

The Effect of Rebirthing Technique on GA-based Size Optimization

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Abstract

The effect of rebirthing technique on the genetic algorithm (GA)-based size optimization is investigated. The GA mimics the principles of nature and it can gradually improve structural design through biological operations such as fitness, selection, crossover and mutation. However, premature optimum has been often detected in the generic GA with continuous design variable. Since then, the so-called rebirthing technique has been proposed to avoid this problem. However, the performance of the rebirthing technique has not been reported. Therefore, the size optimizations of spatial structures are tackled to investigate the performance of the rebirthing technique on the generic GA. From numerical results, it is well proved that the rebirthing technique is very effective to produce the optimum values regardless of the values of parameters used in the GA operations.

Keywords : Genetic Algorithm, Rebirthing Technique, Crossover Rate, Mutation Rate, Selection, Spatial Structures

1. INTRODUCTION

The genetic algorithm (GA) was originally proposed by Holland at the University of Michigan in 1970s. It is a search engine which mimics the principles of nature such as natural selection and survival of the fittest. Therefore, it does not require the gradient information of objective and constraint functions as *priori* in the design optimization problem unlike mathematical programming techniques do. The GAs can facilitate both discrete and continuous design variables in the optimization process so that they are considered as very versatile techniques to deal with most structural optimization problems.

The basic principles of the generic GA (Goldberg, 1989) and its application into the structural optimization problem are well described in open literatures (Adeli and Cheng, 1993; Goldberg and Samtani, 1986; Koumouis, 1994; Lee and Lee, 2007; Saka, 2007). As well known fact, the GA has very simple architecture so that it is involving nothing more than copying strings and swapping partial strings using the operations such as reproduction, crossover, and mutation in the design optimization.

However, it is also reported that GA produces poor optimums when the continuous design variables are used in some optimization problems. Specifically, the GA with continuous design variable can produce the premature optimum so that no further improvement in the objective function could be found over several iterations. Therefore, recently the concept of '*rebirthing*' was introduced in GA to overcome this difficulty (Ghasemi, Hinton and Wood, 1999). However, the capability of the rebirthing technique has not been fully evaluated and provided in open literatures although it has been advocated as a good way of preventing the premature optimum problem.

In this study, the effect of rebirthing on the GA-based size optimization is therefore thoroughly investigated using two spatial structures. In particular, we focus our investigation on the correlation between the rebirthing and the parameters of GA operators such as crossover and mutation.

Finally, we provide two complete benchmark test results obtained by using the GA with/without the rebirthing technique for future study.

2. GENERIC GA

2.1. Selection

The generic GA as illustrated in Figure 1 proceeds the searching in the following manner: An initial population of individuals is randomly generated. The individuals in the current population are decoded and their fitness is evaluated. To form a new population for the next generation, individuals are selected according to their fitness.

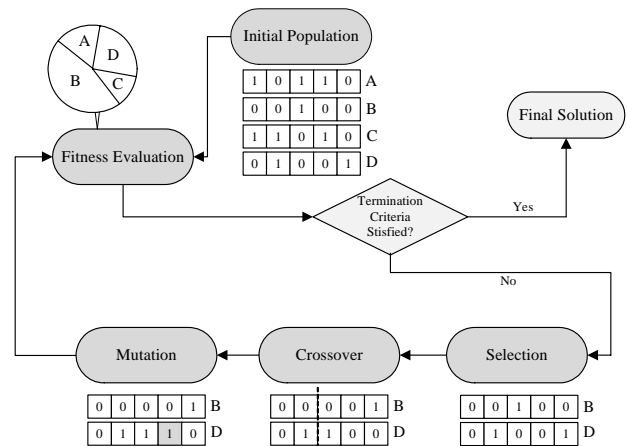


Figure 1. The process of generic GA

One of the simplest criteria for selection is Holland's original fitness-proportionate selection where individuals are selected according to their relative fitness. Therefore, an individual is chosen to be proportional to its relative performance in the population. The fitness level is used to associate a probability of selection with each individual. If F_i is the fitness of the i^{th} individual in the population, its probability of being selected is

$$p_i = \frac{F_i}{\sum_{j=1}^N F_j} \quad (1)$$

where N is the number of individuals in the population.

Thus, good individuals with high fitness have a better chance of ‘reproducing’, while low-fitness ones are likely to disappear. Selection methods such as roulette, tournament, top percent, best, random selection have also been proposed in GA (Yang and Sho, 1987). In particular, selection methods such as stochastic universal sampling or tournament selection have often been used in practice. This is because they have less stochastic noise and easy to implement. In this study, tournament selection is adopted in the optimization process.

2.2. Fitness evaluation

To carry out the design optimization by using GAs, whether dealing with equality or inequality constraints, the objective function F_j has to be converted into some corresponding fitness values in such a way that the best individual has maximum fitness. This is a general case to cater for minimization or maximization of the objective function. Goldberg suggests that for minimization problems, F_j should be subtracted from a large constant value so that all the fitness are of nonnegative values and individuals get fitness values according to their actual merit. In this study, this large constant value is found by adding the maximum and minimum values of F_j together. Therefore, the expression for fitness then becomes

$$\hat{F}_j = F_{j(\max)} + F_{j(\min)} - F_j \quad (2)$$

where \hat{F}_j is the transformed fitness of the j^{th} individual and $F_{j(\max)}$ and $F_{j(\min)}$ are the maximum and minimum values of F_j respectively.

2.3. Crossover

Crossover is a genetic operator used to vary individual from one generation to the next. Crossover may proceed in two steps: First, newly reproduced individuals in the mating pool are randomly mated. Next, each pair of strings of individual undergoes crossing over. There have been different types of crossover such as one point, two point, uniform, non-uniform, arithmetic and heuristic ones (Lee and Lee, 2007). We here introduce the simplest one such as one point crossover.

2.4. Mutation

Mutation is a genetic operator used to maintain genetic diversity from one generation of a population to the next. Mutation is a kind of the random alteration of the value of a string position of the individual and it is need to form new genetic patterns since selection and crossover sometimes cannot effectively search and recombine existing individuals and then fall into the local optimum.

3. REBIRTHING TECHNIQUE

When we try some optimization problems with continuous variables, it was sometimes found that premature optimum can be obtained so that no further improvement in

the objective function could be achieved over several iterations. When such a situation has occurred, it may be possible to alter the bounds of the side constraints S^ℓ and S^u into \hat{S}^ℓ and \hat{S}^u for each design variable. The method of finding the new bounds for each design variable which is named as ‘rebirthing’ can take place as follow: select the fittest design of the converged solution, and for each design variable S_i , choose new bounds using the expression:

$$\hat{S}_i^\ell = S_i^\ell + (S_i^c - S_i^\ell) \times (1 - r) \quad (3)$$

$$\hat{S}_i^u = S_i^u - (S_i^u - S_i^c) \times r$$

where r is a user specified percentage for new bounds.

For example, the rebirthing technique can be illustrated as Figure 2.

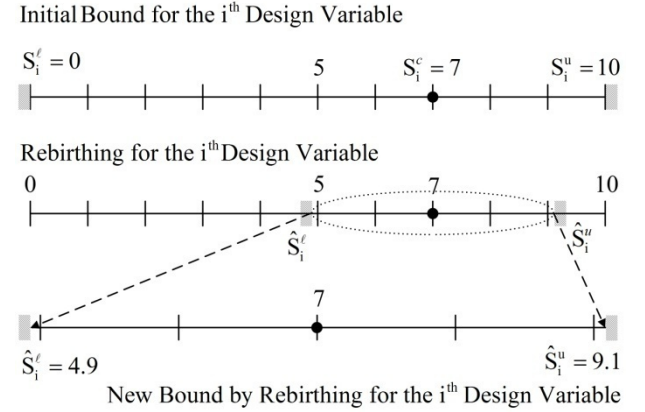


Figure 2. The new bounds calculation by using rebirthing technique

As shown in Figure 2, let us assume that the initial lower and upper bounds for all the design variables are $S_i^\ell = 0.0$ and $S_i^u = 10.0$ respectively and that the value of the current i^{th} design variables S_i^c at the point of rebirthing is equal to 7.0. When the first rebirthing occurs, the lower and the upper bounds S_i^ℓ , S_i^u for the current design variable S_i^c will then be adjusted as $\hat{S}_i^\ell = S_i^\ell + (S_i^c - S_i^\ell) \times (1 - r)$ and $\hat{S}_i^u = S_i^u - (S_i^u - S_i^c) \times r$ respectively. At this time, $r=0.35$ is used as a percentage value of the design variable within which it is allowed to move in either side of its value. Therefore the particular design variable $S_i^c = 7$ will have new lower and upper bounds of $\hat{S}_i^\ell = 4.9$ and $\hat{S}_i^u = 9.1$ respectively. However if its new bound exceeds the initial allowable bound, it have to be adjusted to the initial one.

Note that the same procedure is carried out automatically for the next rebirthing where different (and smaller) lower and upper bounds are automatically computed and allocated to that i^{th} design variable.

In this study, the rebirthing begins when the difference between the fittest designs over 5 generations is very small. In other word, if the values of design variables cannot be improved anymore since the values of design variables available are not fine enough to produce a better design. The implication here is that refining the values of design variables allows further searching for better designs. The process of rebirthing may repeat itself until no further improvement is possible.

4. GA WITH REBIRTHING TECHNIQUE

4.1. Pseudo-code for GA

The GA with rebirthing technique used in this study can be summarized in the following procedure:

```

choose the initial population
evaluate each individual's fitness
determine population's average fitness
repeat
    select best-ranking individuals to reproduce
    mate pairs random
    apply crossover operator
    apply mutation operator
    evaluate each individual's fitness
    determine population's average fitness
    apply rebirthing if it is on
until terminating condition
    
```

4.2. Termination Condition

As we proceed with more generations, there may not be much improvement on the population's fitness and the best individual may not change for subsequent populations. As the generations progress, the population gets filled by more fit individuals, with perhaps a slight deviation from the fitness of the best individual so far found, and the average fitness comes very close to the fitness of the best design. Criteria have to be evolved to decide on the termination of the process. In the present study, two criteria are used and if either one is satisfied, then the process will terminate. One criterion is when the percentage difference between the fittest design and the average of all the designs reaches a target value, namely the convergence percentage value. The other criterion implemented in the program ISADO, is when the fittest design does not change for a certain number of generations the process will terminate. If either of the two criteria is triggered then the process will end.

5. SIZE OPTIMIZATION PROBLEM DEFINITION

The size optimization problem can be defined by the following expression:

$$\begin{aligned} & \text{Minimize } \Psi(\mathbf{s}) \\ & \text{Subject to } g_i(\mathbf{s}) \leq 0 \quad i = 1, \dots, k \end{aligned} \quad (4)$$

where $\mathbf{s} = [s_1, s_2, \dots, s_{ndv}]$ is the vector of design variables, k is the number of constraints and ndv is the number of design variable.

Since the weight minimization of spatial structures is formulated here to see the effect of rebirthing technique, the $\Psi(\mathbf{s})$ can be defined as follows:

$$\Psi(\mathbf{s}) = \sum_{e=1}^{numel} \rho A_e \ell_e \quad (5)$$

where A_e is the cross-section area of the element e , ℓ_e is its length, ρ is the density of the material and $numel$ is the number of element. Constraint function $g_i(\mathbf{s})$ can be expressed in a non-dimensional form as

$$\left| \frac{\sigma_e}{\sigma_{all}} \right| - 1 \leq 0, \quad \left| \frac{u_n}{u_{all}} \right| - 1 \leq 0 \quad (6)$$

where σ_e , u_n is the stress and the displacement in the element e and node n respectively and σ_{all} , u_{all} is the allowable stress and displacement.

6. NUMERICAL EXAMPLES

In order to see the effect of rebirthing technique on GA-based size optimization, several important aspects of the generic GA are also tested. The parameters and aspects used in the present test can be summarized as follows:

- (a) Crossover rate (P_c),
- (b) Mutation rate (P_m),
- (c) Population size (P_s),
- (d) Rebirthing (R).

Each parameter is tested individually to show its effect on the process of GA-based size optimization. However, it should be noted that the other parameters are fixed at certain specified values when one parameter is tested. For the tests, we use two spatial structures such as 25-bar three-dimensional truss and geodesic dome. In this study, 4 cases are tested for each structure as described in Table 1. Each case includes 100 combinations of the parameters P_c and P_m .

Table 1. Four cases for parametric study

| Cases | Descriptions |
|-------|---|
| I | $P_s = 50$, no rebirthing, P_c and $P_m = 0.1 \sim 1.0$ |
| II | $P_s = 50$, rebirthing, P_c and $P_m = 0.1 \sim 1.0$ |
| III | $P_s = 200$, no rebirthing, P_c and $P_m = 0.1 \sim 1.0$ |
| IV | $P_s = 200$, rebirthing, P_c and $P_m = 0.1 \sim 1.0$ |

6.1. 25-bar Truss

A well-known 25-bar truss is used to investigate the effect of rebirthing technique on GA-based size optimization problem. The geometry of 25-bar truss is illustrated in Figure 3. The following material properties are assumed: elastic modulus $E = 1.0 \times 10^7$ ksi (6.89×10^7 MPa) and Poisson's ratio $\nu = 0.3$. The analysis is carried out using 25, 2-node truss FEs with 10 nodal points. The allowable stress and displacement are $\sigma_{all} = 40$ ksi (275.6MPa), $u_{all} = \pm 0.35$ in (0.889cm).

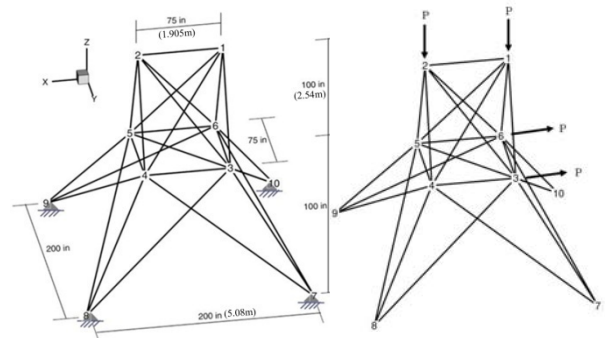


Figure 3. Geometry of 25-bar truss

The point loads are applied to the nodal points 1, 2, 3, 6.

The nodal point loads are summarized in Table 2.

| Node | X | Y | Z |
|------|-------------|---------------|---------------|
| 1 | 1000(0.453) | -10000(-4.53) | -10000(-4.53) |
| 2 | 0 | -10000(-4.53) | -10000(-4.53) |
| 3 | 500(0.226) | 0 | 0 |
| 6 | 600(0.272) | 0 | 0 |

The members of 25-bar truss are linked into 8 groups for design optimization as shown in Figure 4.

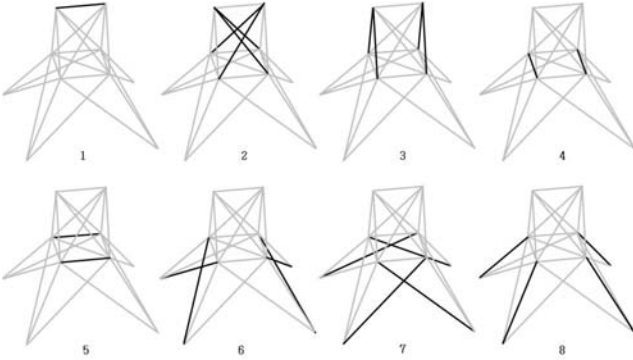


Figure 4. Member grouping and its number

[CASE I: $P_s = 50$; no R; $P_c, P_m = 0.1 \sim 1.0$]: We firstly investigate the effect of changing two main parameters P_c, P_m on the searching ability of the generic GA for the optimum. The 100 combinations of crossover and mutation rates are tested for this case. The population size P_s is taken as 50 without rebirthing technique. Numerical results for this case are illustrated in Figure 5. Note that the objective function values described in Figure 5 are normalized by using the minimum value of objective function Ψ_{min} among those of 100 combinations.

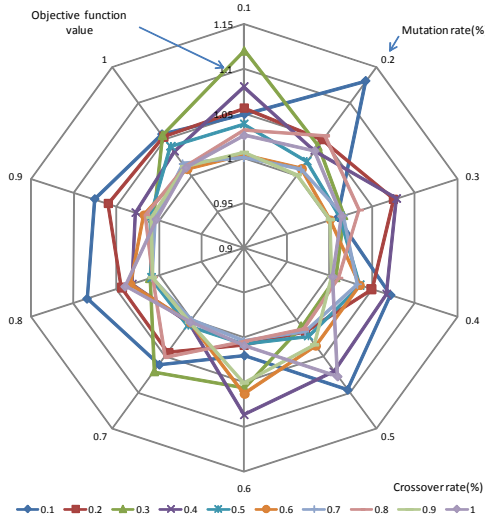


Figure 5. Objective function values distribution for the 100 combinations of crossover and mutation rates ($P_s = 50$ without rebirthing)

From numerical results, it is found to be that the different values of mutation and crossover rates produce widely distributed final values of objective function. In general,

the higher values of the objective function value are detected with low values of crossover rate. However, the larger values of crossover rate above 0.4 can provide good optimum values close to the minimum value Ψ_{min} .

[CASE II: $P_s = 50$; R; $P_c, P_m = 0.1 \sim 1.0$]: In this case, we introduce the rebirthing technique into the CASE I. The effect of the rebirthing technique on the problem with small size population is evaluated. The 100 combinations are tackled and numerical results are illustrated in Figure 6. From numerical results, the optimum values are quite well distributed close to the Ψ_{min} for most values of crossover rate due to the introduction of rebirthing. But the lower values of crossover rate such as $P_c = 0.1, 0.3$ produced a little higher values of objective function far from the Ψ_{min} . In particular, the combinations of lower values of P_c and $P_m=0.1, 0.5, 0.7$ produce poor values of the objective function.

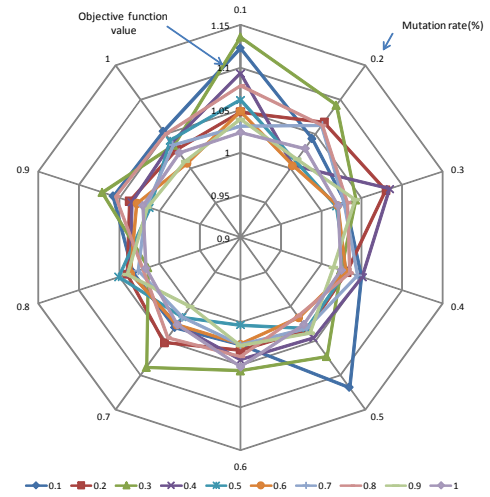


Figure 6. Objective function values distribution for the 100 combinations of crossover and mutation rates ($P_s = 50$ with rebirthing)

[CASE III: $P_s = 200$; no R; $P_c, P_m = 0.1 \sim 1.0$]: We tackle the same truss using the generic GA with a larger population size of $P_s = 200$ without rebirthing technique. The distribution of the final objective function values for this case is presented in Figure 7.

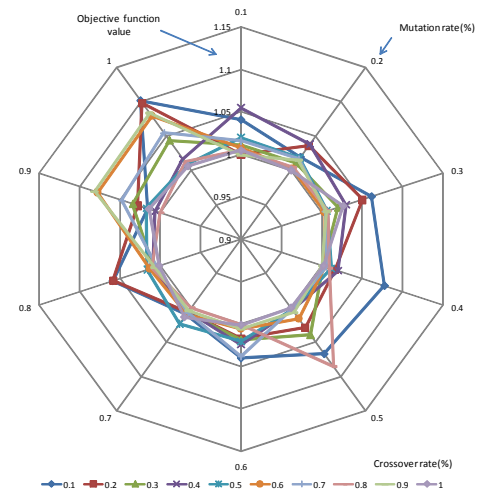


Figure 7. Objective function values distribution for 100 combinations of crossover and mutation rates ($P_s = 200$ without rebirthing)

It is found to be that the lower values of crossover rate produces the higher values of the final objective function. The probability of reaching the minimum final objective function value Ψ_{min} is greatly enhanced compared to the CASE I with only increasing the population size.

[CASE IV: $P_s = 200$; $R; P_c, P_m = 0.1 \sim 1.0$]: We then introduce the rebirthing technique to CASE III and plot numerical results in Figure 8.

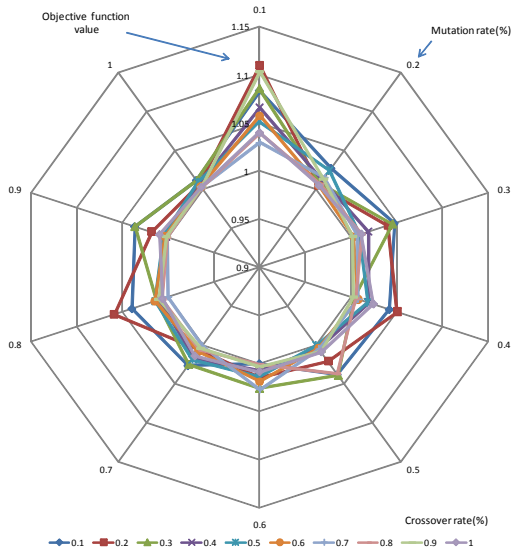


Figure 8. Objective function values distribution for the 100 combinations of crossover and mutation rates ($P_s = 200$ with rebirthing)

From the diagram as shown in Figure 8, the values of final objective function are well posed within $1.05 \times \Psi_{min}$ except several cases, in particular with $P_m = 0.1$.

Now, the minimum final objective function values for all four cases can be summarized as shown in Table 3 with some reference solutions (Zhu, 1986; Rajeev and Krishnamoorthy, 1996; Cai and Thiereu, 1993; Coello, 1994; Thong and Liu, 2001; Kripka, 2004).

| Methods | Weight |
|--|---------------|
| CASE I ($P_c = 0.1$; $P_m = 0.7$) | 471.14(213.7) |
| CASE II ($P_c = 0.7$; $P_m = 0.9$) | 474.12(215.0) |
| CASE III ($P_c = 0.1$; $P_m = 0.9$) | 469.81(213.1) |
| CASE IV ($P_c = 0.8$; $P_m = 0.7$) | 470.49(213.4) |
| Branch and Bound (Zhu, 1986) | 562.93(255.3) |
| Penalty method(Cai & Thiereu, 1993) | 487.41(221.0) |
| GA(Coello, 1994) | 539.78(244.8) |
| GA (Rajeev & Krishnamoorthy, 1996) | 546.01(247.6) |
| Quotient method(Thong & Liu, 2001) | 485.05(220.0) |
| Simulated Annealing (Kripka, 2004) | 484.33(219.6) |

Table 4 provides the final values of design variables for four cases. It is found to be that the best minimum value is obtained from the CASE III with $P_c = 0.1$ and $P_m = 0.8$ without rebirthing. However, the CASE IV also produces almost the same final objective function value to that of the CASE III.

Table 4. Final design variables [unit: in²(cm²)]

| Group | Area of cross-section | | | |
|-------|-----------------------|--------------|--------------|--------------|
| | CASE I | CASE II | CASE III | CASE IV |
| 1 | 0.019(0.12) | 0.089(0.57) | 0.029(0.19) | 0.026(0.17) |
| 2 | 0.049(0.32) | 0.234(1.51) | 0.044(0.28) | 0.051(0.33) |
| 3 | 3.765(24.29) | 3.490(22.52) | 3.575(23.06) | 3.661(23.62) |
| 4 | 0.322(2.08) | 0.120(0.77) | 0.634(4.09) | 0.059(0.38) |
| 5 | 1.507(9.72) | 1.851(11.94) | 1.887(12.17) | 1.935(12.48) |
| 6 | 0.795(5.13) | 0.813(5.25) | 0.761(4.91) | 0.780(5.03) |
| 7 | 0.292(1.88) | 0.254(1.64) | 0.278(1.79) | 0.251(1.62) |
| 8 | 3.853(24.86) | 3.843(24.79) | 3.931(25.36) | 3.869(24.96) |

We also plot all the objective function values for 400 combinations to see its overall distribution for this example. We can see that the distribution of objective function values of the CASE IV has best distribution as plotted in Figure 9. It turns out to be that the design can have more chance of getting the minimum objective function value with the rebirthing. In particular, better minimum final objective values are produced with larger population size $P_s = 200$.

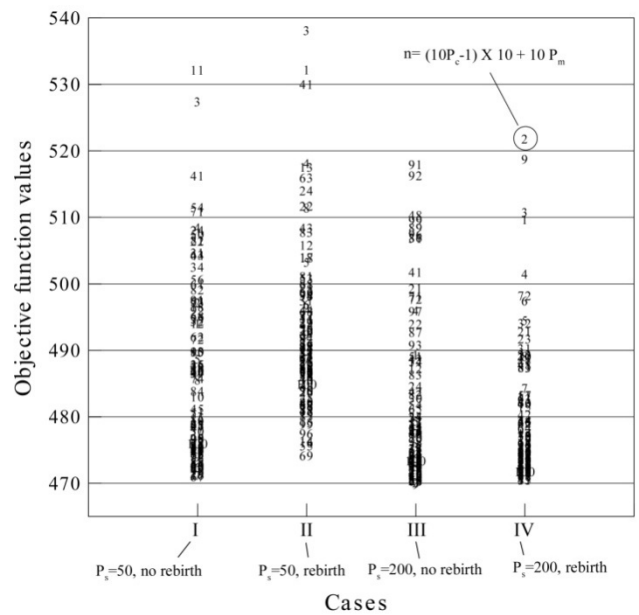


Figure 9. Distribution of the final objective function values for all 400 combinations

6.2. Geodesic Dome (2V)

The effect of rebirthing technique on GA-based size optimization problem is also investigated with a geodesic dome structure. The half-sphere type geodesic dome (Lee and Bae, 2008) is generated for this test. The FE model of geodesic dome is illustrated in Figure 10. The geodesic dome is subjected to a single point load $P = -5000\text{lb}(2.26\text{tf})$ at the top. The radius of domed is the value of $r = 182.9\text{ft}(55.74\text{m})$. The following material properties are assumed: elastic modulus $E=1.0 \times 10^7(6.89 \times 10^7\text{MPa})$ and Poisson's ratio $\nu = 0.3$. The analysis is carried out using 65, 2-node truss FEs with 26 nodal points.

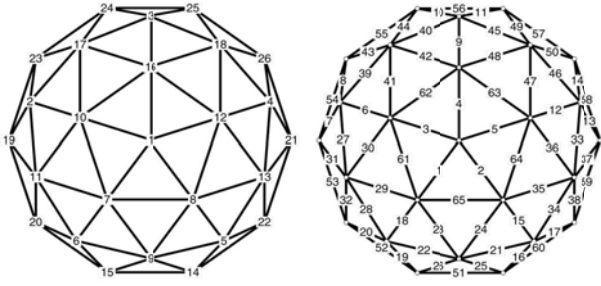


Figure 10. FE model of dome: (left) node number, (right) element number

For design optimization, the allowable stress and displacement are $\sigma_{all} = 30\text{ksi}(206.7\text{Mpa})$, $u_{all} = \pm 0.25\text{in}(0.635\text{cm})$. The members of dome are linked into 7 groups for design optimization as shown in Figure 11.

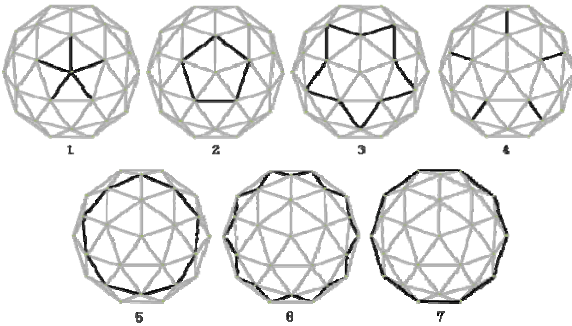


Figure 11. Member grouping and its number

[CASE I: $P_s = 50$; no R; $P_c, P_m = 0.1 \sim 1.0$]: In this test, we also investigated the effect of changing two main parameters P_c and P_m as described in Section 6.1. The 100 combinations of P_m and P_s are also used in this test. The objective function values for 100 combinations are illustrated in Figure 12.

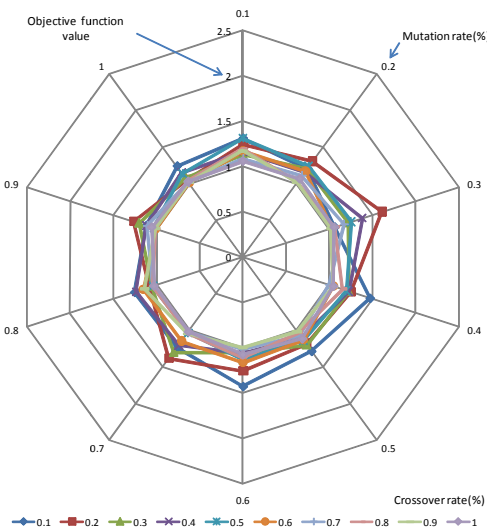


Figure 12. Objective function values distribution for the 100 combinations of crossover and mutation rates ($P_s = 50$ without rebirthing)

Note that the objective function is normalized by using the minimum value of objective function Ψ_{min} among 100 combinations. In this case, the minimum value of the objective function $\Psi_{min} = 228.59\text{lb}(103.6\text{kg})$ is obtained. The higher values of the objective function value are detected with low values of crossover rate. However, the cases with larger value of crossover rate above 0.5 can produce good optimum values close to the minimum values Ψ_{min} .

[CASE II: $P_s = 50$; R; $P_c, P_m = 0.1 \sim 1.0$]: The rebirthing technique is introduced into the same problem with the population size ($P_s = 50$) to see the effect of the rebirthing technique. The 100 combinations are tested and the results are illustrated in Figure 13. From numerical results, the overall optimum distributions are clearly enhanced with most values of crossover rate. However, the cases with lower crossover rate such as $P_c = 0.1$ which produce very poor values of objective function far from the Ψ_{min} . In general, the combinations of lower values of P_c and P_m produce poor final values of the objective function.

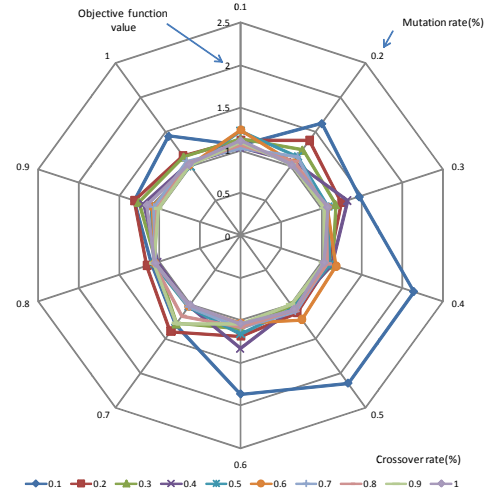


Figure 13. Objective function values distribution for the 100 combinations of crossover and mutation rate ($P_s = 50$ with rebirthing)

[CASE III: $P_s = 200$; no R; $P_c, P_m = 0.1 \sim 1.0$]: The same geodesic dome is then optimized with a larger population size of $P_s = 200$ without rebirthing. Numerical results are illustrated in Figure 14. In this case, most combinations produce better final objective function values than those produced by CASE II. From Figure 14, it is found to be that the values of objective function are well posed within $1.2 \times \Psi_{min}$ except 4 combinations.

In this case, the final objective function values are turned out to be near the best possible objective function value Ψ_{min} with most values of crossover rate.

At this point, we can evaluate the performance of generic GA according to the population size without rebirthing. From the comparison between CASE I and CASE III, we can find that the larger population size can provide more homogeneous distribution of minimum objective function values and more chances of reaching the minimum optimum value (Ψ_{min}). With the increase of popula-

tion size from $P_s = 50$ to $P_s = 200$, the possibility to get to the values of $1.1 \times \Psi_{\min}$ can be enhanced from 49 to 77 out of 100 combinations. However, it produces slightly different distribution of optimum objective function values compare to the case with small population size case $P_s = 50$.

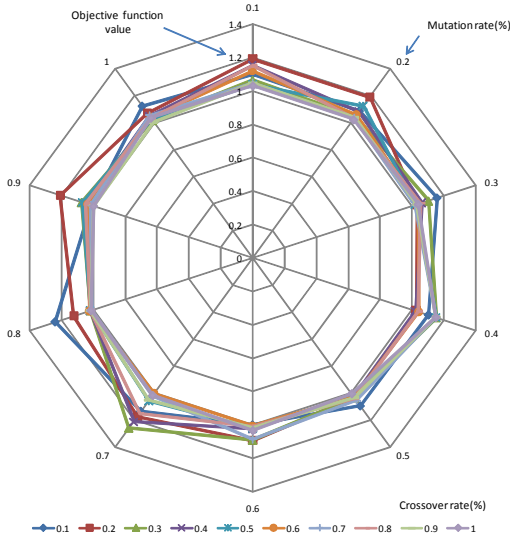


Figure 14. Objective function values distribution for the 100 combinations of crossover and mutation rate ($P_s = 200$ without rebirthing)

[CASE IV: $P_s = 200$; R; $P_c, P_m = 0.1 \sim 1.0$]: We introduce the rebirthing to the CASE III to see the effect of rebirthing technique with large population size. The optimization results are illustrated in Figure 15.

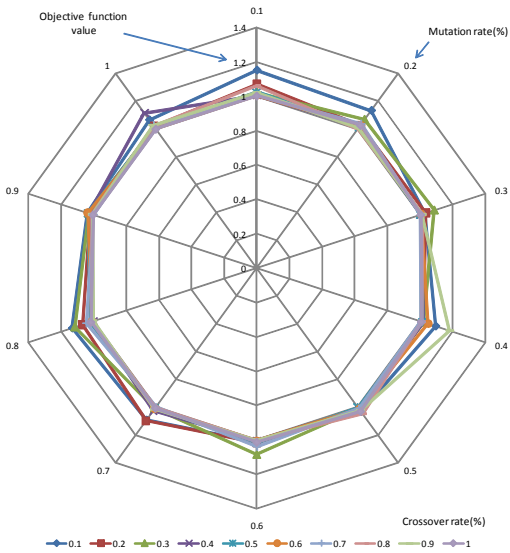


Figure 15. Objective function values distribution for the 100 combinations of crossover and mutation rate ($P_s = 200$ with rebirthing)

There are huge improvements in the final optimum values for 100 combinations with the population size $P_s = 200$ and rebirthing as shown in Figure 15. The possibility of achieving the value of $1.1 \times \Psi_{\min}$ is enhanced from 77 to 94 out of 100 combinations. Therefore, most final objective function values exist near the minimum objective value.

We also plot the final objective function value for all 400 combinations in Figure 16. The possibility to get to the minimum final objective function value is greatly increased with introduction of rebirthing technique, in particular with the population size $P_s = 200$ as shown in Figure 16. Therefore, if new structure is required to be optimized, the rebirthing technique can definitely provide more chance of getting the better optimum solution. The final minimum objective function values for geodesic dome with 4 all cases are described in Table 5.

Table 5. Final minimum objective function values [weight unit: lb(kg)]

| Cases | | Weight |
|-------|-----------------------------|---------------|
| I | $P_s = 50$; no rebirthing | 228.59(103.6) |
| II | $P_s = 50$; rebirthing | 231.78(105.1) |
| III | $P_s = 200$; no rebirthing | 228.25(103.5) |
| IV | $P_s = 200$; rebirthing | 228.30(103.5) |

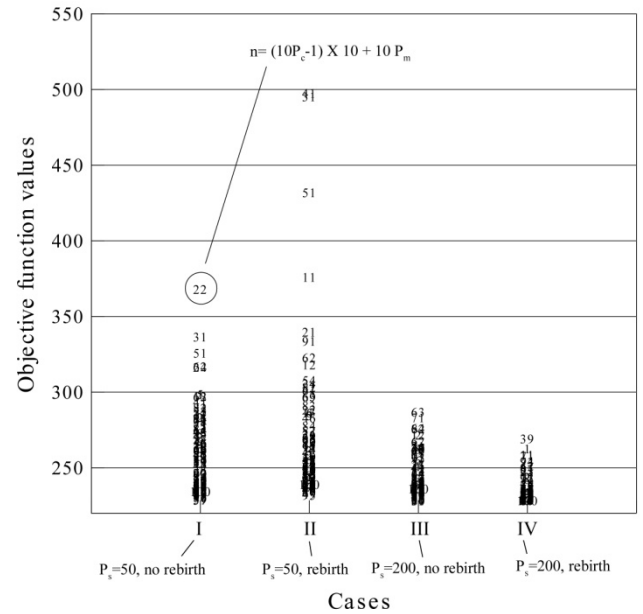


Figure 16. Distribution of the final objective function values for 400 combinations

7. CONCLUSIONS

The GA-based size optimization process with rebirthing technique is proposed to find optimum member pattern for three dimensional spatial structures. In particular, the effect of rebirthing technique on GA-based size optimization is thoroughly investigated. The benchmark tests with 4 cases having all 800 combinations are tackled to provide the correlation of the rebirthing technique with the GA parameters such as crossover, mutation and population size. Here, some specific conclusions are now drawn from the benchmark tests:

- (1) Higher mutation and crossover rates, easier to get the optimum solution. The recommended value of the

rates of mutation and crossover is generally the value larger than 0.5 to find optimum objective function values in GA-based size optimization for spatial structures.

- (2) The present GA with rebirthing technique shows very good searching ability regardless of the parameters used in GA-based size optimization and it clearly shows more chance of getting to the optimum objective function value than the generic GA has.
- (3) The rebirthing technique is turned out to be more effective in the problem with large population size.

Finally, it is found to be that the proposed technique is proven to be simple and extremely applicable for design optimization. In particular, it can provide a good performance and a stable convergence in design optimization of spatial structures. A set of benchmark tests is provided to show the overall capability of the proposed methodology on size optimization of spatial structures and it is provided as reference solution for future studies.

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