

Time-Cost Trade-Off by Lead-Time Adjustment in the PDM Network

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Abstract

Since the late 1980s, the schedule technique applied to the construction industry around the world has rapidly changed from the traditional ADM (Arrow Diagramming Method) to the PDM (Precedence Diagramming Method) technique. The main reason for this change is to overcome the limits and inconveniences of the traditional ADM technique. The time-cost trade-off is one of the core scheduling techniques to establish the best optimized combination plan in terms of a relationship between the cost and schedule. However, most of the schedule-related textbooks and research papers have discussed and proposed applications of a time-cost trade-off technique based only on the Finish to Start relationship. Therefore, there are almost no consideration and discussion of problems or restrictions that emerge when the time-cost trade-off technique is applied to the PDM network that has overlapping relationships. This paper proposes the lead-time adjustment method as a methodology for overcoming some restrictions that are encountered when the time-cost trade-off technique is applied to the overlapping relationships of the PDM network.

Keywords : Time-Cost Trade-Off, Lead-Time Adjustment, Overlapping Relationships, PDM Network

1. INTRODUCTION

1.1 BACKGROUND

Since the late 1980s, the schedule technique applied to the construction industry around the world has rapidly changed from the traditional ADM (Arrow Diagramming Method) to the PDM (Precedence Diagramming Method) technique. The main reason for this change is to overcome the limits and inconveniences of the traditional ADM technique. However, the ADM is still the main schedule technique applied to the construction industry and the basic schedule technique for most schedule-related textbooks and exams for professional engineer licensed by government by the Korean government which is very different from the global trend.

The main disadvantage of the ADM technique is that it does not allow overlapping relationships between preceding and succeeding activities. Therefore, this technique can not exactly represent the construction processes that are actually performed on construction sites, and largely drop off in efficiency of the schedule management due to its managerial inconvenience. On the other hand, the PDM technique has advantages of exactly representing any type of relationship between the activities by allowing overlapping relationships between the preceding and succeeding activities, and improving the efficiency of schedule management by decreasing the number of activities in the network. Due to the advantages of PDM, most popular schedule management software such as P3, Artemis, and Open Plan in the world abandoned the ADM technique and has been applying only the PDM technique for the time-cost trade-off is one of the crucial scheduling techniques to establish the best optimized cost and time combination plan. This is importantly accounted as a method to establish the best optimized schedule not only to fit the normally planned period into the contractual period

of the project but also to recover schedule delay in terms of mutual relationships between the cost and time in the activities during project implementation. Until recently, however, most schedule-related textbooks and research papers have discussed and proposed applications of time-cost trade-off technique based only on the Finish to Start relationship. Therefore, there are almost no consideration and discussion of problems or restrictions that emerge when the time-cost trade-off technique is applied to the PDM network that has overlapping relationships.

This paper proposes the lead-time adjustment method as a methodology for overcoming some restrictions that are encountered when the time-cost trade-off technique is applied to the overlapping relationships of the PDM network.

1.2 RESEARCH PROCESS AND METHOD

This research is limited to the PDM network that has the overlapping relationships, and has been performed as the following process and method.

The first step is to review about the theories of the time-cost trade-off technique and overlapping relationships.

The second step is to analyze various restrictions and influences in case of the activity time compression by applying the time-cost trade-off technique to the four logical relationships in the PDM network, and extract the need for the lead-time adjustment method to overcome those restrictions.

The third step is to propose the lead-time adjustment method of overlapping relationships and apply them to the example PDM network.

The fourth step is to compare and analyze the results and effects of the lead-time non-adjustment and lead-time adjustment methods applied to the sample PDM network.

2. REVIEW OF THE TIME-COST TRADE-OFF TECHNIQUE AND OVERLAPPING RELATIONSHIPS

2.1 TIME-COST TRADE-OFF TECHNIQUE

In many cases the contractual time requirements have been established by an engineer or owner without regard to a reasonable assessment of the activity durations. It is then imperative that a scheduler attempt to adjust the activity durations and sequences to fit within the contractual requirements, even if inefficient adjustments such as multiple shifts, overtime, or large crew sizes are required. (Callahan, 1992)

In the most initial schedules, the completion of all or part of the project must be shortened to fit the normally planned period into the contractual period of the project. The easiest way to reduce the duration of a project is to eliminate unnecessary restraints between the various activities. Another way is to re-sequence activities such as dividing them into smaller activities that can be performed concurrently and thus reduce the overall duration by overlapping various operations. However, the best way to adjust the duration of a project is to apply the time-cost trade-off technique.

The time-cost trade-off is a core theory of the CPM (Critical Path Method) that establishes the best time-cost combination plan in terms of mutual relationships between the activity's cost and time. The procedure for the time-cost trade-off technique is summarized as follows.

The first is to find a crash point which is to perform an activity in the shortest time, and a normal point, which is to perform an activity at the lowest cost. The second is to determine a cost slope which is a linear assumption line connecting a normal point to a crash point. The third is to reduce the activity's duration or time compression on the critical path in order of the lowest cost slope up to the required period.

2.2 OVERLAPPING RELATIONSHIPS

Until the late 1980s since the CPM was introduced in 1956, most construction projects applied the ADM technique to their scheduling. The ADM network consists of an arrow which represents an activity, and an event (or a node) which represents the activity's start and finish. As a basic logic of the ADM scheduling is based on the assumption that a succeeding activity can start only after the completion of all preceding activities, it does not allow overlapping relationships between activities. However, because work situations in which succeeding activities can start without the completion of preceding activities frequently happen in construction projects, it is impossible to exactly represent the actual sequences of construction activities by the ADM.

The PDM network proposed by Professor Fondahl in 1961 consists of a node that represents an activity and a line that represents the connection between the preceding and succeeding activities. Ponce-Campos proposed four logical types that make overlapping relationships between the activities of the PDM network possible in his doctoral dissertation in 1971.

As any kind of overlapping relationships between activities can be represented by the four logical types, recently

most CPM schedules have been established by the PDM technique. An overlapping period between the activities in the PDM network is represented by the lead-time that is defined as the period of time between the connection points of any preceding and succeeding activity, (Harris, 1978).

3. RESTRICTION ANALYSIS OF TIME-COST TRADE-OFF IN OVERLAPPING RELATIONSHIPS

3.1 RESTRICTIONS OF TIME-COST TRADE-OFF IN FOUR LOGICAL RELATIONSHIPS

When all of the preceding and succeeding activities lie on the critical path of the PDM network that has overlapping relationships, and their durations should be compressed in order of the lowest cost slope by the time-cost trade-off, the following restrictions happen at the four logical relationships.

3.1.1 Finish to Start Relationshipship

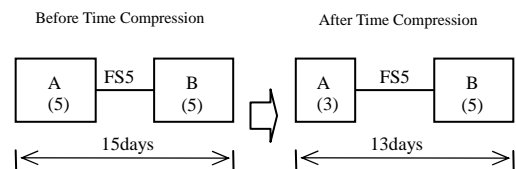


Figure 1. Time Compression of the FS Relationship

The Finish to Start (FS) is a relationship in which the succeeding activity starts after the preceding activity has finished. When the finish of the preceding activity is moved earlier during the FS relationship, the project duration is compressed because the start of the succeeding activity can be moved equally earlier. Therefore, the FS relationship has no restriction on the application of time-cost trade-off. Figure 1 shows an example of the project duration being compressed equally when the preceding activity is moved earlier in the FS relationship.

3.1.2 Start to Start Relationshipship

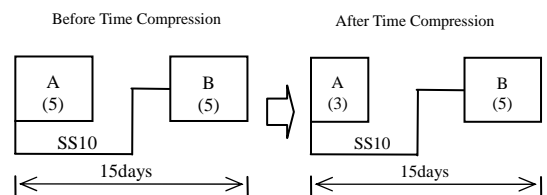


Figure 2. Time Compression of the SS Relationship

The Start to Start (SS) is a relationship in which the succeeding activity starts after the preceding activity has started and passed as much as the lead-time. When the duration of the preceding activity is compressed at the SS relationship, the finish of the preceding activity is moved earlier, but the start of the preceding activity cannot be moved. This means that the start of the succeeding activity cannot be moved consequently because it is connected to the start of the preceding activity by the lead-time. Therefore, the compression of the preceding activity of the SS

relationship does not affect to the compression of the total project duration at all. Figure 2 shows an example of the project duration not being compressed although the finish of the preceding activity is moved earlier at the SS relationship.

3.1.3 Finish to Finish Relationship

The Finish to Finish (FF) is a relationship in which the succeeding activity finishes after the preceding activity has finished and passed as much as the lead-time. When the duration of the preceding activity is compressed in the FF relationship, the finish of the preceding activity is moved earlier and the finish of succeeding activity is moved equally. However, when the duration of the succeeding activity is compressed, the finish of the succeeding activity cannot be moved earlier because it is connected to the finish of the preceding activity by the lead-time. This means that the lead-time of the FF relationship is restricted to moving the finish of the succeeding activity earlier. Figure 3 shows an example that the finish of succeeding activity cannot be moved earlier due to the lead-time of the FF relationship.

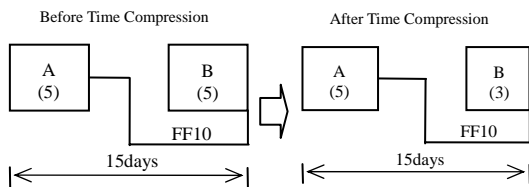


Figure 3. Time Compression of the FF Relationship

3.1.4 Start to Finish Relationship

The Start to Finish (SF) is a relationship in which the succeeding activity finishes after the preceding activity started and passed as much as the lead-time. When the duration of the preceding activity is compressed in the SF relationship, the finish of the preceding activity is moved earlier but the start of preceding activity cannot be moved. In addition, when the duration of the succeeding activity is compressed in the SF relationship, the finish of the succeeding activity cannot be moved earlier because it is connected to the start of the preceding activity by the lead-time. Therefore, the compression of preceding and succeeding activities in the SF relationship do not affect the compression of project duration due to the lead-time of the SF relationship. Figure 4 shows an example when the lead-time of the SF relationship is restricted to the compression of the project duration.

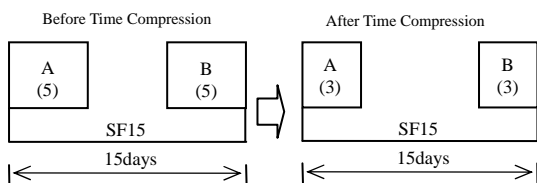


Figure 4. Time Compression of the SF Relationship

3.2 A CASE STUDY OF THE TIME-COST TRADE-OFF IN THE PDM NETWORK

It is necessary to confirm that the lead-time of the SS, FF and SF logical relationships restrict the compression of the activity durations when the time-cost trade-off is applied in the PDM network. The sample PDM network in this paper is shown in Figure 5. It has 8 activities and 3 overlapping logical relationships (the SS, FF and SF), and a critical path of A-C-F-H as a result of the time computation. The activity's legend in Figure 6 shows an activity ID, early start date (ESD), early finish date (EFD), late start date (LSD), late finish date (LFD), free float (FF), total float (TF), crash and normal work duration, and cost slope of activity, and represents a logical relationship, lead-time, and link lags¹ on the line of preceding and succeeding activities. The asterisk (*) in the front of the activity ID represents the impossibility of time compression according to the restrictions on the four logical relationships.

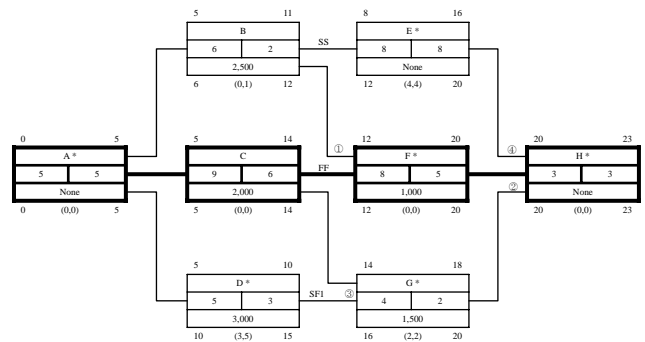


Figure 5. An example of the PDM Network

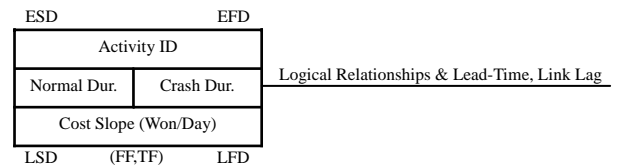


Figure 6. Activity's Legend

The time-cost trade-off on the sample PDM network is performed as the following steps.

The first step of the time-cost trade-off is as follows. The first step is to select an activity F which has 3 days of compressible work duration and the lowest cost slope of 1.5 million won per day on the critical path A-C-F-H. It looks like activity F can be compressed by only 2 days because the path G-H has 2 days of lag value. However, an activity F, which is a succeeding activity in the FF relationship, cannot be compressed at all because the finish time of the succeeding activity in the FF relationship cannot be moved earlier due to the lead-time connected with its preceding activity as the 3.1.3 clause. The next is to select activity C which has 3 days of compressible work duration and the second lowest cost slope of 2 million won per day on the critical path A-C-F-H. It looks like that ac-

¹ A link lag is defined as the difference between the early start date of an activity and the early finish date of the preceding activity. (Harris, 1978)

tivity C can be compressed by a whole 3 days because the path E-H has 4 days of lag value. However, because activity D and G connect with the SF relationship, the finish of activity G has no change due to the lead-time of the SF relationship as the 3.1.4 clause. Therefore, the path C-G-H that has activity G cannot be compressed although the finish of activity C can be moved earlier. As a result, activity C can be compressed by only 1 day within the lag value of path B-F. After the first step of the time-cost trade-off, the total project duration is compressed by 1 day from 23 to 22, the total cost for time compression is 2 million won, and the path A-B-F-H is added to the critical path. The first step result of the time-cost trade-off is shown in Figure 7.

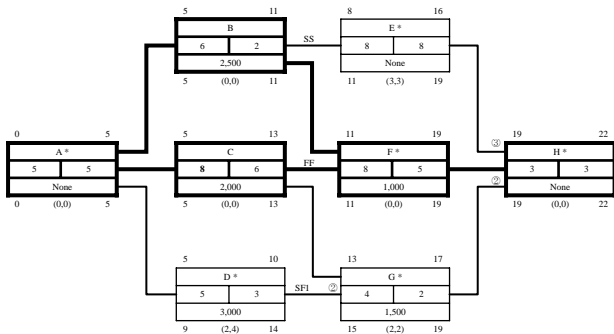


Figure 7. The First Step Result of Time-Cost Trade-off

The second step of the time-cost trade-off is as follows. The first is to select the combination of activity B on the critical path A-B-F-H and activity C on the critical path A-C-F-H, which has the lowest combined cost slope of 4.5 million won per day. However, because activity B and E connect with the SS relationship, the start of activity E has no change due to the lead-time of SS relationship as the 3.1.2 clause. The combination of activities, B and C can be compressed by 2 days because activity C has 2 days of compressible work duration. After the second step of the time-cost trade-off, the total project duration becomes 20 days, and the total cost for time compression increases to 11 million won which is the sum of 2 million won and 9 million won for the first and second time compression respectively. After the second step, further time compression on the network is impossible because all activities on the critical path A-C-F-H have asterisk marks. The second step result of the time-cost trade-off is shown on the Figure 8.

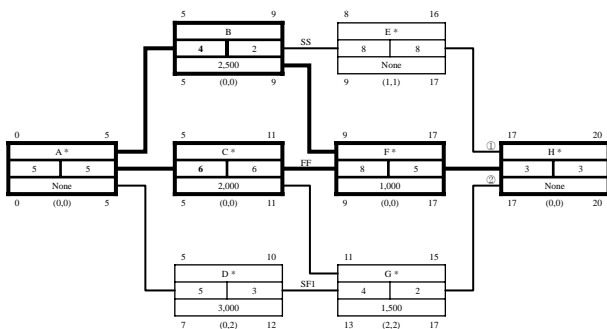


Figure 8. The Second Step Result of Time-Cost Trade-off

3.3 NEED FOR LEAD-TIME ADJUSTMENT OF THE TIME-COST TRADE-OFF IN OVERLAPPING RELATIONSHIPS

A case study of the sample network shows that only 3 days are compressed even if the total number of compressible days on the network is 6 which is the sum of 3 days of activity C and 3 days of activity F. The main reason for the insufficient time compression is because the lead-time of the overlapping relationships restricts the compression of the activity durations when the time-cost trade-off is applied. Therefore, the lead-time between the preceding and succeeding activities must be adjusted according to the compression of the activity duration. For instance, if the finish of the preceding activity by the time compression at the SS relationship is moved earlier, the start of the succeeding activity should be moved equally because the start of the succeeding activity is co-related to the duration of the preceding activity and the lead-time of the SS relationship. In other words, if the duration of the preceding activity during the SS relationship is compressed and the finish of the preceding activity is moved earlier, it is so rational that the lead-time should be compressed and the start of succeeding activity should be moved earlier as much as the time compression. Also during the FF relationship, if the duration of the succeeding activity is compressed, it is quite right that the finish of the succeeding activity is moved as much as the time compression because the finish of the succeeding activity is co-related with the duration of the succeeding activity and the lead-time of the FF relationship. In the SF relationship, if the duration of the preceding or succeeding activity is compressed, it is very reasonable that the start or finish of the succeeding activity should be moved earlier as much as the time compression by the same logics applied to the SS and FF relationships. The result of the case study suggests a need to upgrade the traditional time-cost trade-off technique in order to apply it more realistically and reasonably to the overlapping relationship.

4. THE LEAD-TIME ADJUSTMENT IN OVERLAPPING RELATIONSHIPS

4.1 The Lead-Time Adjustment Method by Compression Days

This research proposes the lead-time adjustment method by compression days when the time-cost trade-off technique is applied. This method is to shorten the lead-time as much as the compression days when the durations of the preceding and succeeding activities on the overlapping relationships are compressed. However, the time compression of activity by the proposed method is performed until the lead-time becomes zero "0" because it cannot be below zero or minus. The lead-time adjustment method of the activities connected with the SS, FF, and SF relationships is as follows.

4.1.1 Start to Start Relationship

When the duration of the preceding activity in the Start to Start relationship is compressed, the lead-time con-

connected with the SS relationship is shortened, and the start of the succeeding activity is moved earlier as much as the compression days of the preceding activity. Figure 9 shows an example that the lead-time is shortened and the start of the succeeding activity is moved earlier by 2 days when the duration of the preceding activity is compressed by 2 days in the SS relationship.

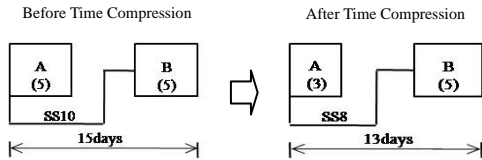


Figure 9. Time Compression of the SS Relationship by the Lead-Time Adjustment Method

4.1.2 Finish to Finish Relationship

When the duration of the succeeding activity in the Finish to Finish relationship is compressed, the lead-time connected with the FF relationship is shortened and the finish of succeeding activity is moved earlier as much as the compression days of the succeeding activity. Figure 10 shows an example in which the lead-time is shortened and the finish of the succeeding activity is moved earlier by 2 days when the duration of the succeeding activity is compressed by 2 days in the FF relationship.

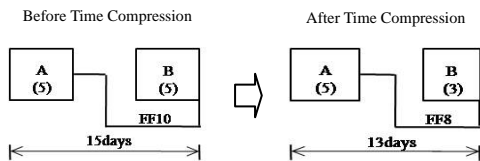


Figure 10. Time Compression of the FF Relationship by the Lead-Time Adjustment Method

4.1.3 Start to Finish Relationship

When the duration of the preceding or succeeding activity at the Start to Finish relationship is compressed, the lead-time connected with the SF relationship is shortened, and the start of succeeding activity is moved earlier as much as the compression days of the preceding or succeeding activity. Figure 11 shows an example in which the lead-time is shortened and the start of the succeeding activity is moved earlier by 4 days when the durations of the preceding and succeeding activities are compressed by 2 days in the SF relationship.

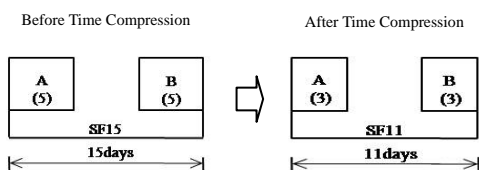


Figure 11. Time Compression of the SF Relationship by the Lead-Time Adjustment Method

4.2 A CASE STUDY OF TIME-COST TRADE-OFF BY LEAD-TIME ADJUSTMENT IN THE PDM NETWORK

The time-cost trade-off applied by the lead-time adjustment method to the sample PDM network in Figure 3 is performed as the following steps.

The first step is to select activity F which has 3 days of compressible work duration and the lowest cost slope of 1 million won per day on the critical path A-C-F-H. Activity F can be shortened by 2 days only because the path G-H has 2 days of lag value. In addition, the lead-time of the FF relationship between activity C and F is compressed by 2 days from 6 to 4 by the rule of lead-time adjustment method as the 4.1.2 clause. After the first step of the time-cost trade-off, the total project duration is compressed by 2 days from 23 to 21, and the total cost for time compression is 2 million won, and the path A-C-G-H is added to the critical path. The first step result of the time-cost trade-off applied by the lead-time adjustment method is shown in Figure 12.

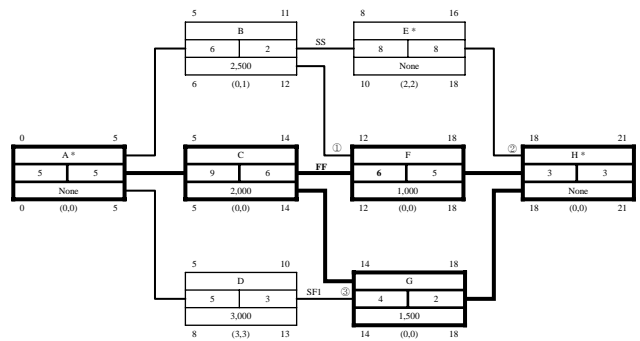


Figure 12. The First Step Result of Time-Cost Trade-off Applied by the Lead-time Adjustment Method

The second step is to select activity C which has 3 days of compressible work duration and the lowest cost slope of 2 million won per day on the critical paths of A-C-F-H and A-C-G-H. Activity C can be shortened by 1 day only because the path B-F has 1 day of lag value. After the second step of the time-cost trade-off, the total project duration is compressed by 1 day from 21 to 20, the total cost for time compression increases to 4 million won, and the path A-B-F-H is added to the critical path. The second step result of the time-cost trade-off is shown in Figure 13.

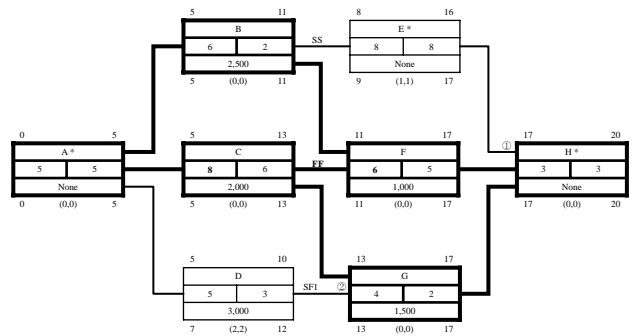


Figure 13. The Second Step Result of Time-Cost Trade-off Applied by the Lead-time Adjustment Method

The third step is to select the combination of activities F and G on the critical paths A-C-F-H, A-C-G-H, and A-B-F-H, which have the lowest combined cost slope of 2.5 million won per day. Activities F and G can be shortened by 1 day because the compressible day of activity F remains 1 day and path E-H has 1 day of lag value. In addition, the lead-time of the FF relationship between activities C and F is compressed from 4 to 3 days by the rule of the lead-time adjustment method as the 4.1.2 clause. After the third step of the time-cost trade-off, the total project duration is compressed by 1 day from 20 to 19, the total cost for time compression increases to 6.5 million won, and the path A-B-E-H is added to the critical path. The third step result of the time-cost trade-off applied by the lead-time adjustment method is shown in Figure 14.

The fourth step is to select the combination of activities B and C on the critical paths A-C-F-H, A-C-G-H, A-B-F-H, and A-B-E-H, which have the lowest combined cost slope of 4.5 million won per day.

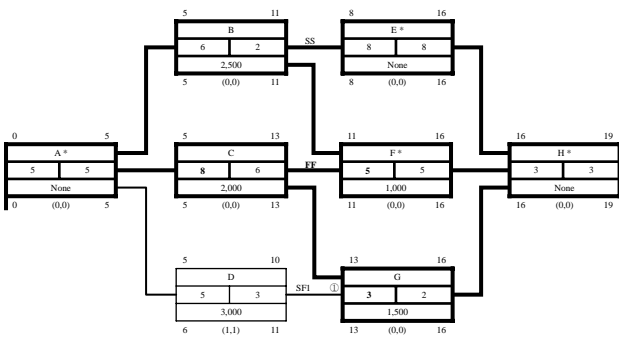


Figure 14. The Third Step Result of Time-Cost Trade-off Applied by the Lead-time Adjustment Method

Activities B and C can be shortened by 1 day because the path D-G has 1 day of lag value. In addition, the lead-time of the SS relationship between activities B and E is compressed from 3 to 2 days by the rule of the lead-time adjustment method as the 4.1.1 clause. After the fourth step of the time-cost trade-off, the total project duration is compressed by 1 day from 19 to 18, and the total cost for time compression increases to 11 million won, and the path A-D-G-H is added to the critical path. The fourth step result of the time-cost trade-off applied by the lead-time adjustment method is shown in Figure 15.

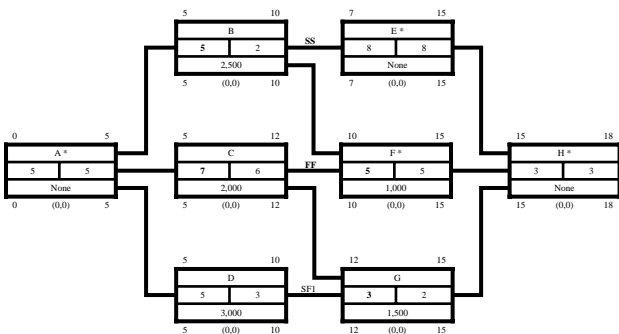


Figure 15. The Fourth Step Result of Time-Cost Trade-off Applied by the Lead-time Adjustment Method

The fifth step is to select the combination of activities B, C, and D on the critical paths A-C-F-H, A-C-G-H, A-B-F-H, A-B-E-H, and A-D-G-H which have the lowest combined cost slope of 7.5 million won per day. Activities B, C, and D can be shortened by 1 day because the compressible day of activity C remains 1 day. In addition, the lead-time of the SS relationship between activities B and E is compressed from 2 to 1 day, and the lead-time of the SF relationship between activities D and G is compressed from 10 to 9 days by the rules of the lead-time adjustment method as the 4.1.2 and 4.1.3 clauses, respectively. After the fifth step of the time-cost trade-off, the total project duration is compressed by 1 day from 18 to 17, and the total cost for time compression increases to 18.5 million won. The fifth step result of the time-cost trade-off applied by the lead-time adjustment method is shown in Figure 16.

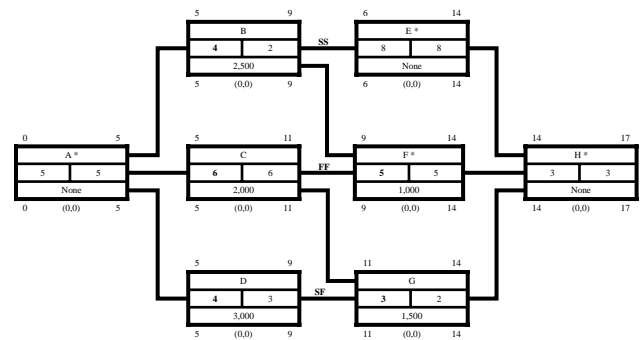


Figure 16. The Fifth Step Result of Time-Cost Trade-off Applied by the Lead-time Adjustment Method

After the first through fifth steps of the time-cost trade-off by the lead-time adjustment method, the sample network cannot be further compressed because all activities on the critical path A-C-F-H have the asterisk marks.

4.3 COMPARISON BETWEEN THE TIME-COST TRADE-OFF BY NO LEAD-TIME ADJUSTMENT AND THE LEAD-TIME ADJUSTMENT

The two case studies obviously show the different results such that a total of 6 days on the critical path can be compressed by the lead-time adjustment method while only a total of 3 days can be compressed by the traditional method although a total of 7 days as the sum of 4 days of an activity B and 3 days of activity F on the path A-B-F-H can be compressed. In conclusion, it is confirmed that the traditional time-cost trade-off has some restrictions in compressing the activity duration due to the lead-time of the overlapping relationships in comparison with the time-cost trade-off by the lead-time adjustment method. This means that an objective of the time-cost trade-off to establish the best optimized cost and schedule combination plan can be achieved more actively and reasonably by the lead-time adjustment method on the overlapping relationships.

5. CONCLUSION

The time-cost trade-off is one of the core scheduling

techniques to establish the best optimized combination plan in terms of a relationship between the cost and schedule. Especially, it is importantly accounted as a method to establish the best optimized schedule not only to fit the normally planned period into the contractual period of project but also to recover the schedule delay in terms of mutual relationships between the cost and schedule in the activities during project implementation. However, most schedule related-textbooks and research papers have discussed and proposed the applications of time-cost trade-off technique based only on the Finish to Start relationship. Therefore, there are almost no consideration and discussion of the problems or restrictions that emerge when the time-cost trade-off technique is applied to the PDM network that has overlapping relationships.

Based on the analysis of some restrictions that are encountered when the time-cost trade-off technique is applied to the PDM network with the SS, FF, and SF relationships, this paper proposes the lead-time adjustment method as a methodology to overcome them. It is analyzed that the proposed method can shorten the most parts of the compressible durations of critical activities on the network rather than applying the traditional method. This means that an objective of the time-cost trade-off to establish the best optimized cost and schedule combination plan can be achieved more actively and reasonably by the lead-time adjustment method. The current research is focused only on the lead-time adjustment by compression days, but future research will be performed on another possible method such as the lead-time adjustment method by compression proportion.

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