Line-Source Scattering from Slant Strips

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Abstract

Electromagnetic scattering from slant strips excited by a line source is investigated. Boundary conditions are applied to obtain simultaneous equations for discrete modal coefficients. Computations are performed to illustrate the effects of line-source scattering on radiation patterns.

Key words : Mode-Matching, Slant Strips.

Ⅰ. Introduction

Multiple slant strips are used as polarizers and frequency-selective surfaces for antennas, microwaves, and optical devices. Scattering from slant strips was studied in [1]~[4], when a uniform plane wave was incident on strips. However, if the source location is near slant strips, the uniform plane wave assumption is not always satisfied, and wave sphericity must be accounted for. Such cases often arise in practice when a polarization grid is in close proximity of an antenna feed[5]. Therefore, it is of interest to investigate the effects of source location on the radiation patterns from slant strips. In this work, a line-source excitation is assumed to investigate scattering from slant strips. We utilize the mode-matching technique, which was used previously in [6]. For the sake of completeness, some of the final results in [6] are given in this paper. Salient features of the effects of source location on the scattering and radiation patterns are illustrated next.

Ⅱ. Theory

Consider electromagnetic scattering from multiple slant strips, as shown in Fig. 1. A time convention $e^{-i\omega t}$ is suppressed throughout analysis. A y-oriented electric current $\bar{J}=\hat{y}J\delta(x-x')\delta(z-z')$ is placed in region (Ⅰ), where $\delta(\cdot)$ denotes a Dirac delta function. The strips are assumed to be infinitely long in the y-direction, and the field variation is independent of y. The total field in region (Ⅰ) consists of the incident and scattered fields. A Green's function is used to obtain a primary field from the electric line source. The primary field E_y^i is

$$E_y^i(x,z)=\frac{i\omega\mu J}{2\pi}\begin{cases}\int_{-\infty}^{\infty}\frac{\sin\kappa z}{\kappa}e^{i\kappa z'+i\zeta(x-x')}d\zeta & 0\le z<z' \\ \int_{-\infty}^{\infty}\frac{\sin\kappa z'}{\kappa}e^{i\kappa z+i\zeta(x-x')}d\zeta & z>z'\end{cases} \tag{1}$$

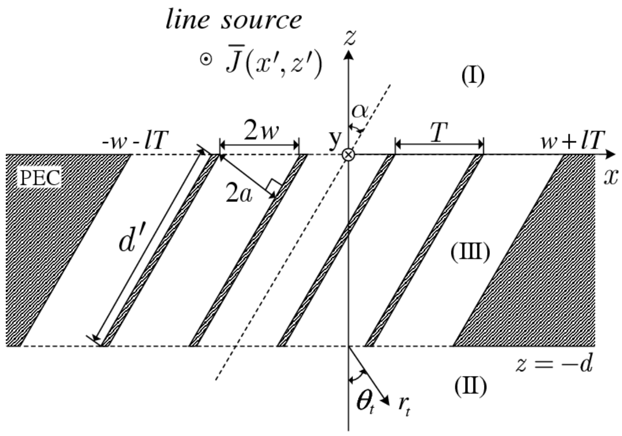

Fig. 1. Multiple slant strips excited by a line source.

where $\kappa_1=\sqrt{k_1^2-\zeta^2}$ and k_1 is the wavenumber in region (I). The scattered fields in regions (Ⅰ), (Ⅱ), and (Ⅲ) can be written in terms of modal coefficients. In particular, we note that the modal coefficients $A_m^{l\pm}$, C_n^r, D_n^r, E_n^r, and F_n^r are used in (1), (3), (4), (9), and (10) of [6]. We use Green's second theorem[6] and the coordinate transformations to deal with the field in the slant region (Ⅲ). The relevant coordinate transformations are written as (see Fig. 2)

$$\begin{pmatrix}x_l\\ z_l\end{pmatrix}=\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}u_l\\ v_l\end{pmatrix} \tag{2}$$

$$\begin{pmatrix}x_l'\\ z_l'\end{pmatrix}=\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}u_l\\ v_l+d'\end{pmatrix} \tag{3}$$

After applying Green's second theorem to $O_lA_lB_l$ and $O_l'A_l'B_l'$, we obtain [6]

$$\mp i2a\xi_q A_q^{l\pm}=\sum_{n=-\infty}^{\infty}\left[C_n^l\Xi^{\pm}-D_n^l\frac{q\pi}{2w}(-1)^n\Lambda_q(\chi_n^{\pm})\right] \tag{4}$$

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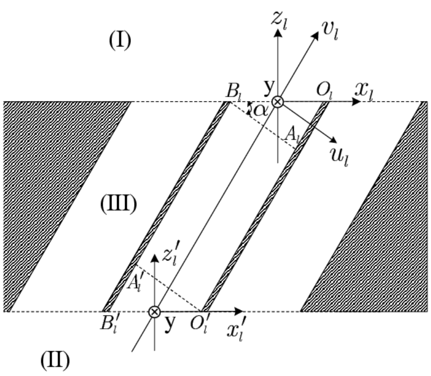


Fig. 2. Coordinate configuration.

$$\mp i2a\xi_q e^{\pm i\frac{\xi_q d}{\cos\alpha}} A_q^{l\pm} = \sum_{n=-\infty}^{\infty}\left[E_n^l \Xi^\pm - F_n^l \frac{q\pi}{2w}(-1)^n \Lambda_q\left(\chi_n^\pm\right)\right] \tag{5}$$

The expressions ξ_q, $\Xi^\pm$, $\Lambda_q(\zeta)$, and $\chi_n^\pm$ are available in [6]. In order to determine the unknown modal coefficients, we enforce the boundary conditions of E_y and H_x continuities. The boundary conditions at $z=0$ and $z=-d$ yield

$$D_m^r = \frac{\omega\mu_3 J}{4\pi w} I_m^r - \frac{i\mu_3}{4\pi w\mu_1}\sum_{l=L_1}^{L_2}\sum_{n=-\infty}^{\infty} C_n^l I_{nm}^{1lr} \tag{6}$$

$$F_m^r = \frac{i\mu_3}{4\pi w\mu_2}\sum_{l=L_1}^{L_2}\sum_{n=-\infty}^{\infty} E_n^l I_{nm}^{2lr} \tag{7}$$

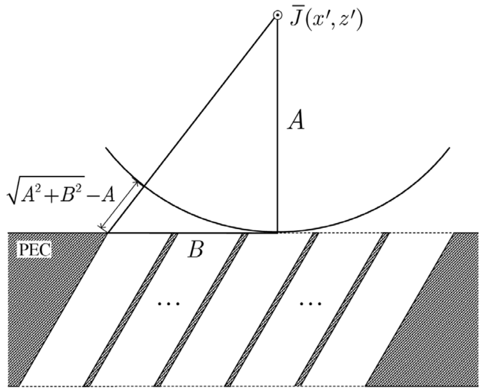
where

$$I_m^r = \int_{-\infty}^{\infty} e^{i\kappa z' - i\zeta x' + irT\zeta} M_m(-\zeta)\, d\zeta \tag{8}$$

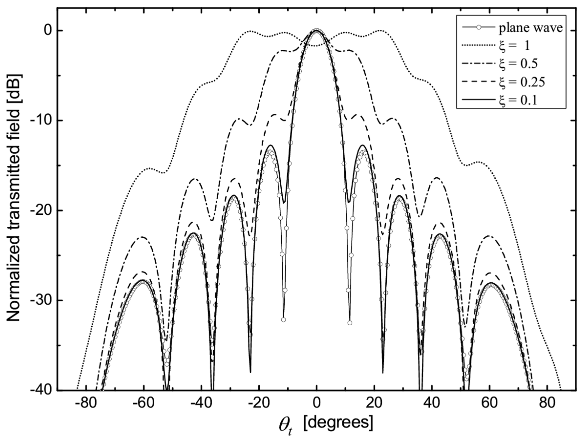
The expressions $I_{nm}^{1lr}, I_{nm}^{2lr}$, and $M_m(\zeta)$ are available in [6]. The unknown coefficients $A_m^{l\pm}$, C_n^r, D_n^r, E_n^r, and F_n^r are determined by simultaneously solving (4), (5), (6), and (7). The far-zone-transmitted field in region (Ⅱ) is

$$E_y^{II}(x,z) \simeq \sqrt{\frac{2k_2}{\pi r_t}}\cos\theta_t \sin(wk_2\sin\theta_t)\, e^{i\left(k_2 d\tan\alpha\sin\theta_t + k_2 r_t - \frac{\pi}{4}\right)} \cdot \sum_{l=L_1}^{L_2}\sum_{n=-\infty}^{\infty}\frac{E_n^l e^{-ilTk_2\sin\theta_t}}{\left(k_2\sin\theta_t - \frac{n\pi}{w}\right)} \tag{9}$$

where k_2 is the wavenumber in region (Ⅱ). Numerical computations are performed to investigate the effects of source location on scattering. Fig. 3 shows the angular behavior of far-zone-transmitted fields, where the normalized path length is given by $\xi=\left(\sqrt{A^2+B^2}-A\right)/\lambda$. The transmitted field of the uniform plane wave at $\theta_i=0°$ is also shown for comparison. As ξ decreases (i.e., A increases), the main beam becomes narrower and approaches a uniform plane wave case. Fig. 4 illustrates the



(a)



(b)

Fig. 3. (a) Wave incidence configuration. (b) Angular behavior of far-zone transmitted field versus ξ: w=5 mm, T=10.2 mm, d=10 mm, α=30°, N=15, x'=0, $\varepsilon_{r1}=\varepsilon_{r2}=\varepsilon_{r3}=1$, $\mu_1=\mu_2=\mu_3=\mu_0$.

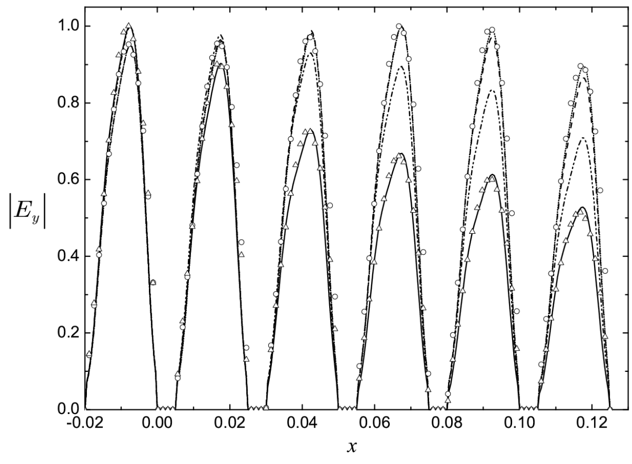


Fig. 4. Normalized magnitude of E_y at $z=-d$ versus x (mm): w=10 mm, T=25 mm, d'=20 mm α=30°, N=11, x'=0, $\varepsilon_{r1}=\varepsilon_{r2}=\varepsilon_{r3}=1$, $\mu_1=\mu_2=\mu_3=\mu_0$. ⋯⋯ plane wave, —— ξ=1, — · · ξ=0.5, - - - ξ=0.1, ○ △ Comsol Multiphysics

normalized magnitude of E_y at $z=-d$. The comparison between our results and those of COMSOL Multiphysics[7] shows good agreement. As ξ decreases, the E-field amplitude approaches a uniform plane wave case, as expected. It is seen that the plane wave approximation is only valid when the distance A from the strips to the source is much larger than the strip size B, such as $\xi \ll 1$. If antennas with polarization grids do not satisfy the condition $\xi \ll 1$, the uniform plane wave incidence assumption fails and wave sphericity effects should be included in the antenna pattern analysis.

Ⅲ. Conclusion

A rigorous formulation of electromagnetic scattering from slant strips, when the antenna feed is close to the slant strips, has been presented. The formulation in this paper must be used for antenna pattern calculations if the antenna feed is in close proximity of strips. The normalized path length ξ can be used as a criterion of the wave sphericity effect.

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