity potential. The facing side of the circular plate is in contact with an inviscid and incompressible fluid. The fluid movement due to vibration of the plate is described by the spatial velocity potential that satisfies the Laplace equation:

$$\nabla^2\Phi(r,\theta,x,t)=\frac{\partial^2\Phi(r,\theta,x,t)}{\partial t^2}/c^2 \tag{5}$$

where c is the speed of sound in fluid. It is possible to separate the function Φ with respect to r by observing that in the radial direction the vessel that supports the edges of the plate is assumed to be rigid, as in the case of the plate in complete contact. Thus:

$$\Phi(r,\theta,x,t)=\mathrm{i}\omega\phi_j(r,\theta,x)\exp(\mathrm{i}\omega t), j=1,2 \tag{6}$$

For the bounded fluid, the boundary condition along the cylindrical vessel wall assures zero fluid velocity in the radial direction given by:

$$\left.\frac{\partial\phi_j}{\partial r}\right|_{r=R_1}=0 \tag{7}$$

Since the plate thickness is neglected, the compatibility conditions at the fluid interface with the plate yield:

$$\left.\frac{\partial \phi_1(r,\theta,x)}{\partial x}\right|_{x=0} = w(r,\theta) \tag{8a}$$

$$\left.\frac{\partial \phi_2(r,\theta,x)}{\partial x}\right|_{x=0} = w(r,\theta) \tag{8b}$$

When gravity is neglected, it is useful to introduce the Rayleigh quotient in order to calculate the coupled natural frequencies of the circular plate submerged in the ideal fluid.

$$\omega^2 = \frac{V_d}{T_d + T_F} \tag{9}$$

where V_d is the potential energy of the plate and T_d and T_F are the reference kinetic energies of the plate and the fluid, respectively. In order to perform numerical calculations for each fixed n value, a sufficiently large finite M number of terms must be considered in all the previous sums of the expanding term m. For this purpose, a vector q of the unknown parameters is introduced as:

$$\mathbf{q} = \{q_1 \quad q_2 \quad q_3 \cdots\cdots \quad q_M\}^T \tag{10}$$

Now, it is necessary to know the reference kinetic energies of the plate and containing fluids in order to calculate the coupled natural frequencies of the circular plate in contact with fluid. Using the hypothesis of irrotational movement of the fluid, the reference kinetic energy of the fluid can be evaluated from its boundary motion.

$$\begin{aligned} T_F &= \frac{1}{2}\rho_o \int_0^{2\pi} \int_0^{R_1} w\, \phi_2(r,-d_1)\, r\, dr\, d\theta \\ &+ \frac{1}{2}\rho_o \int_0^{2\pi} \int_0^{R_1} w\, \phi_1(r,d_2)\, r\, dr\, d\theta \end{aligned} \tag{11}$$

The reference kinetic energy of the circular plate is presented:

$$T_d = \frac{1}{2}\rho h \int_0^{2\pi} \int_0^{R_1} r W_{nm}^{\ 2}\, dr\, d\theta \tag{12}$$

The maximum potential energy of the plate can be approximated by :

$$V_d \approx \frac{1}{2} D \lambda_{nm}^{\ 4} \int_0^{2\pi} \int_0^{R_1} W_{nm}^{\ 2}\, r\, dr\, d\theta \tag{13}$$

The correspondence between the reference total kinetic energy of each mode multiplied by its square circular frequency and the maximum potential energy of the same node are used. In order to find natural frequencies and mode shpaes of the plate in contact with fluid, the Rayleigh quotient for the plate vibration coupled with ideal fluid is used. Minimizing Rayleigh quotient $V_d/(T_d+T_F)$ with respect to the unknown parameters q_m, the non-dimensional Galerkin equation can be obtained and an eigenvalue problem and the natural frequencies ω can be calculated [4].

3. ANALYSIS

3.1 Theoretical Analysis

On the basis of the preceding analysis, the natural frequencies of a circular plate submerged in fluid are found numerically using MathCAD. In order to check the validity and accuracy of the results from the theoretical study, finite element analyses are also performed and frequency comparisons between them are carried out for the fluid-coupled system.

The circular plate is connected to a fixed closed-type container that is made of carbon steel. The plate is made of aluminum, having a radius of 150 mm and a thickness of 3 mm. The physical properties of the material are as follows: Young's modulus = 69.0 GPa, Poisson's ratio = 0.3, and mass density = 2700 kg/m^3. Water is used as the contained fluid, having a density of 1000 kg/m^3. The speed of sound in water is 1483 m/s, which is equivalent to the bulk modulus of elasticity of 2.2 GPa.

The frequency equations derived in the preceding sections involve an infinite series of algebraic terms. Before exploring the analytical method to obtain the natural frequencies of the fluid-coupled plate, it is necessary to conduct convergence studies and establish the number of terms required in the series expansions involved. In the numerical calculation, the Bessel-Fourier expansion term is set at 200 and the expanding term for the admissible function is set at 40, which gives an exact enough solution by convergence.

3.2 Finite Element Analysis

Finite element analyses using the commercial computer code ANSYS 11.0 [6] are performed to verify the analytical results for the theoretical study. The results from finite element method are used as the baseline data. Several different three-dimensional models are developed for plates with and without holes as shown in Fig. 3. The fluid region is divided into a number of 3-dimensional contained fluid elements (FLUID80) with eight nodes having three degrees of freedom at each node. The fluid element FLUID80 is particularly well suited for calculating hydrostatic pressures and fluid/solid interactions. The circular plate is modeled as elastic shell elements (SHELL63) with four nodes.

The perimeter nodes of the plate are coupled with the nodes of the container, which are fixed in all six degrees of freedom. The fluid movement at top and bottom of the container is considered to be constrained in the vertical direction for the bounded surface fluid case. The vertical velocities of the fluid element nodes adjacent to each surface of the wetted circular plate coincide to those of plate so that the model can simulate Eqs. (8a) and (8b).

Several cases of finite element analyses are performed depending on the existence of the hole, the size of hole, and the eccentricity. The Block Lanczos method is used for the eigenvalue and eigenvector extractions to calculate 500 frequencies including fluid modes [7]. It uses the Lanczos algorithm where the Lanczos recursion is performed with a block of vectors. This method is as accurate as the subspace method, but faster. The Block Lanczos method is especially powerful when searching for eigenfrequencies in a given part of the eigenvalue spectrum of a given system. The convergence rate of the eigenfrequencies will be about the same when extracting modes in the midrange and higher end of the spectrum as when extracting the lowest modes.

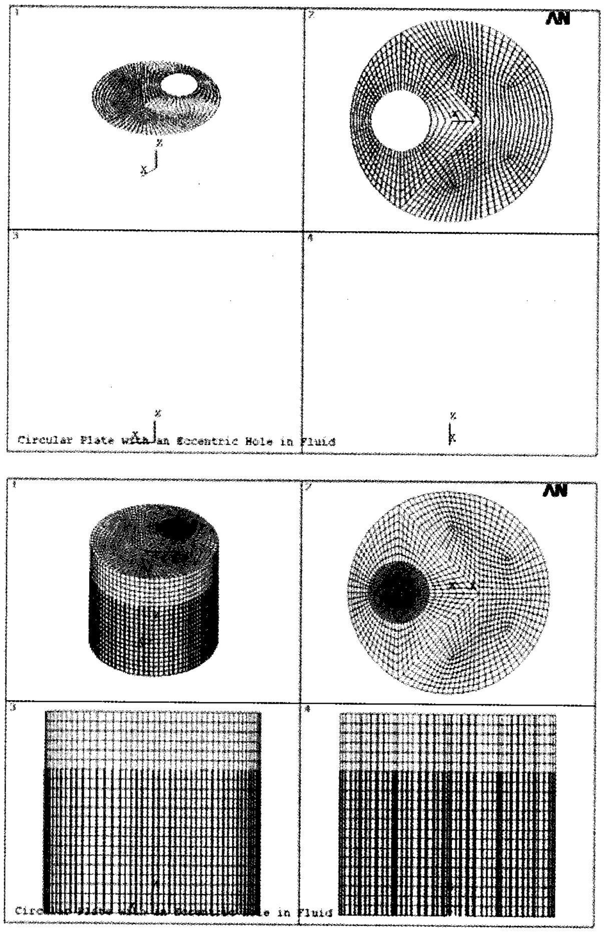

Fig. 3. Finite Element Model of Plate with Eccentric Hole Submerged in Fluid

4. RESULTS AND DISCUSSION

4.1 Validation of Theory

The frequency comparisons between the analytical solution developed here and that found through the finite element method are shown in Figs. 4 and 5 for plates in air and submerged in fluid, respectively. The symbol m' in the figures represents the number of nodal circles of the wet mode; the symbol n means the number of nodal diameter. The frequency differences of plates in air are almost negligible, as shown in Fig. 4. But the largest discrepancy of 5.9 % in $m' = 4$, $n = 4$, as shown in Fig. 5, is obtained for the plate submerged in fluid. As the mode number increases, the discrepancy becomes large, but can be reduced by using a sufficient number of nodes in the radial direction in the finite element modeling. Also, the compressibility of the fluid was found to reduce the natural frequency of the lower wet modes in the case of a fluid-filled cylindrical shell [8]. Therefore, discrepancies in Fig. 5 may, also, be caused by the assumption that the water is incompressible in theory. But the frequency comparisons between theoretical and finite element analysis results are generally found to be in good agreement, within 6 %.

4.2 Fluid Effect

The effect of the fluid if the plate is submerged in it can be investigated by comparing the frequencies of the plate in air and that submerged in fluid.

Frequencies of the plate in air and submerged in fluid are shown in Figs. 6 and 7, respectively; their normalized frequencies are shown in Fig. 8. As shown in the figures, the frequencies increase as mode numbers increase and the

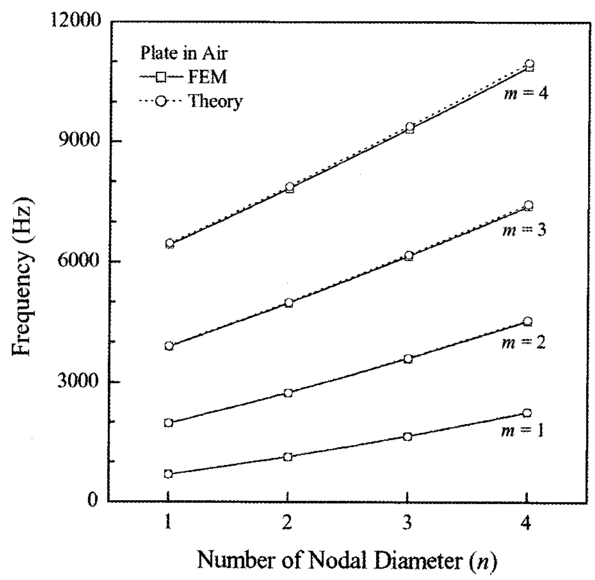

Fig. 4. Comparison of Frequencies between FEM and Theory for Plate in Air

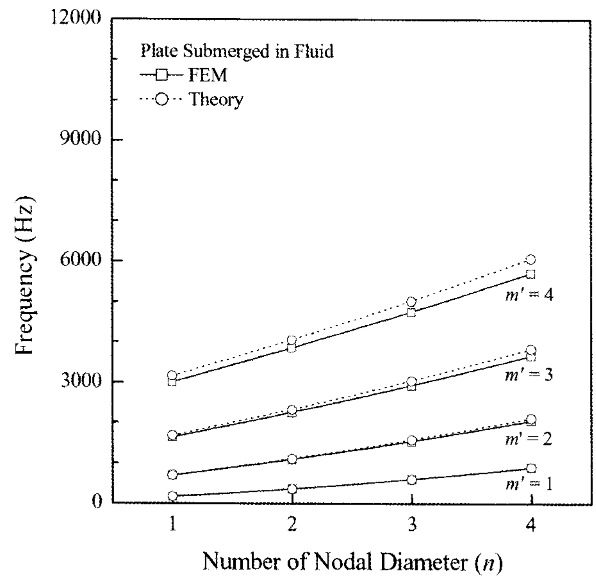

Fig. 5. Comparison of Frequencies between FEM and Theory for Plate Submerged in Fluid

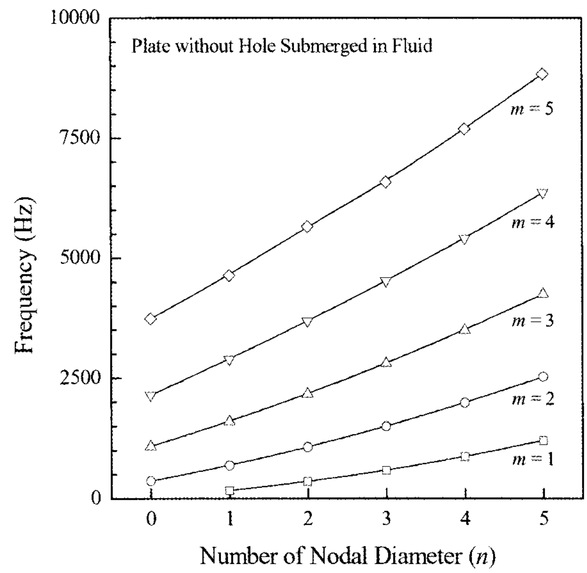

Fig. 7. Frequencies of Plate without Hole Submerged in Fluid

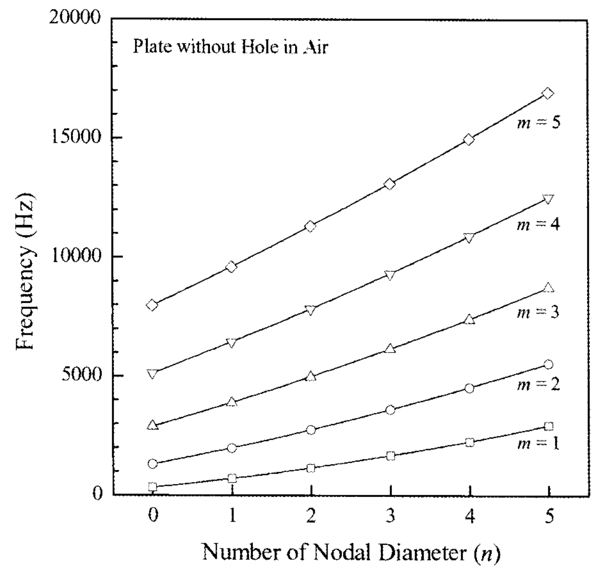

Fig. 6. Frequencies of Plate without Hole in Air

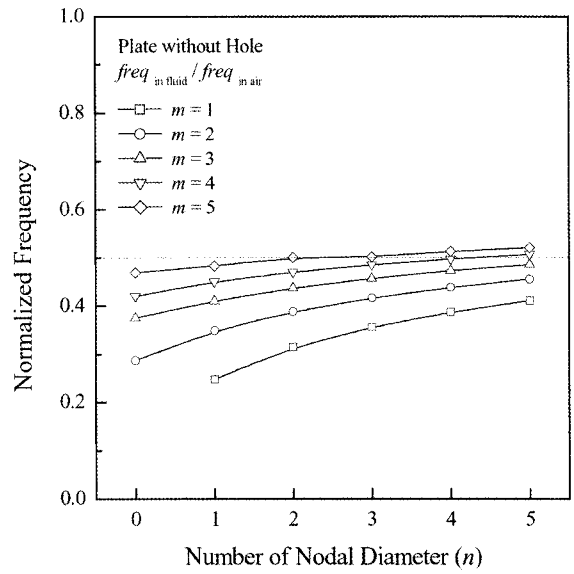

Fig. 8. Normalized Frequencies of Plate Submerged in Fluid with Respect to Plate in Air

trends of both are the same irrespective of the environment conditions. If the plate is submerged in fluid, the frequencies decrease to values of less than 0.5. The extent of the decrease is more severe in the lower modes. This is explained by the fact that for the lower modes the fluid moves more significantly to compensate for the mode shapes. For the higher modes, the fluid movement is relatively small in amplitude and therefore affects the modes less significantly.

Frequencies of the plate with a concentric hole in air and submerged in fluid are shown in Figs. 9 and 10, respectively, and their normalized frequencies are shown in Fig. 11. As shown in the figures, the frequencies increase as mode number increases and both of their trends are the same irrespective of the environment conditions. If the plate is submerged in fluid, the frequencies decrease to values of less than 0.5. The extent of the decrease is more severe in the lower modes. This is explained by the fact that for the lower modes the fluid moves more significantly to compensate for the mode shapes. For the higher modes, the fluid movement is relatively small in amplitude and therefore affects the modes less significantly. This kind of phenomenon is almost the same as that indicated in

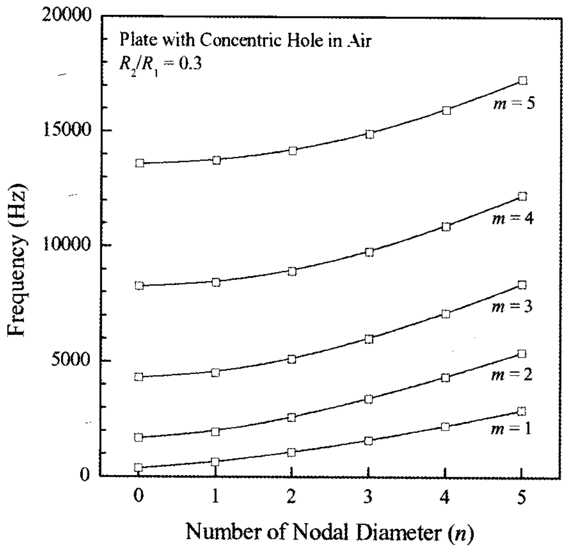

Fig. 9. Frequencies of Plate with Concentric Hole in Air

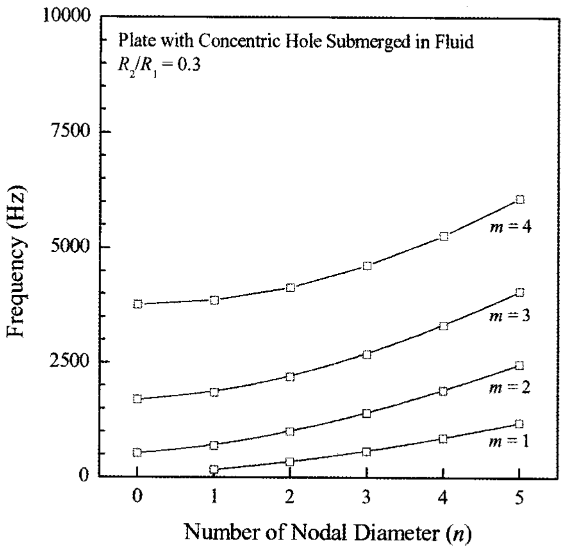

Fig. 10. Frequencies of Plate with Concentric Hole Submerged in Fluid

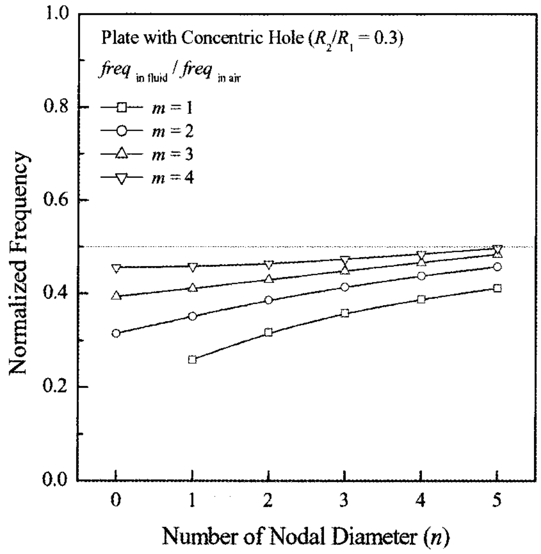

Fig. 11. Normalized frequencies of plate with concentric hole submerged in fluid with respect to plate in air

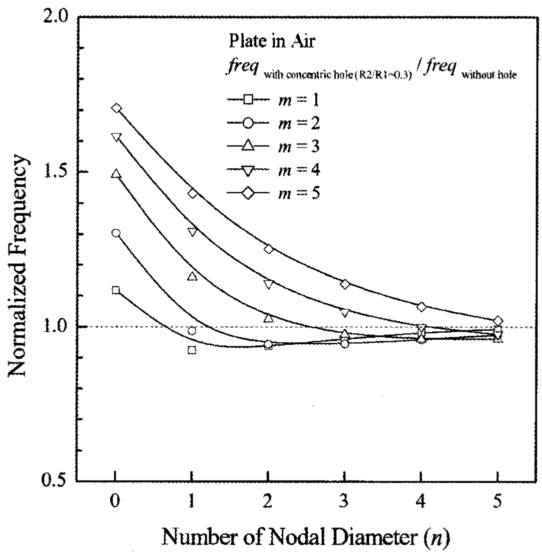

Fig. 12. Normalized Frequencies of Plate with Concentric Hole w.r.t. without Hole in Air

the plate without a hole, but comparing normalized frequencies of the Figs. 8 and 11, the case of a plate without a hole is affected by the fluid a little bit, especially for the lower modes. This is related to the separation of the fluid by the upper and lower portions due to the insertion of the solid plate. If there is a hole in the plate, the upper and lower fluid can move to the other part and therefore the mass effect can decrease.

4.3 Hole Effect

Figure 12 shows normalized frequencies of plate with concentric hole with respect to without hole in air condition. The radius ratio of hole to plate is 0.3 and it indicates that including hole increases the frequencies generally. This is explained by the fact that including the hole means a decrease in the mass, resulting in a decrease of the frequencies. But this is not always true because the hole effect also modifies the stiffness of the plate and therefore the frequency changes due to the hole are a combination of the mass and stiffness changes. As the mode numbers increase the normalized frequency tends to converge to unity, which means that for the higher modes the concentric hole effect is almost negligible.

If there are multiple holes in the plate, the situation is much more complex. The stiffness of the plate decreases more than the mass and therefore the frequencies of a perforated plate are usually smaller than those of the solid plate [9]. This is the opposite situation from that of the plate with a single hole.

Figure 13 shows normalized frequencies of a plate with a concentric hole with respect to a plate without a hole when the plate is submerged in fluid. In this case, including the hole increases the frequencies a little bit but is not as significant as in the air condition. For the lower modes, the frequencies decrease by including the concentric hole, which means that the decrease of the mass by including the hole is compensated for by the fluid surrounding the plate. As the mode numbers increase, the normalized frequency tends to converge to unity, which means that for the higher modes the concentric hole effect is almost as negligible as in that in the air condition.

When comparing frequencies of the plate in air and that submerged in fluid, the frequencies decreased to about 0.2 to 0.5 as shown in Figs. 8 and 11 for cases without and with hole, respectively. The decreasing rate in the fluid condition increased if there is a hole in the plate, resulting in a decreasing ratio of frequencies of about 0.5 to 1.1.

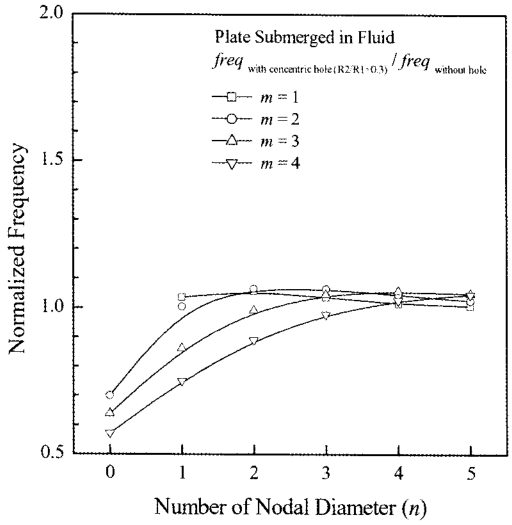

Fig. 13. Normalized Frequencies of Plate with Concentric Hole w.r.t. without Hole in Fluid

4.4 Eccentricity Effect

For the plate with an eccentric hole, the doublet frequency splits into two distinct values: one is the symmetric mode, which has a symmetric shape of the modes with respect

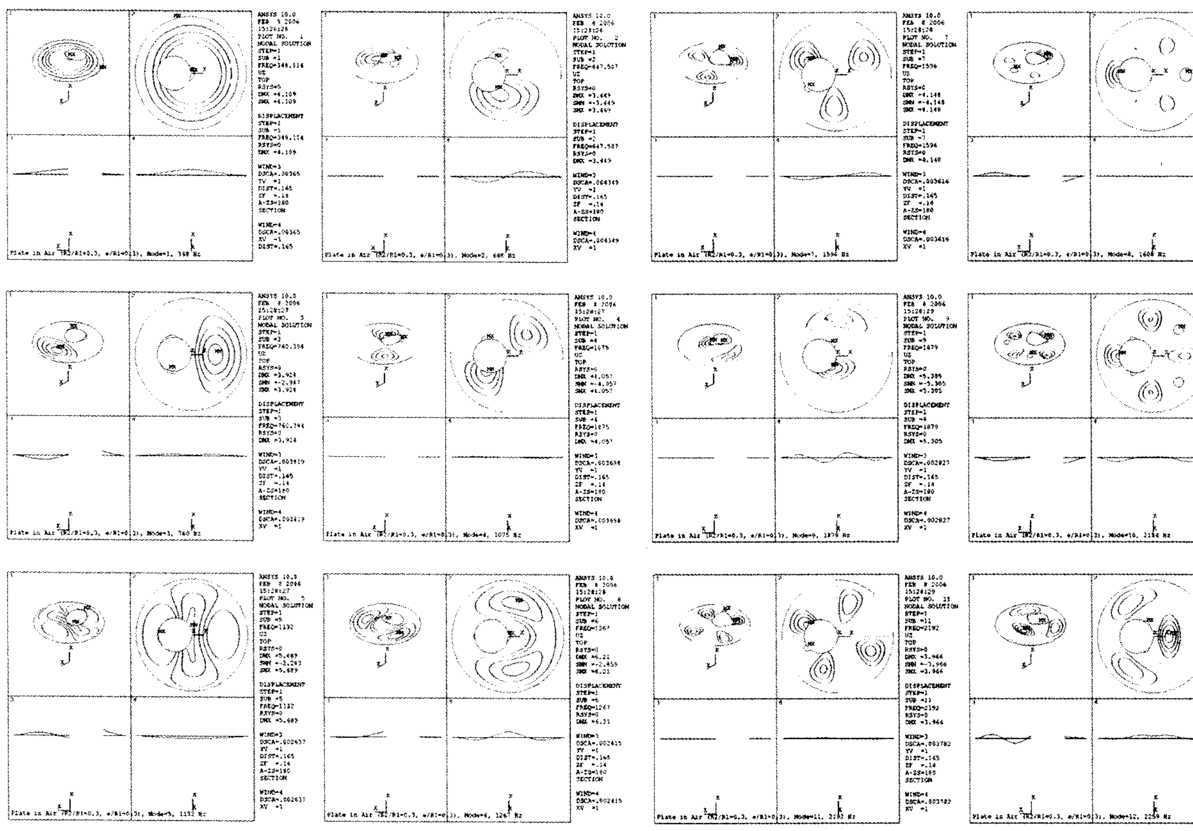

Fig. 14. Mode Shapes of Plate with an Eccentric Hole in Air for $e/R_1 = 0.3$

to the line connecting the center of the plate and the center of the hole; the other is the asymmetric mode, which has an asymmetric shape of the modes. Figure 14 shows the mode shapes of the plate with an eccentric hole (R_2/R_1=0.3) in air for $e/R_1 = 0.3$.

Frequency comparisons between eccentricity are shown in Fig. 15. For the radial mode number $m = 1$, asymmetric modes are not changed too much with respect to the eccentricity. But the frequencies of the symmetric mode increase with increasing eccentricity and decrease beyond a certain value of eccentricity. For example, symmetric mode (1,1) has the peak frequency at eccentricity 30% and symmetric mode (1,3) has the peak frequency at eccentricity 50%. Therefore, with the increasing modes the peak frequencies are obtained with the increasing eccentricity. The radial mode number $m = 2$ has the same characteristics as $m = 1$ but the distinction is not clear.

It was found from the study of the eccentric shells that the effect of the eccentricity on the frequencies is more severe on the appearance of transition modes or disappearance of certain modes than on the frequency changes [10]. Therefore, it was recommended to investigate the modal characteristics rather than frequency itself to know how much eccentricity there is in the shells with fluid-filled annulus. This may be applied to the plate with an eccentric hole with some modifications. The eccentricity of the hole has a slight effect on the asymmetric modes but the symmetric modes have appeared with the increasing eccentricity. Therefore, it is also recommended to investigate the modal characteristics rather than frequency itself to know how much eccentricity there is in the plate with an eccentric hole.

4.5 Hole Size Effect

The effect of hole size can be investigated by comparing

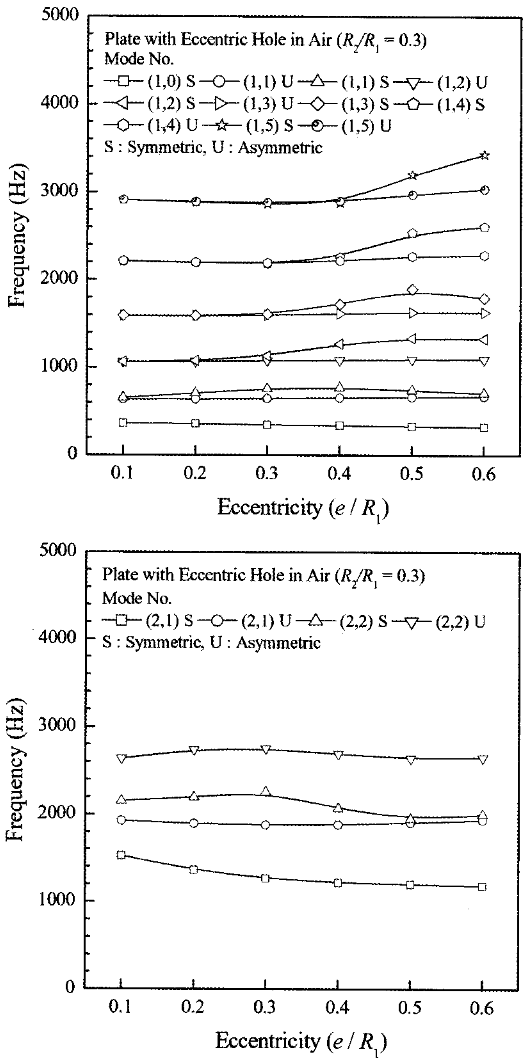

Fig. 15. Frequencies of Plate with an Eccentric Hole in Air with Respect to Eccentricity

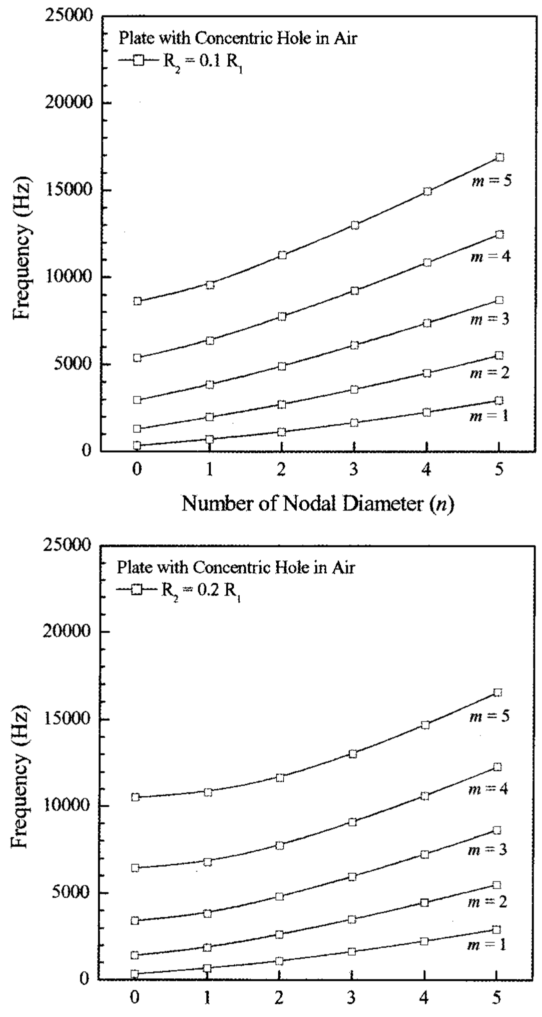

Fig. 16. Frequencies of Plate with a Concentric Hole in Air

the frequencies of the plate with a concentric hole with respect to the size of the hole radius. Figure 16 shows the frequencies of plate with a concentric hole in air for various sizes of hole. As the mode number increases, the frequencies increases but as the hole size increases, the extent of the increase becomes less significant.

Figure 17 shows the frequencies of plate with a concentric hole in air with respect to hole radius. As the ratio of hole to the radius of the plate increases, the frequencies of the lower number of nodal diameter are almost the same; for the ratio of hole size less than 20% of the plate the frequencies are almost the same irrespective of the hole size except for the circumferential mode number of 2. This kind of situation is evident for the higher number of circumferential modes. Or, for the circumferential number of mode 5, the effect of the hole size is found to be negligible, which indicates that for the hole size smaller than 40% of the plate the effect of mass and stiffness reduction is almost the same and therefore the frequencies are found to be unaffected by including the hole in the center of the plate for less than the radial mode number 3.

Summarizing this, the effect of hole size is negligible for higher circumferential and lower radial modes. Another thing to be noted is that for hole sizes larger than 40% of the plate, the frequencies increase slightly with respect to the number of nodal diameter, especially for higher circumferential modes.

5. CONCLUSIONS

An analytical method to estimate the frequencies of a circular plate submerged in fluid is developed using the finite Fourier-Bessel series expansion and Rayleigh-Ritz

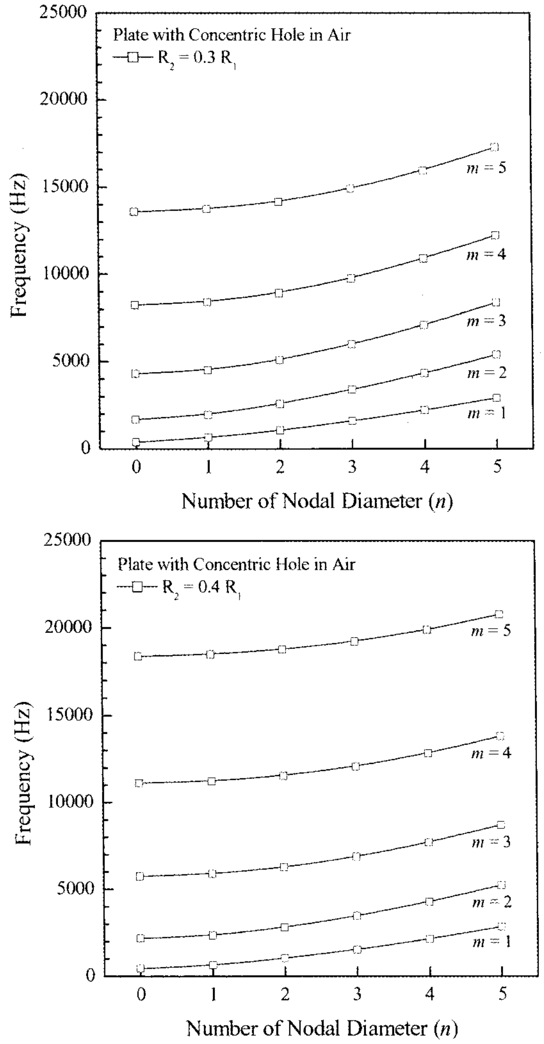

Fig. 16. Frequencies of Plate with a Concentric Hole in Air (Cont'd)

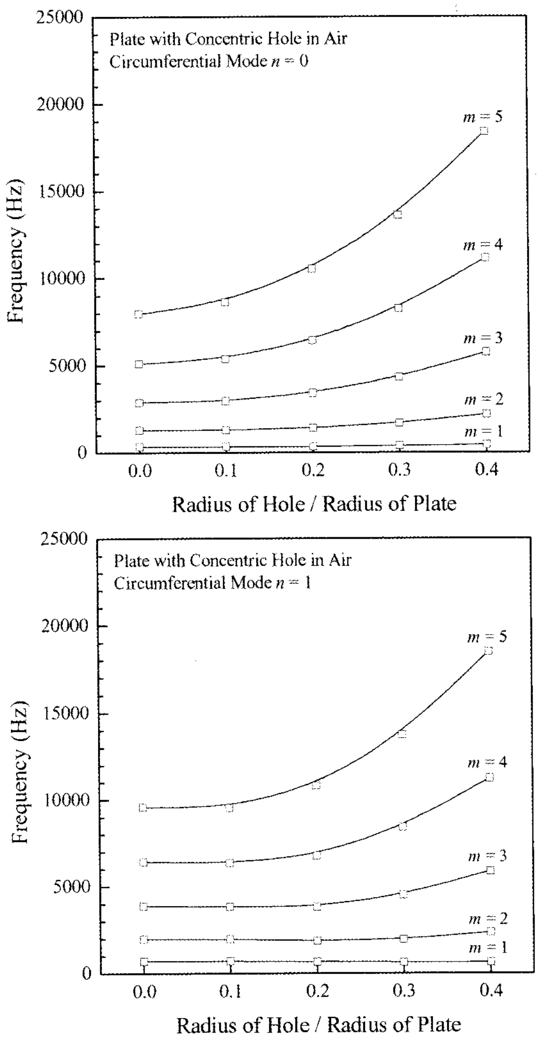

Fig. 17. Frequencies of Plate with a Concentric Hole in Air with Respect to Hole Radius

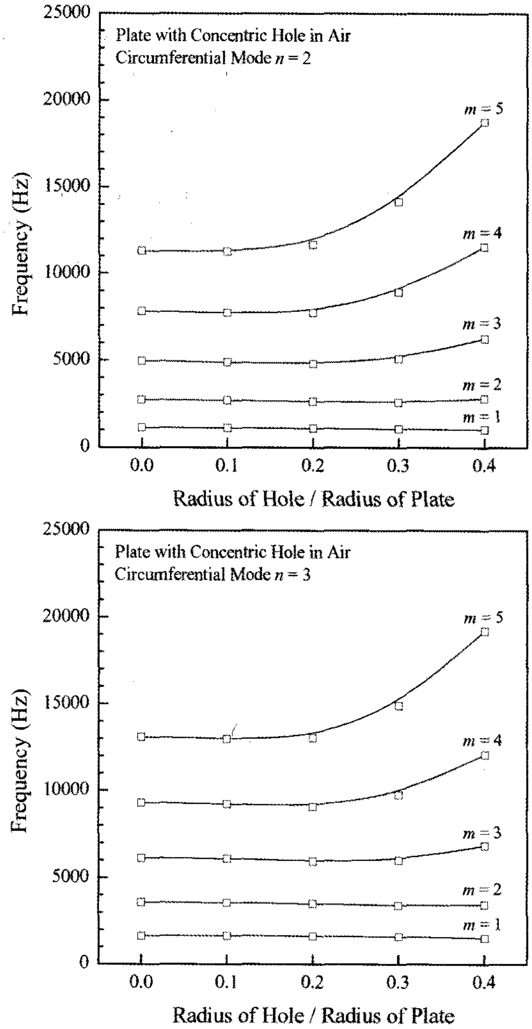

Fig. 17. Frequencies of Plate with a Concentric Hole in Air with Respect to Hole Radius (Cont'd)

method. The effects of fluid, hole, eccentricity, and hole size on the modal characteristics are investigated; results can be used as guidance for the modal analysis and damage detection of a circular plate with a hole.

The following conclusions were generated:

- To verify the validity of the analytical method developed, finite element method is used and the frequency comparisons between theoretical and finite element analysis results are found to be in good agreement.
- If the plate is submerged in fluid, the frequencies decrease to the values of less than 0.5. The extent of the decrease is more severe in the lower modes. Also, the case of a plate without a hole is slightly affected by the fluid, especially for the lower modes.
- Including a hole generally increases the frequencies. As the mode numbers increase, the normalized frequency tends to converge to unity, which means that for the higher modes the concentric hole effect is almost negligible. Including a hole in the submerged fluid increases the frequencies slightly but for the lower modes the frequencies decrease by including a concentric hole.
- The eccentricity of the hole has a slight effect on the asymmetric modes but the symmetric modes appear with the increasing eccentricity. Therefore, it is also recommended to investigate the modal characteristics rather than frequency itself to know how much eccentricity there is in the plate with an eccentric hole.
- The effect of hole size is negligible for higher circumferential and lower radial modes. For hole sizes larger than 40% of the plate radius, the frequencies increase slightly with respect to the number of nodal diameter especially for higher circumferential modes.

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