The Effects of Product Line Rivalry: Focusing on the Issue of Fighting Brands

竞争产品线的影响:关注战斗品牌

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Abstract

Firms produce various products that differ by function, design, color, etc. Product proliferation occurs for three different reasons. When there exist economies of scope, the unit cost for a product is lower when it is produced in conjunction with another product than when it is produced separately. Second, consumers are heterogeneous in the sense that they have different tastes, preferences, or price elasticities. A firm can earn more profit by segmenting consumers into different groups with similar characteristics. For example, product proliferation helps a firm increase profits by satisfying various consumer needs more precisely. The third reason for product proliferation is based on strategy. Producing a number of products can not only deter entry by providing few niches, but can also cause a firm to react efficiently to a low-price entry. By producing various products, a firm can reduce niches so that potential entrants have less incentive to enter. Moreover, a firm can produce new products in response to entry, which is called fighting brands. That is, when an entrant tries to attract consumers with a low price, an incumbent introduces a new lower-quality product while maintaining the price of the existing product.

The drawback of product proliferation, however, is cannibalization. Some consumers who would have bought a high-price product switch to a low-price product. Moreover, it is possible that proliferation can decrease profits when a new product is less differentiated from a rival's than is the existing product because of more severe competition.

Many studies have analyzed the effect of product line rivalry in the areas of economics and marketing. They show how a monopolist can solve the problem of cannibalization by adjusting quality in a market where consumers differ in their preferences for quality. They find that a consumer who prefers high-quality products will obtain his or her most preferred quality, but a consumer who has not such preference will obtain less than his or her preferred quality to reduce cannibalization

This study analyzed the effects of product line rivalry in a

duopoly market with two types of consumers differentiated by quality preference. I assume that the two firms are asymmetric in the sense that an incumbent can produce both high- and low-quality products, while an entrant can produce only a low-quality product.

The effects of product proliferation can be explained by comparing the market outcomes when an incumbent produces both products to those when it produces only one product. Compared to the case in which an incumbent produces only a high-quality product, the price of a low-quality product tends to decrease in a consumer segment that prefers low-quality products because of more severe competition. Prices, however, tend to increase in a segment with high preferences because of less severe competition.

It is known that when firms compete over prices, it is optimal for a firm to increase its price when its rival increases its price, which is called a strategic complement. Since prices are strategic complements, we have two opposing effects. It turns out that the price of a high-quality product increases because the positive effect of reduced competition outweighs the negative effect of strategic complements. This implies that an incumbent needs to increase the price of a high-quality product when it is also introducing a low-quality product. However, the change in price of the entrant's low-quality product is ambiguous.

Second, compared to the case in which an incumbent produces only a low-quality product, prices tend to increase in a consumer segment with low preferences but decrease in a segment with high preferences. The prices of low-quality products decrease because the negative effect outweighs the positive effect.

Moreover, when an incumbent produces both kinds of product, the price of an incumbent's low-quality product is higher, even though the quality of both firms' low-quality products is the same. The reason for this is that the incumbent has less incentive to reduce the price of a low-quality product because of the negative impact on the price of its high-quality product.

In fact, the effects of product line rivalry on profits depend not only on changes in price, but also on sales and cannibalization. If the difference in marginal cost is moderate compared to the difference in product quality, the positive effect of product proliferation outweighs the negative effect, thereby increasing the profit. Furthermore, if the cost difference is very large (small), an incumbent is better off producing only a low (high) quality product.

Moreover, this study also analyzed the effect of product line

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rivalry when a firm can determine product characteristics by focusing on the issue of fighting brands. Recently, Korean air and Asiana airlines have established budget airlines called Jin air and Air Busan, respectively, to confront the launching of budget airlines such as Hansung airline and Jeju air, among others. In addition, as more online bookstores have entered the market, a leading off-line bookstore Kyobo began its own online bookstore.

Through fighting brands, an incumbent with a high-quality product can increase profits by producing an additional low-quality product when its low-quality product is more differentiated from that of the entrant than is its high-quality product.

Keywords: product line rivalry, cannibalization, self-selection, fighting brands, product proliferation

摘要

公司生产不同功能,设计,颜色的产品。 产品扩散的出现有三个不同的原因。当存在规模经济,当这种产品和别的产品一起生产时,单位成本比单独生产要低 。二,消费者是异构的,即它们具有不同的品味,喜好,或价格弹性。一家公司可赚取细分为具有类似特点的不同群体的消费者更多的利润。例如,产品扩散通过更准确地满足不同消费者的需要来帮助公司增加利润。产品扩散的第三个原因是基于战略。生产一定数量的产品,不仅可以阻止通过提供给一些少数市场的产品进入。通过生产各种产品进入。通过生产各种产品,公司可以减少利基,使潜在进入者有较少进入的诱因。此外,企业可以生产新产品来应对进入,我们称之为战斗品牌。也就是说,当一个进入者试图以低廉的价格吸引消费者,已存在者介绍新的低质量的产品,同时保持现有产品的价格。

产品扩散的缺点是同型装配。一些买了高价位的产品的消费者会转向低价位的产品。此外,当新产品与对手现有的产品的异化程度不高时,由于激烈的竞争,产品扩散会降低利润。

许多研究已经在经济分析和市场营销等领域的产品线竞争的影响。它们展示了一个垅断者可以通过调整质量来解决市场中的消费者对质量的偏好不同的同型装配的问题。他们发现, 喜欢高品质的产品的消费者将获得他或她最喜欢的质量, 但没有这方面的偏好的消费者将获得比他或她所喜爱的质量低的产品。

本研究分析了产品的竞争在一个双头垅断市场, 两种不同类型的消费者对质量偏好的影响。我假设这两家公司将在这个意义上的不对称,一个运营商可以同时生产高,低质量的产品,而一个进入者只能产生低质量的产品。

产品扩散的影响是可以通过比较市场结果来解释,当已存在的运营商生产两种产品和只生产一种产品时。在这个案例中,当已存在的运营商只生产高品质的产品,由于激烈的竞争,在喜欢低质量产品的消费群中低质量的产品价格趋于下降。但由于缺乏竞争,在喜欢高质量产品的消费群中价格会上涨。

据了解,当企业在进行价格竞争时,理想状况是当公司的对手提高价格时,此公司也提高价格,这被称为战略补充。由于价格是战略性的补充,我们有两种相反的效果。事实证明,一个高品质的产品价格上升,因为竞争力减弱的积极作用超过了战略互补的负面影响。这意味着,已存在的运营商 推出了低质量的产品时还需要增加高品质产品的价格。然而,在进入者

的低质量产品的价格变化是模糊的。

二,此案例中,已存在的运营商只生产低品质的产品,在偏好低质量的消费群中价格往往增加。但 在偏好高质量的消费群中价格往往下降。低质量产品的价格下降是因为负面影响大于正面影响。

而且, 当已存在的运营商生产两种产品时, 其低质量产品的价格往往较高, 尽管两家的低品质的产品质量一样。 此原因由于对高品质产品价格的负面影响, 运营商没有较大的动机去降低低品质产品的价格。

事实上, 竞争的产品线对利润的影响不仅取决于价格变化, 还取决于销售和同型装配。 如果在边际成本同产品质量的差 异相比是适中的话, 产品扩散的积极影响大于负面影响, 从而增加利润。此外, 如果成本差异是非常大(小), 运营商最好只生产一种低(高)质量的产品。

而且,本研究还分析了当公司通过关注战斗品牌来决定产品特征时,竞争产品线的影响。最近,大韩航空和韩亚航空公司建立了廉价航空线路,分别是Jin线路和釜山线。 来应对Hansung航空和济州航空。另外,很多网上书店也进入市场,例如处于领先地位的实体书店Kyobo已经有了自己的网上书店。

通过战斗品牌,在它的低品质产品跟新成员比起来有差别时,一个具有高品质产品的运营商通过生产更多的低质量产品可以增加利润。

关键词:竞争产品线,同型装配,自主选择,战斗品牌,产品扩散

I. Introduction

It is argued that firms produce differentiated products when there exist economies of scope. Product differentiation also provides firms a chance to increase profits by offering different products to consumer segments with different characteristics (Lee 2006; Kwak, Lee and Nam 2006; Park and Kim 1999; Cho and Shim 1997). In this case, firms usually charge different prices for each product, which is called price discrimination (Tirole 1998; Kim, Kim and Shin 2007).

The other reason for product lines is based on strategy. First, product lines can be used as an entry barrier because a firm can reduce niche so that potential entrants have less incentive to enter (Schmalensee 1978). Second, firms often expand product lines in response to competition. It is common that firms introduce a lower quality product called "fighting brand" when an entrant tries to attract consumers with a low price.

When firms produce different products, however, they might experience the profit decrease because some consumers who would have bought a high-price product switch to a low-price product, which is called cannibalization. Thus, it is strategically important for firms to find ways to lessen the problem of cannibalization (Fudenberg and Tirole 1984; Judd 1985).

Some studies have analyzed the effect of product line rivalry in the areas of economics and marketing (Mussa and Rosen 1978; Moorthy 1984). They show how a monopolist can solve the problem of cannibalization using quality in a

market where consumers are differentiated by the preference for quality. Other studies have discussed the effects of product line rivalry in a duopoly market (Desai 2001). Desai analyzed a market with two firms where consumers differed not only in quality preference, but also in taste preference. He finds that, under some conditions, the cannibalization problem does not affect the firms' quality choices, and that each firm provides each consumer with its preferred quality. For example, as the taste preference of a low-preference consumer weakens, the prices of low-quality products decrease. The cannibalization problem is more severe as high-preference consumers experience more incentive to buy the low-quality product. Hence, as the taste preference of the low (high) preference consumer is strengthened (weakened), the cannibalization problem is reduced and each consumer can obtain his or her preferred quality.

Some studies have discussed firms' incentives for proliferation using a game-theoretic approach (Gilbert and Matutes 1993). Using a market in which consumers differ in both quality and taste preference, they show under which conditions firms have an incentive to specialize or proliferate their product lines. For a simultaneous game, both firms produce both products in a symmetric equilibrium. For a sequential game, the market outcome depends on the extent of consumer heterogeneity. If taste preference is weaker than quality preference, each firm specializes in a different quality. If taste preference is stronger than quality preference, firms produce both products. This is because proliferation profits depend on the degree of taste preference and are lower with decreased degree.

I analyzed the effect of product proliferation in a duopoly market where consumers are heterogeneous in terms of product quality and characteristics. Unlike Desai, I assume that the two firms are asymmetric in the sense that an incumbent can produce both high- and low-quality products, while an entrant can produce only a low-quality product. The effects of product proliferation on profits depend on the effects on price, the amount of sales, and cannibalization. If the difference in marginal cost is moderate compared to the difference in product quality, the positive effect of product proliferation outweighs the negative effect, thereby increasing profits.

Moreover, I also analyzed the effect of product line rivalry when a firm can determine product characteristics by focusing on the issue of fighting brands.

By fighting brands, an incumbent with a high-quality product can increase profits by producing an additional low-quality product when its low quality product is more differentiated from the entrant's than is its high quality product.

The paper is organized as follows. In the next section, I will introduce the model. In Section III, I explain the effects of product proliferation when the product characteristics are exogenous for both firms. In Section IV, I relax the assumption of exogenous product characteristics and assume that an incumbent can determine the characteristics of a low-quality product, while the characteristics of its high-quality product and an entrant's low-quality product are given. I show when

and how an incumbent introduces a low-quality product in response to a low-price entry. Finally, I conclude and discuss possible extensions of this study for the future.

II. Model

I analyze a market where products differ not only in quality, but also with regard to product characteristics. The former is called vertical and the latter horizontal differentiation. There are two types of consumers who are differentiated by quality valuation. A consumer in segment P is willing to pay $\theta^r q$, and a consumer in segment A is willing to pay $\theta^a q$ for a product with quality q where $\theta^a < \theta^p$. I use a location model so that consumers in each segment are uniformly distributed along the line segment [0, 1]. When a consumer at x buys a product from a firm at t, his transportation cost is $k(t-x)^2 \cdot 1$ Quadratic transportation cost causes the demand curve to be continuous. The number of consumers in each segment is N^m where m = P, A. I assume that θ or q are sufficiently large so that every consumer buys one unit of a product.

Firm 1 can produce both a high (product H1)- and low (product L1)-quality product, but firm 2 produces only a low (product L2)-quality product. The value of product j is q^j . \prod_i is firm 1's profit, and p_i is the price of product j of firm i where i=1,2 and j=H,L. The marginal cost of a high-quality product is c, and that of a low-quality product is assumed to be 0 without a loss of generality.

Firms play a two-stage game. In the first stage, firm 1 decides which product to produce. In the second stage, the two firms compete over prices. I will find a pure strategy equilibrium that is sub-game perfect, and thus I will derive the second stage equilibrium first. I will then analyze the two different stages according to whether product characteristics are exogenous or endogenous.

III. Exogenous Product Characteristics

I assume not only that the characteristics of every product are exogenous but also that firm 1's products have the same characteristics. For simplicity, I assume that product H1 and L1 are located at a^1 and L2 is located at $b^2 = 1 - a^1$ where $0 \le a^1 < 1/2$. In the second stage, three cases depend on firm 1's decision about product choice.

3.1. Firm 1 produces only a high-quality product (A1)

Let x_{H1L2}^{p} and y_{H1L2}^{A} in Figure 1 be the location of a marginal consumer who is indifferent between buying product H1

 $^{^{1)}}$ Quadratic transportation cost causes the demand curve to be continuous.

and L2 in segment P and A, respectively. Since $\theta^P q^H - p_1^H - k(x_{H1L2}^P - a^1)^2 = \theta^P q^L - p_2^L - k\{(1-a^1) - x_{H1L2}^P\}^2$, we have the following equation.

$$x_{H1L2}^{P} = \frac{1}{2(1 - 2a^{1})k} \left[(1 - 2a^{1})k + \theta^{P}(q^{H} - q^{L}) - (p_{1}^{H} - p_{2}^{L}) \right]$$
(1)

Equation (1) implies that consumers along $[0, x_{H1L2}^p]$ in segment P will buy product

H1 and those along $\left(x_{H1L2}^{P},1\right]$ will buy product L2. Similarly, the location of a marginal consumer at segment A is as follows.

$$y_{H1L2}^{A} = \frac{1}{2(1-2a^{1})k} \left[(1-2a^{1})k + \theta^{A}(q^{H} - q^{L}) - (p_{1}^{H} - p_{2}^{L}) \right]$$
(2)

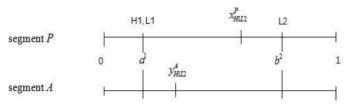


Fig. 1. Consumer Purchasing Behavior (case A1)

Since consumers are uniformly distributed along the line, x_{H1L2}^{P} and y_{H1L2}^{A} are the market shares in each segment. Thus, each firm's profit is

$$\Pi_{1} = (p_{1}^{H} - c) [N^{P} x_{H1L2}^{P} + N^{A} y_{H1L2}^{A}] \qquad \Pi_{2} = p_{2}^{L} [N^{P} (1 - x_{H1L2}^{P}) + N^{A} (1 - y_{H1L2}^{A})]$$

By the first-order conditions, $\partial \Pi_1 / \partial p_1^H = \partial \Pi_2 / \partial p_2^L = 0$, we have²)

$$p_i^{H} = \frac{1}{3} \left[2c + 3(1 - 2a^i)k + \frac{(N''\theta'' + N''\theta^i)}{(N'' + N'^i)} (q'' - q^i) \right] \\ \qquad p_2^{L} = \frac{1}{3} \left[c + 3(1 - 2a^i)k - \frac{(N''\theta'' + N'\theta^i)}{(N'' + N'^i)} (q'' - q^i) \right] \\ \qquad p_3^{L} = \frac{1}{3} \left[c + 3(1 - 2a^i)k - \frac{(N''\theta'' + N''\theta^i)}{(N'' + N'^i)} (q'' - q^i) \right] \\ \qquad p_4^{L} = \frac{1}{3} \left[c + 3(1 - 2a^i)k - \frac{(N''\theta'' + N''\theta^i)}{(N'' + N'')} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + 3(1 - 2a^i)k - \frac{(N''\theta'' + N''\theta^i)}{(N'' + N'')} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + 3(1 - 2a^i)k - \frac{(N''\theta'' + N''\theta^i)}{(N'' + N'')} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + 3(1 - 2a^i)k - \frac{(N''\theta'' + N''\theta^i)}{(N'' + N'')} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + 3(1 - 2a^i)k - \frac{(N''\theta'' + N''\theta^i)}{(N'' + N''\theta^i)} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + 3(1 - 2a^i)k - \frac{(N''\theta'' + N''\theta^i)}{(N'' + N''\theta^i)} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + \frac{(N''\theta'' + N''\theta^i)}{(N'' + N''\theta^i)} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + \frac{(N''\theta'' + N''\theta^i)}{(N'' + N''\theta^i)} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + \frac{(N''\theta'' + N''\theta^i)}{(N'' + N''\theta^i)} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + \frac{(N''\theta'' + N''\theta^i)}{(N'' + N''\theta^i)} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + \frac{(N''\theta'' + N''\theta^i)}{(N'' + N''\theta^i)} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + \frac{(N''\theta'' + N''\theta^i)}{(N'' + N''\theta^i)} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{1}{3} \left[c + \frac{(N''\theta'' + N''\theta^i)}{(N'' + N''\theta^i)} (q'' - q^i) \right] \\ \qquad p_5^{L} = \frac{(N''\theta'' + N''\theta'')}{(N'' + N''\theta'')} (q'' - q'') \\ \qquad p_5^{L} = \frac{(N''\theta'' + N''\theta'')}{(N'' + N''\theta'')} (q'' - q'') \\ \qquad p_5^{L} = \frac{(N''\theta'' + N''\theta'')}{(N'' + N''\theta'')} (q'' - q'') \\ \qquad p_5^{L} = \frac{(N''\theta'' + N''\theta'')}{(N'' + N''\theta'')} (q'' - q'') \\ \qquad p_5^{L} = \frac{(N''\theta'' + N''\theta'')}{(N'' + N''\theta'')} (q'' - q'') \\ \qquad p_5^{L} = \frac{(N''\theta'' + N''\theta'')}{(N'' + N''\theta'')} (q'' - q'') \\ \qquad p_5^{L} = \frac{(N''\theta'' + N''\theta'')}{(N'' + N''\theta'')} (q'' - q'') \\ \qquad p_5^{L} = \frac{(N''\theta'' + N''\theta'')}{(N'' + N''\theta'')} (q'' - q'') \\ \qquad p_5^{L} = \frac{(N''\theta'' + N''\theta'')}{(N'' + N''\theta'')} (q'' - q'') \\ \qquad p_5^{L} = \frac{(N''\theta'' + N''\theta'')}{(N'' + N''\theta'')} (q'' - q'') \\ \qquad p$$

By substituting these equilibrium prices in (1) and (2), we have

$$\begin{split} x_{mll2}^{P} &= \frac{1}{2} + \frac{1}{6(1-2a^{l})k} \left[-c + \frac{N^{l}\theta^{l} + 3N^{l}\theta^{l} - 2N^{l}\theta^{l}}{N^{P} + N^{d}} (q^{ll} - q^{l}) \right], \qquad y_{mll2}^{d} &= \frac{1}{2} + \frac{1}{6(1-2a^{l})k} \left[-c + \frac{-2N^{l}\theta^{l} + 3N^{l}\theta^{l} + N^{l}\theta^{l}}{N^{P} + N^{d}} (q^{ll} - q^{l}) \right] \\ \Pi_{1} &= \frac{(N^{P} + N^{A})}{18(1-2a^{1})k} \left[3(1-2a^{1})k - c + \frac{(N^{P}\theta^{P} + N^{A}\theta^{A})}{(N^{P} + N^{A})} (q^{ll} - q^{l}) \right]^{2} \\ \Pi_{2} &= \frac{(N^{P} + N^{A})}{18(1-2a^{1})k} \left[3(1-2a^{1})k + c - \frac{(N^{P}\theta^{P} + N^{A}\theta^{A})}{(N^{P} + N^{A})} (q^{ll} - q^{l}) \right]^{2} \end{split}$$

3.2. Firm 1 produces only a low-quality product (A2)

Since both firms 1 and 2 produce a low-quality product, this case is symmetric. The location of the marginal consumer in both segments is

$$\frac{N^{p}\theta^{p}+3N^{4}\theta^{p}-2N^{4}\theta^{4}}{N^{p}+N^{4}}(q^{H}-q^{L})-3(1-2a^{l})k < c < \frac{-2N^{p}\theta^{p}+3N^{p}\theta^{4}+N^{4}\theta^{4}}{N^{p}+N^{4}}(q^{H}-q^{L})+3(1-2a^{l})k \tag{F1}$$

$$x_{LLL2}^{p} = y_{LLL2}^{A} = \frac{1}{2(1 - 2a^{1})k} \left[(1 - 2a^{1})k - (p_{1}^{L} - p_{2}^{L}) \right]$$
(3)

By the same procedure as in the previous case, we have the following market outcomes.

$$p_1^L = p_2^L = (1 - 2a^1)k, \quad x_{L1L2}^P = y_{L1L2}^A = \frac{1}{2}$$

$$\Pi_1 = \Pi_2 = \frac{1}{2}(1 - 2a^1)(N^P + N^A)k$$

3.3. Firm 1 produces both a high- and a low-quality product (A3)

When firm 1 produces both kinds of product, consumers have three kinds of products available. Table 1 shows the net value (total value-price-transportation cost) of a consumer located at x for each product.

Table 1. Net Value

| ĺ | | H1 | L1 | L2 |
|---|-----------|---------------------------------------|---------------------------------------|---|
| | segment P | $\theta^P q^H - p_1^H - k(x - a^1)^2$ | $\theta^P q^L - p_1^L - k(x - a^1)^2$ | $\theta^{P} q^{L} - p_{2}^{L} - k(1 - a^{1} - x)^{2}$ |
| | segment A | $\theta^A q^H - p_1^H - k(x - a^1)^2$ | $\theta^A q^L - p_1^L - k(x - a^1)^2$ | $\theta^{A}q^{L} - p_{2}^{L} - k(1 - a^{1} - x)^{2}$ |

According to Table 1, consumers in segment P prefer product H1 to L1 if $p_1^H - p_1^L \le \theta^P(q^H - q^L)$, and consumers in segment A prefer L1 to H1 if $p_1^H - p_1^L > \theta^A(q^H - q^L)$. Thus, if

$$\theta^{A}(q^{H} - q^{L}) < p_{1}^{H} - p_{1}^{L} \le \theta^{P}(q^{H} - q^{L})$$
(4)

holds, the self-selection condition is satisfied in the sense that consumers along $\left[0,x_{H1L2}^{P}\right]$ in segment P buy product H1, and consumers along $\left[0,y_{L1L2}^{A}\right]$ in segment A buy product L1. The firms' profits are then

$$\begin{split} \Pi_{1} &= (p_{1}^{H} - c) \left[N^{P} x_{H1L2}^{P} \right] + p_{1}^{L} \left[N^{A} y_{L1L2}^{A} \right], \\ \Pi_{2} &= p_{2}^{L} \left[N^{P} \left(1 - x_{H1L2}^{P} \right) + N^{A} \left(1 - y_{L1L2}^{A} \right) \right] \end{split}$$

where x_{H1L2}^P and \mathcal{Y}_{L1L2}^A are the same as in equations (1) and (3). By $\partial \Pi_1/\partial p_1^H=\partial \Pi_1/\partial p_1^L=\partial \Pi_2/\partial p_2^L=0$, the profit maximizing prices are

$$\begin{split} p_1^H &= (1-2a^1)k + \frac{1}{6(N^P + N^A)} \Big[(2N^P + 3N^A)\theta^P (q^H - q^L) + (4N^P + 3N^A)c \Big] \\ p_1^L &= (1-2a^1)k - \frac{N^P}{6(N^P + N^A)} \Big[\theta^P (q^H - q^L) - c \Big] \\ p_2^L &= (1-2a^1)k - \frac{N^P}{3(N^P + N^A)} \Big[\theta^P (q^H - q^L) - c \Big] \end{split}$$

and the equilibrium market share and profits are

$$x_{H1L2}^{P} = \frac{1}{2} + \frac{2N^{P} + 3N^{A}}{12(1 - 2a^{1})(N^{P} + N^{A})k} \Big(\theta^{P}(q^{H} - q^{L}) - c\Big) \\ y_{L1L2}^{A} = \frac{1}{2} - \frac{N^{P}}{12(1 - 2a^{1})(N^{P} + N^{A})k} \Big(\theta^{P}(q^{H} - q^{L}) - c\Big)$$

$$\Pi_1 = \frac{1}{770 - 2a^{\dagger}(\sqrt{N^F} + N^A)^2 F} \left[N^A \left[\delta(1 - 2a^{\dagger})(N^F + N^A)k - N^F (\theta^F (q^H - q^L) - c) \right]^2 + N^F \left[\delta(1 - 2a^{\dagger})(N^F + N^A)k + (2N^F + 3N^A)(\theta^F (q^H - q^L) - c) \right]^2 \right]$$

If we substitute equilibrium prices in (4), the necessary condition for case A3 to exist is The condition for both firms to

 $^{^{\}rm 2)}$ The condition for two firms to have positive sales in both consumer segments for A1 is

$$\Pi_{2} = \frac{1}{18(1 - 2a^{1})(N^{P} + N^{A})k} \left[3(1 - 2a^{1})(N^{P} + N^{A})k - N^{P}(\theta^{P}(q^{H} - q^{L}) - c) \right]^{2}$$

have positive sales in both segments in this case is³⁾

$$Max \left[(2\theta^{A} - \theta^{P})(q^{H} - q^{L}), \theta^{P}(q^{H} - q^{L}) - \frac{6(N^{P} + N^{A})}{2N^{P} + 3N^{A}} (1 - 2a^{1})k \right] \prec c \leq \theta^{P}(q^{H} - q^{L})$$

$$(2\theta^{A} - \theta^{P})(q^{H} - q^{L}) < c \leq \theta^{P}(q^{H} - q^{L})$$
(5)

The higher is $(q''-q^L)$ or $(\theta^P-\theta^A)$, the more likely it is that firm 1 will produce both products. This is because as $(q''-q^L)$ or $(\theta^P-\theta^A)$ increases, the consumer in segment P has more incentive to buy a high-quality product, which reduces the cannibalization problem.

Comparing the market outcomes in the three cases leads to the following lemma.

<Lemma 1>

(i) -
$$p_1^H(A3) > p_1^H(A1)$$
, $p_1^L(A2) > p_1^L(A3)$ and $p_2^L(A2) > p_2^L(A3) 34$
- In case A3, $[p_1^H(A3) - c] > p_1^L(A3) > p_2^L(A3)$ (6)
(ii) $y(A1) < y(A3) < x(A2) = y(A2) = \frac{1}{2} < x(A3) < x(A1)$.

The effects of product proliferation can be explained by comparing the market outcomes in cases A3 to A1 and A2. The effects of proliferation on prices are as follows. First, compared to A1, in A3 the price of a low-quality product tends to decrease in segment A because competition is more severe. Prices, however, tend to increase in segment P because competition is less severe.

It is known that when firms compete over prices, it is optimal for a firm to increase its price when its rival increases its price, a process that is called strategic complements (Fudenberg and Tirole 1984). Since prices are strategic complements, we have two opposing effects. It turns out that $p_1^H(A3) > p_1^H(A1)$ since the positive effect from less competition outweighs the negative effect from strategic complements. This implies that firm 1 needs to increase the price of a high-quality product when it introduces a new low-quality product.5) However, the change in p_2^L is ambiguous.

Second, compared to A2, in A3, prices tend to increase in

$$\begin{split} \mathit{Max} \bigg[(2\theta^{\scriptscriptstyle A} - \theta^{\scriptscriptstyle P})(q^{\scriptscriptstyle H} - q^{\scriptscriptstyle L}), \theta^{\scriptscriptstyle P}(q^{\scriptscriptstyle H} - q^{\scriptscriptstyle L}) - \frac{6(N^{\scriptscriptstyle P} + N^{\scriptscriptstyle A})}{2N^{\scriptscriptstyle P} + 3N^{\scriptscriptstyle A}} (1 - 2a^{\scriptscriptstyle 1})k \bigg] \prec c \leq \theta^{\scriptscriptstyle P}(q^{\scriptscriptstyle H} - q^{\scriptscriptstyle L}) \,. \end{split}$$

A3, profit maximizing prices given that
$$p_1^H = p_1^H(A1)$$
 are
$$p_1^L = \frac{9k(1-2a_1)(N^H+N^L)^2 + 2N^H(N^H+N^L)c - N^H(2N^H\theta^F+3N^L\theta^F-N^L\theta^A)(q^H-q^L)}{3(N^H+N^L)(4N^H+3N^L)}$$

segment A but decrease in segment P. Since the negative effect outweighs the positive effect for a low-quality product, $p_1^L(A2) > p_1^L(A3)$ and $p_2^L(A2) > p_2^L(A3)$.

Moreover, even though the qualities of products L1 and L2 are the same, as in (6). This is because firm 1 has less incentive to $p_1^L(A3) > p_2^L(A3)$ reduce the price of a low-quality product because of the negative impact on the price of its high-quality product H1.

Lemma 1(ii) explains the sales effect of product proliferation, which is shown in Figure 2. Compared to case A1, firm 1 can satisfy consumer needs in segment A more properly in A3, meaning that y(A1) < y(A3). However, x(A3) < x(A1) since $p_1^H(A3) > p_1^H(A1)$ by Lemma 1(i). By the same logic, x(A2) < x(A3). Since p_2^L decreases more than p_1^L in A3, we have y(A3) < y(A2).

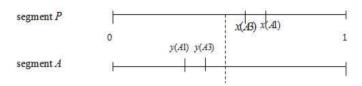


Fig. 2. Consumer Purchasing Behavior (case A3)

Thus far, I analyzed the effects of product proliferation on prices and sales. Proliferation gives rise to the cannibalization problem as shown in Figure 2. Consumers along [0, y(A1)] in segment A buy product H1 in A1, but product L1 in A3. Since product H1 has a higher per-unit margin by (6), firm 1's profit from these consumers will decrease by producing a low-quality product.

Consequently, the effects of product proliferation on profits depend on the effects on price, sales, and cannibalization. Figure 3 shows that the positive effect is predominant when the cost difference between the high- and low-quality product is about the same as the quality difference . If the cost difference is very large (small), firm 1 will be better off producing only a low (high)-quality product. This leads to the following proposition.

<Proposition 1> (i) If $c > \theta^p(q^H - q^L)$, firm 1 produces only a low-quality product.

(ii) If $\alpha < c \le \theta^p(q^n - q^n)$, firm 1 produces both a high- and a low-quality product where

$$\alpha < \frac{N^P \theta^P + N^A \theta^A}{N^P + N^A} (q^H - q^L)_{6}$$

(iii) If $c \le \alpha$, firm 1 produces only a high-quality product.

Proof: see the Appendix.

 $^{^{\}rm 3)}$ The condition for both firms to have positive sales in both segments in this case is

⁻ If c $^{2}\frac{N^{r}\theta^{r}+N^{A}\theta^{4}}{N^{r}+N^{A}}(q^{u}-q^{t})$, A2 and A3 are possible as we can see in Figure 3 and $p_{2}^{L}(A2)>p_{2}^{L}(A3)$.

⁻ If $\theta^4(q''-q^L) \le c < \frac{N^P \theta^P + N^A \theta^A}{N^P + N^A}(q'''-q^L)$, A1 and A3 are possible and $p_2^L(A1) \ge p_2^L(A3)$.

⁻ If $c < \theta^A(q^H - q^L)$, A1 and A3 are possible and $p_2^L(A1) < p_2^L(A3)$

⁵⁾ When firm 1 cannot increase the price of a high-quality product, it is less possible for it to make more profits through product proliferation. In

 $p_{2}^{L} = \frac{3k(1-2a_{1})(N''+N^{L})(2N''+3N^{L})+4N''(N''+N^{L})c-2N''(2N''\theta''+3N^{L}\theta''-N^{L}\theta'')(q''-q^{L})}{3(N''+N^{L})(4N''+3N^{L})}$

 $^{^{6)}}$ It is very difficult to derive the value of α . If $\alpha<(2\theta^{A}-\theta^{P})(q^{H}-q^{L})$, then $\Pi_{\rm I}({\it A}3)$ and $\Pi_{\rm I}({\it A}1)$ do not intersect since $\Pi_{\rm I}({\it A}3)\!>\!\Pi_{\rm I}({\it A}1)$. Figure 3 assumes that $\alpha>(2\theta^{A}-\theta^{P})(q^{H}-q^{L})$

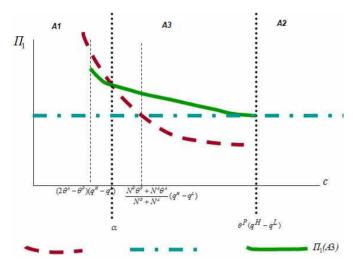


Fig. 3. Profits of firm 1

IV. Endogenous Product Characteristics

In this section, I relax the assumption of exogenous product characteristics. However, it is very complicated to deal with the case where the characteristics of every product are endogenous.

Thus, I analyze a model where firm 1 can decide the characteristics of product L1, while those of product H1 and L2 are given. This model is useful for analyzing the effect of fighting brands. The when the market is covered, firm 1 sells product H1 to segment P only before firm 2's entry if $\frac{N''}{N^{2}} > Max \left[\frac{\partial^{4} g'' - k(1-a'')^{2} - c}{(\partial^{2} - \partial^{2})g''} \cdot \frac{\partial^{4} g^{L} - k(1-a'')^{2}}{(\partial^{2} - \partial^{2})g''} \right]$ holds. Thus, when the above condition is satisfied with equation (5), firm 1 will introduce fighting brands (product L1) in response to firm 2's entry with a low-quality product (L2). Johnson and Myatt (2003) show under what circumstances an incumbent will reduce its product line in response to entry. As in the previous section, products H1 and L2 are located at a^{H} and $b^{2} = 1 - a^{H}$. Assume that product L1 is located at a^{L} where $0 \le a^{L} < 1/2$. As we can see in Figure 4, there are three cases depending on the value of

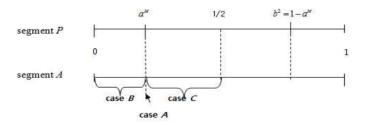


Fig. 4. Consumer Purchasing Behavior bases on a^L

$$a^{L}$$
 (A) $a^{L} = a^{H}$ (B) $0 \le a^{L} < a^{H}$ and (C) $a^{H} < a^{L} < 1/2$.

Since case A has been discussed in the previous section, I will analyze the

remaining two cases. Table A1 in the Appendix shows that there are six possible equilibria in case B. By similar logic, it can be shown that the same is true in case C. To make the analysis easier, I will use the following assumptions.

Assumption 1: When c is moderate, firm 1 produces a high- and a low-quality product.

Assumption 2: When $a^L \neq a^H$, a pure strategy equilibrium exists which satisfies the self-selection constraint, and $\frac{\partial \Pi_1}{\partial a^L} < 0$ when a^L is close to a^H .

Assumption 1 guarantees that firm 1 produces both products. By proposition 1, when C is small, firm 1 produces only product H1, thus $\Pi_1(Al) = \Pi_1(Bl) = \Pi_1(Cl)$. Meanwhile, when C is large, firm 1 produces only product L1. Lemma 2 (iii) and Lemma 3 (i) in the Appendix show that firm 1's profit is maximized at $a^L = 0$, $a^L = a^H + \varepsilon$ in B2 and C2, respectively, and $\Pi_1(B2; a^L = 0) > \Pi_1(A2) > \Pi_1(C2; a^L = a^H + \varepsilon)$. This implies that when firm 1 producesonly a low-quality product, it maximizes profit in B2, which reinforces the result of the maximum product differentiation explained by Shaked and Sutton (1982).

When Assumption 1 holds, it is very difficult to derive the optimal value of a^L and show under which case firm 1's profits are maximized. Thus, for the ease of analysis, we need assumption 2. The numerical example below shows that these assumptions can be satisfied.

The assumption of the existence of an equilibrium satisfying the self-selection constraint ensures that equilibriums exist in cases B3 and C3 out of B3-B6 and C3-C6 in Table A1. In B3 and C3, the market outcomes are the same.⁸⁾. That is, the locations of the marginal consumers are

$$x_{HIL2}^{P} = \frac{1}{2k(1-2a^{H})} \left[k(1-2a^{H}) + \theta^{P}(q^{H}-q^{L}) - (p_{1}^{H}-p_{2}^{L}) \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H})^{2} - (a^{L})^{2} \right\} - (p_{1}^{L}-p_{2}^{L}) \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H})^{2} - (a^{L})^{2} \right\} - (p_{1}^{L}-p_{2}^{L}) \right] \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H})^{2} - (a^{L})^{2} \right\} - (p_{1}^{L}-p_{2}^{L}) \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H})^{2} - (a^{L})^{2} \right\} - (p_{1}^{L}-p_{2}^{L}) \right] \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H})^{2} - (a^{L})^{2} \right\} - (p_{1}^{L}-p_{2}^{L}) \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H})^{2} - (a^{L})^{2} \right\} - (p_{1}^{L}-p_{2}^{L}) \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H})^{2} - (a^{L})^{2} \right\} - (p_{1}^{L}-p_{2}^{L}) \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H})^{2} - (a^{L})^{2} \right\} - (p_{1}^{L}-p_{2}^{L}) \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H})^{2} - (a^{L})^{2} \right\} - (p_{1}^{L}-p_{2}^{L}) \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H})^{2} - (a^{L})^{2} \right\} - (p_{1}^{L}-p_{2}^{L}) \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H})^{2} - (a^{L})^{2} \right\} - (p_{1}^{L}-p_{2}^{L}) \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H}-a^{L})^{2} + (a^{L})^{2} \right\} - (p_{1}^{L}-a^{L})^{2} \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H}-a^{L})^{2} + (a^{L})^{2} + (a^{L})^{2} \right\} \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H}-a^{L})^{2} + (a^{L})^{2} + (a^{L})^{2} + (a^{L})^{2} + (a^{L})^{2} \right\} \right] \quad y_{LIL2}^{A} = \frac{1}{2k(1-a^{H}-a^{L})} \left[k \left\{ (1-a^{H}-a^{L})^{2} + (a^{L})^{2} + (a^{L})^{2$$

and the firms' profits are

$$\Pi_1 = (p_1^H - c)[x_{H1L2}^P N^P] + p_1^L[y_{L1L2}^A N^A] \qquad \Pi_2 = p_2^L \left[(1 - x_{H1L2}^P) N^P + (1 - y_{L1L2}^A) N^A \right]$$

By the first order conditions, we have

$$\begin{split} p_2^L &= \frac{1-a^H-a^L}{3\left[\left(1-a^H-a^L\right)N^P+\left(1-2a^H\right)N^A\right]}\left[\left(1-2a^H\right)\left(3N^P+\left(3+a^H-a^L\right)N^A\right)k - \left\{\theta^P(q^H-q^L)-c\right\}N^P\right] \\ p_1^L &= \frac{1}{2}\left[p_2^L+\left(\left(1-a^H\right)^2-\left(a^L\right)^2\right)k\right] \\ p_1^H &= \frac{1}{2}\left[p_2^L+\left(1-2a^H\right)k+\theta^P(q^H-q^L)+c\right] \end{split}$$

It can be shown that $\frac{\partial p_1^L}{\partial a^L}, \frac{\partial p_2^L}{\partial a^L}, \frac{\partial p_1^H}{\partial a^L} < 0$. Intuitively, as a^L increases, the price of a low-quality product decreases because products L1 and L2 are less differentiated. This lowers the price of a high-quality product since the prices are strategic

 $^{^{7)}}$ When the market is covered, firm 1 sells product H1 to segment P only before firm 2's entry if $\frac{N''}{N''} > Mar \left[\frac{\sigma' q'' - A(1-a'')^2 - c}{(\sigma' - \sigma') q''} \frac{\sigma'' - A(1-a'')^2}{(\sigma' - \sigma') q''} \right]$ holds. Thus, when the above condition is satisfied with equation (5), firm 1 will introduce fighting brands (product L1) in response to firm 2's entry with a low-quality product (L2). Johnson and Myatt (2003) show under what circumstances an incumbent will reduce its product line in response to entry.

⁸⁾ The difference between the two cases is a necessary condition for self-selection. When $a^L < a^H$, $c < \theta^P (q^H - q^L) - \frac{1}{2} (a^H)^2 - (a^L)^2 \frac{1}{2}$ should hold, but when $a^L > a^H$, $c > (2\theta^d - \theta^P)(q^H - q^L) + \frac{1}{2} (a^L)^2 - (a^H)^2 \frac{1}{2}$ should hold.

complements. Meanwhile, since $\left|\frac{\partial p_1^{\prime\prime}}{\partial a^L}\right| = \frac{1}{2} \left|\frac{\partial p_2^{\prime}}{\partial a^L}\right|$, it is true that $\frac{\partial x_{\mu_{1L2}}^{\prime\prime}}{\partial a^L} < 0$. However, the sign of $\frac{\partial y_{\mu_{1L2}}^{\prime\prime}}{\partial a^L}$ is ambiguous.

Assumption 2 implies that firm 1's profit decreases as a^L increases, since the negative price effect outweighs the positive sales effect. In other words, this assumption ensures that the theory of maximum product differentiation by Shaked and Sutton is valid in the market with asymmetric firms. As a result, we have $\Pi_1(B3) > \Pi_1(A3) > \Pi_1(C3)$ at the optimal $(a^L)^*$.

Consequently, when assumption 1 and 2 hold, firm 1 chooses a^L , which satisfies $a^L < a^H$. Since $a^L < a^H$, we have $a^L < a^H < b^2$. The strategic implication of this inequality is that when firm 1 introduces fighting brands in response to the entry of a low-quality product, it should locate the fighting brand (a^L) further from the entrant's product (b^2) than its existing high-quality product (a^H) . This leads to <Proposition 2>.

Proposition 2 When assumptions 1 and 2 hold, firm 1 introduces a low-quality product that is more differentiated from firm 2's product than is its high-quality product.

Numerical example

Table 2 shows under which case firm 1's profit is maximized for c=2.0, 2.2 when k=1, $a^H=0.2$ $q^H=10$, $q^L=8$, $\theta^P=1.2$, $\theta^A=1$, $N^P=500$, $N^A=100.9$) This table shows that firm 1's profit is maximized in case B. More specifically, when c=2.0, firm 1 can maximize profit in B3 where $(a^L)^*=0.10$) Thus, when c=2.0, assumptions 1 and 2 are satisfied. When c=2.2, firm 1 maximizes profit in B2 where $(a^L)^*=0$.

Table 2. Numerical Example

| $(k=1, a^H=0.3, q^H=10, q^L=8, \theta^P=1.2, \theta^A=1, N^P=500, N^A=100)$ | | | | | | | | |
|---|----|-----------|-----------------|-----------|-----------------|--|--|--|
| | | c = 2.0 | | c = 2.2 | | | | |
| | | $(a^L)^*$ | $\Pi_1 (\Pi_2)$ | $(a^L)^*$ | $\Pi_1 (\Pi_2)$ | | | |
| | Al | • | 195.93 (62.59) | • | 148.15 (94.82) | | | |
| $a^L = a^H$ | A2 | 0.3 | 120 (120) | 0.3 | 120 (120) | | | |
| | A3 | 0.3 | 200.09 (62.59) | 0.3 | 156.69 (88.98) | | | |
| | B1 | • | 195.93 (62.59) | • | 148.15 (94.82) | | | |
| | B2 | 0 | 170.10 (254.10) | 0 | 170.10 (254.10) | | | |
| | В3 | 0 | 207.07 (70.55) | 0 | 162.95 (99.57) | | | |
| $a^L < a^H$ | B4 | • | NE | • | NE | | | |
| | B5 | 0.10 | 196.33 (62.59) | • | NE | | | |
| | В6 | • | NE | • | NE | | | |
| | C1 | • | 195.93 (62.59) | • | 148.15 (94.82) | | | |
| | C2 | 0.3 + ε | 119.99 (119.99) | 0.3+ € | 119.99 (119.99) | | | |
| $a^L > a^H$ | C3 | 0.3 + ε | 200.09 (62.59) | 0.3+ € | 156.69 (88.98) | | | |
| u > u | C4 | • | NE | • | NE | | | |
| | C5 | • | NE | 0.45 | 123.38 (67.69) | | | |
| | C6 | • | NE | • | NE | | | |

⁻ the fractions of profits are rounded off to two decimal places

V. Conclusions

Firms produce various products for several reasons, one of which may be market strategy. I analyzed the effects of product proliferation when an incumbent can produce both a high-and a low-quality product, but an entrant can produce only a low-quality product. First, compared to the case in which an incumbent produces a high (low)-quality product, the price of a high (low)-quality product increases (decreases) when an incumbent introduces an addition low-quality product. The price of the low-quality product of the incumbent is higher than that of the entrant.

Second, the effect of a product line rivalry on profit depends on its effects on price, sales, and cannibalization. If the cost difference between the high- and low-quality products is moderate compared to the quality difference, proliferation can increase profits.

Finally, I analyze the effects of proliferation when an incumbent can decide the characteristics of a low-quality product by focusing on the issue of fighting brands. When an incumbent introduces a low-quality product in response to entry, it should make its low-quality product more differentiated from the entrant's than is its current high-quality product.

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Appendix

Proof of <Proposition 1>: $\Pi_1(A2) = \frac{1}{2}(1-2a^1)(N^P+N^A)k$ and does not depend on C. $\frac{d^2\Pi_1(A1)}{dc^2} > 0$ and $\Pi_1(A1)$ is minimized at $c = 3(1-2a^1)k + \frac{N^P\theta^P + N^A\theta^A}{N^P + N^A}(q^H - q^L)$. Thus, $\Pi_1(A1)$ decreases in the range of (F1) in footnote 2). In addition, $\Pi_1(A1) = \Pi_1(A2)$ when $c = \frac{N^P\theta^P + N^A\theta^A}{N^P + N^A}(q^H - q^L)$.

Meanwhile, $\frac{d \Pi_1(AB)}{dc^2} > 0$ and $\Pi_1(AB)$ is minimized at $c = \theta^P(q^H - q^L) + \frac{12(1 - 2a^1)(N^P + N^A)^2k}{(2N^P + 3N^A)^2 + N^PN^A}$. Thus, $\Pi_1(AB)$ decreases in the range of (5), and $\Pi_1(AB) = \Pi_1(AB)$ at $c = \theta^P(q^H - q^L)$. Finally, $\Pi_1(AB)$ and $\Pi_1(AB)$ either do not intersect in the range of (5), or they intersect in the range where $c < \frac{N^P\theta^P + N^A\theta^A}{N^P + N^A}(q^H - q^L)$

Q.E.D.

< Lemma 2>11) If an equilibrium exists in the range where

⁻ NE: no existence of a pure strategy equilibrium

⁹⁾ For the given parameter values, equation (5) is equivalent to 1.6 < c < 2.4. At c = 2.333, $\Pi_1(A3) > \Pi_1(A1) = \Pi_1(A2)$.

¹⁰⁾ Thus, when , assumptions 1 and 2 are satisfied

¹¹⁾ Proofs of lemma 2 and 3 are available upon request.

- $0 \le a^L < a^H$,
- (i) In B1, B4 and B5, $p_1^H(B1) = p_1^H(B4) = p_1^H(B5)$, $p_2^L(B1) = p_2^L(B4) = p_2^L(B5)$ and profits are
 - $\Pi_2(B1) = \Pi_2(B4) = \Pi_2(B5),$
 - $Max[\Pi_1(B1), \Pi_1(B4), \Pi_1(B5)] \ge \Pi_1(A1)$ by $\Pi_1(A1) = \Pi_1(B1)$.
- (ii) In B3 and B6, $p_1^H(B3) = p_1^H(B6)$, $p_1^L(B3) = p_1^L(B6)$, $p_2^L(B3) = p_2^L(B6)$ and profits are
 - $\Pi_2(B3) = \Pi_2(B6),$
 - When assumption 2 holds, $Max[\Pi_1(B3),\Pi_1(B6)] > \Pi_1(A3)$.
- (iii) $\Pi_1(B2)$ is maximized at $a^L = 0$ and $\Pi_1(B2; a^L = 0) > \Pi_1(A2)$.
- **<Lemma 3>** If an equilibrium exists in the range where $a^H < a^L < 1/2$,
- (i) In C2, C4 and C5, $p_1^L(C2) = p_1^L(C4) = p_1^L(C5)$, $p_2^L(C2) = p_2^L(C4) = p_2^L(C5)$ and profits are
 - $\Pi_{1}(C2) = \Pi_{1}(C4) = \Pi_{1}(C5),$
- $\Pi_1(C2)$ is maximized at $a^L = a^H + \varepsilon$, and $\Pi_1(C2; a^L = a^H + \varepsilon) < \Pi_1(A2)$.
- (ii) In C3 and C6, $p_2^L(C3) = p_2^L(C6)$ and profits are,
 - When assumption 2 holds, $\Pi_1(C3) < \Pi_1(A3)$.
- (iii) $\Pi_1(A1) = \Pi_1(C1)$

Lemma 2 implies that when assumption 2 holds, $\Pi_1(B) \ge \Pi_1(A)$. That is, firm 1's profit is higher when $a^L < a^H$ than when $a^L = a^H$. Lemma 3 implies that $\max[\Pi_1(C1), \Pi_1(C2), \Pi_1(C3)] \le \Pi_1(A)$ when assumption 2 holds.

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