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프로세스 품질 개선과 셋업 절감을 고려한 연속 및 불연속 배송 환경에서의 최적 불완전 품질 재고 모형*

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Optimal Imperfect-Quality Inventory Models for Continuous and Discrete Shipping with Process Improvement and Setup Reduction

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■ Abstract ■

Intelligent investment in setup cost reduction and process reliability improvement is crucial to an emerging integrated lean six sigma practice today. This study examines a cost-minimizing problem of jointly determining production lot size, setup cost reduction, and process reliability improvement decisions for a manufacturer with an imperfect production process. We develop models for previously untapped discrete shipping in a supply chain context as well as continuous shipping and solve them optimally using differential calculus and nonlinear programming. We also conduct analytic and numerical sensitivity analyses to provide various important managerial insights into practices.

Keyword : Inventory, Imperfect Process, Process Reliability Improvement, Setup Reduction, Lean Six Sigma

1. Introduction

Recently an emerging integrated lean six sigma program has received an enormous amount of attention in practice, as found by global master firms including Lockheed Martin, Honeywell Aer-

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ospace, Procter and Gamble and Toyota Motor Company [10]. Particularly notable is General Electric (GE), which has achieved tremendous success by going through three major phases of the process reengineering initiative (i.e., six sigma, e-business digitization, and lean) since early 1990s. The essence of their lean six sigma program lies in reducing not only lead time including setup time and related costs (with lean) but also process variability (with six sigma) in order to better match demand and supply through process synchronization (with digitization). Their current effort is centered on integrating and applying these practices to their internal and external supply chains, where the shipping of lots produced in imperfect production lines occurs continuously and discretely in meeting demands. For the successful execution of this lean six sigma program, it is imperative to make an intelligent investment decision to reduce lead time and related cost, particularly setup time and cost, and improve process reliability, along with lot sizing. Thus, one of the important practical research questions to address is what level of investment a firm needs to make in reducing setup cost and improving process reliability along with lot sizing in order to minimize total relevant cost.

With this motivation from a recent practice perspective, we revisit previous relevant research areas : imperfect-quality inventory management and lot sizing, process reliability improvement and setup reduction. More specifically, this paper aims to develop imperfect-quality inventory models of simultaneously determining optimal production lot size and investment levels of setup cost reduction and process reliability improvement. We examine situations where the shipping of demands occurs not only continuously as in the classic economic production quantity (EPQ) setting but also discretely as in a supplier-buyer supply chain environment.

In the area of imperfect-quality inventory models for an imperfect production process yielding defective units, most of the previous researches have focused on finding a production lot size and other variables in situations where process reliability or defective proportion is given by a function or a value [1, 11, 14, 19, 20, 21, 24, 27-31, 33, 39, 43, 45] or on determining a process reliability level with other variables [3-5, 22, 23, 26, 38, 40] (see also Yano and Lee [42] for a review). Some studies have examined production systems which deteriorate in every cycle as run cycles progress after an initial state involving no error [11, 14, 19-21, 24, 26-28, 30, 39, 43], while others have investigated imperfect production processes with no deterioration over time [1, 3-5, 22, 23, 31, 33, 38, 45]. Mostly, these studies have examined cost-minimization models that reflect the internal effect of defects which increase operating costs. These include inspection cost [14, 19-21, 24, 27-29, 31, 33, 39, 43, 45], investment cost as a function of a production process reliability [18-25], additional production cost to compensate for defect production [1, 11], rework cost [14, 19-21, 24, 26, 30, 39, 40, 43], and restoration cost of a machine or system [14, 19-21, 24, 27, 28, 39, 43]. Further, many previous inventory models dealt with different ways to handle defective units, including instantaneous rework [14, 19-21, 24, 26-30, 39, 43], scrap at no cost [1, 3-5, 11, 22, 23, 33, 38, 45], or salvage [31].

Related to six sigma or total quality management practices, process reliability improvement has been known to have significant impact on a firm's investment decision, production lot size, product quality, setup and inventory holding costs, and profitability [7-9]. Further, it has been shown that the investment for process reliability improvement decreases external failure costs of return, warranty and goodwill penalty from quality dissatisfaction by causing less defects. This reduces internal costs of disposing of defects such as rework, scrap and revenue loss from the salvage of defective items [13] as well.

Further, in the practice under the lean or just-in-time (JIT) philosophy, setup reduction has been known to have a positive effect on a firm's costs, flexibility and profitability [12, 32, 36, 41]. More specifically, Porteus [25] and Spence and Porteus [35] analytically studied the impact of setup cost and time reduction on lot size and effective capacity in the classic economic order quantity (EOQ) model with continuous shipping. Billington [2] extended the classic economic production quantity (EPQ) model by including setup cost as a function of capital expenditure. Kim and Arinze [18] developed a knowledge-based decision support system for setup reduction investment.

Finally, more directly related to our research are a few analytic studies of simultaneously determining lot sizing and investment decisions for process reliability improvement and setup cost reduction investment for an imperfect production process. Since the pioneering work by Porteus [26] for a deteriorating process environment, Cheng [4] investigated this simultaneous determination problem for an imperfect production process with no deterioration over time, using geometric programming, and recently Leung [23] generalized this problem.

From the literature review and practices above, we can identify some crucial points which serve as underpinning of our research. First, from a practice point of view, it is natural and imperative to investigate key decision variables related to lean and six sigma practices together, as evidenced by recent integrated lean six sigma practices and considering the fact that the lean/JIT manufacturing challenge is built upon quality improvement (see e.g., Deming [8] ; Hall [12] ; Suzaki [36] ; Shingo [34]). Second, from an academic standpoint, there have been only a few analytic studies of simultaneously determining lot sizing, process quality improvement and setup cost reduction decisions for an imperfect production process [4, 23, 26]. And third, most of the previous relevant studies have examined the continuous shipping (EOQ or EPQ) environment, although most of the supply chain practices exhibit discrete shipping environments where produced lots are shipped discretely from a supplier to a buyer.

Given this motivation, we propose analytic models which determine an optimal production lot size along with optimal investment levels of process reliability improvement and setup cost reduction. Built upon the previous extended EOQ or EPQ studies for an imperfect production process with continuous shipping, we also examine a discrete shipping environment in which the shipping of an entire lot occurs discretely at the end of each production cycle. This setting is frequently found in a dyadic supplier-buyer supply chain including a component assembly plant [17] or serial two-stage production and inventory systems [16, 37] to name a few. Then we develop an optimal solution approach based on nonlinear programming and differential calculus. We also perform both analytic and numerical sensitivity analyses to provide key managerial insights into practices.

The significance of this study may lie in not only jointly determining optimal lot size and investment levels of the prerequisites (i.e., process reliability improvement and setup cost reduction) for important lean six sigma practices today, but also examining previously untapped discrete shipping environments in dyadic supply chains as well as continuous shipping. The rest of the paper is organized as simultaneous imperfect-quality inventory optimization models for continuous shipping and for discrete shipping, and numerical examples and optimal solutions along with managerial implications from sensitivity analyses, followed by concluding remarks with future research directions.

2. Imperfect-quality Inventory Model for Continuous Shipping

The problem we investigate is to determine optimal cost-minimizing production lot size and investment levels for setup cost reduction and process reliability improvement for a stable but imperfect production process with continuous shipping of non-defects to satisfy demands as in the economic production quantity (EPQ) model. The costs include setup and inventory holding costs, quality failure costs from defect production, and investment opportunity costs needed for setup cost reduction and process reliability improvement in an integrated lean six sigma practice.

To facilitate the discussion, we use the following notation :

Q : production lot size per setup (*decision variable* ; unit/cycle)

S : cost per setup (*decision variable* ; \$/setup), where $S \le S_0$ (= current level)

R : process reliability level or non-defect rate (*decision variable*), where $0 < R_0$ (= current level) $\le R \le 1$

d : demand rate (unit/unit time)

V : total production given R (unit/unit time), satisfying demand = : d/R

r : production rate (unit/unit time), where $r > d/R_0 \ge d/R$ (= V)

c : unit production (including inspection) cost (\$/unit)

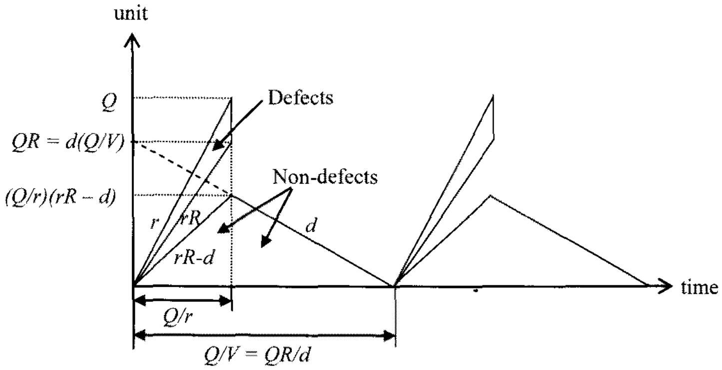
i : inventory holding cost rate fraction (/unit/unit time)

l : unit failure cost (\$/unit)

K_S : capital investment (\$/unit time) for setup cost reduction from S_0 to S = : $aS^{-\sigma}-aS_0^{-\sigma}$, and

K_R : capital investment (\$/unit time) for process reliability improvement from R_0 to R = : $mR^{\rho}-mR_0^{\rho}$, where $\rho > 1$ for K_R to be convex to ensure more investment in an increasing manner for more process reliability improvement, and where $aS_0^{-\sigma}$ and mR_0^{ρ} insure that no investment is needed at the current setup cost and reliability level.

[Figure 1] depicts the behavior of the imperfect-quality inventory model for continuous shipping as in previous studies [1, 3-5, 22, 23, 31, 33, 38, 45]. In each production cycle (Q/V), a production line incurs setup cost (S) and produces a lot (Q) at a unit production (and inspection) cost (c). Due to the imperfect nature of the production process, each lot produced contains defects with a defective proportion of (1-R). During each production run, the lot is in-



[Figure 1] Continuous shipping inventory model for an imperfect process

spected and sorted out into defects and non-defects with perfect (100%) inspection. Those screened defects are assumed to be disposed at the end of each production run. And it is also assumed that only non-defective units (RQ) are continuously replenished and shipped to meet the demands (d) in each cycle. Thus, the total production volume per unit time (V) needs to be d/R, where $d/R \le d/R_0$ and R_0 denotes the current level of process reliability before investment for improvement. The production rate (r) is assumed to be greater than d/R_0, to ensure no shortages of satisfying the demands. And each defect produced incurs unit cost of failure (l) including scrap or environmental concerns related to disposing of a defective unit. Although the defect disposition can take various forms such as rework, salvage to a second market and scrap, we do not include all the details, since the focus of this paper is on investment decisions on setup cost reduction and process reliability improvement (see Yoo [44] for the modeling of incorporating various defect disposition options with different inspection methods).

Regarding inventory holding, due to continuous shipping as shown in [Figure 1], the maximum inventory becomes $(Q/r)(rR-d)$ for non-defects similarly as in the EPQ model with $R = 1$, while that for defects becomes $Q(1-R)$. Thus the average inventory per cycle of Q/V (= QR/d) becomes $Q^2(R^2/d-2R/r+1/r)/2$, consisting of cycle inventory of non-defects ($Q^2(R^2/d-R/r)/2$, i.e. lower triangle) and defects ($(Q^2(1-R)/r)/2$, i.e. upper triangle). So the average inventory per unit time becomes $[Q^2(R^2/d-2R/r+1/r)/2](V/Q) = Q(R^2/d-2R/r+1/r)(d/R)/2$. Further, the investment costs of setup cost reduction and process reliability improvement are defined as opportunity costs (iK_S, iK_R) in power function forms, incremental from the current levels (see Porteus [25,26] ; Spence and Porteus [35] ; Cheng [4] ; Lee et al. [22] ; Leung [23]).

Below is the continuous shipping model for simultaneously finding optimal production lot size or extended EPQ (Q^*), reduced setup cost (S^*) and improved process reliability level (R^*) which minimize total costs per unit time (TC), consisting of total production (TPC), inventory (TIC) (i.e., setup (TSC) and inventory holding (THC)), failure (TLC), and investment for setup cost reduction (*TUC*) and process reliability improvement (*TRC*).

$$\text{Min } TC(Q,\ S,\ R) = TPC + TSC + THC + TLC + TUC + TRC$$

$$= cV + \frac{SV}{Q} + ic\frac{Q^2}{2}\left(\frac{R^2}{d} - \frac{2R}{r} + \frac{1}{r}\right)\frac{V}{Q} + l(1-R)V + iK_S + iK_R$$

$$= \frac{(c+l(1-R))d}{R} + \frac{Sd}{RQ} + \frac{icd}{2}\left(\frac{R^2}{d} - \frac{2R}{r} + \frac{1}{r}\right)\frac{Q}{R} + iaS^{-\sigma} \cdot$$

$$+ imR^{\rho} - (iaS_0^{-\sigma} + imR_0^{\rho}) \qquad (1)$$

$$\text{s.t. } S \le S_0 \text{ and } d/r < R_0 \le R \le 1 \qquad (2)$$

Proposition 1 : The following is true :

a : $TC(Q, S, R)$ is strictly convex and therefore has a unique global minimum.

b : $$Q(S,\ R) = \sqrt{\frac{2S}{ic(R^2/d - 2R/r + 1/r)}}, \qquad (3)$$

$$TIC(S,\ R) = \sqrt{2icSd^2\left(\frac{R^2}{d} - \frac{2R}{r} + \frac{1}{r}\right)\frac{1}{R^2}}, \text{ and} \qquad (4)$$

$$TC(S,\ R) = \frac{(c+l(1-R))d}{R} \qquad (5)$$

$$+ \sqrt{2icSd^2\left(\frac{R^2}{d} - \frac{2R}{r} + \frac{1}{r}\right)\frac{1}{R^2}} + iaS^{-\sigma} + imR^{\rho}$$

$$- (iaS_0^{-\sigma} + imR_0^{\rho}).$$

c : $Q(S, R)$ is monotonically increasing concave in S, while it decreases in R, given $d/r < R_0 \le R \le 1$.

$TIC(S, R)$ is monotonically increasing concave in S, and decreasing convex (concave) in R in the range of R satisfying $R^2(3-2R)/d > (<) (3R-1)(2-R)/r$, given $d/r < R_0 \le R \le 1$.

d : $$S(R) = \left(\frac{2\sigma^2 ia^2}{cd^2}\right)^{\frac{1}{2\sigma+1}}\left(\frac{R^2}{R^2/d - 2R/r + 1/r}\right)^{\frac{1}{2\sigma+1}}$$

and (6)

$$Q(R) = \left(\frac{2}{ic}\right)^{1/2}\left(\frac{2\sigma^2 ia^2}{cd^2}\right)^{\frac{1}{2(2\sigma+1)}} \qquad (7)$$

$$\left(\frac{R}{(R^2/d - 2R/r + 1/r)^{\sigma+1}}\right)^{\frac{1}{2\sigma+1}}$$

e : $S(R)$ is monotonically increasing in R. $Q(R)$ decreases in R outside the range of R, while it increases in the range of R, where the range of R, given $d/r < R_0 \le R \le 1$, is computed as

$$\frac{\sigma/r - \sqrt{(\sigma^2/r + (2\sigma+1)/d)/r}}{(2\sigma+1)/d} < R < \frac{\sigma/r + \sqrt{(\sigma^2/r + (2\sigma+1)/d)/r}}{(2\sigma+1)/d}.$$

Proof : See <Appendix A>.

Observe from Proposition 1(b) that the production lot size $Q(S, R)$ and total inventory cost $TIC(S, R)$ with $R = 1$ are identical to the EPQ solutions. Also note from Proposition 1(c) that $Q(S, R)$ increases in S given a fixed R, while it decreases in R given a fixed S. This result corresponds to a partial integration situation where only one investment option is considered. In the fully integrated, simultaneous optimization involving both investment options, however, it is interesting to find that the lot size $Q(R)$ can either increase or decrease in R depending on its range due to the tradeoff between reduced setup cost $S(R)$ and process reliability level R, given that $S(R)$ increases in R exhibiting the tradeoff between the two investment options, as shown in Proposition 1(e).

Now since $TC'_Q(Q, S, R) = TC'_S(Q, S, R) = TC'_R(Q, S, R) = 0$ in (A1-A3) in Appendix A are not solvable simultaneously but $TC(Q, S, R)$ is strictly convex (see Proposition 1(a)), we obtain the global minimum solutions by solving the following $TC(R)$ reduced from $TC(Q, S, R)$ in (1), by substituting $S(R)$ in (6) into $TC(S, R)$ in (5) :

$$\text{Min}\, TC(R) = TPC(R) + TIC(R) + TLC(R) + TUC(R) + TRC(R)$$

$$= \frac{cd}{R} + (2\sigma ia)^{\frac{1}{2\sigma+1}}(2icd^2)^{\frac{\sigma}{2\sigma+1}}\left(\frac{R^2/d - 2R/r + 1/r}{R^2}\right)^{\frac{\sigma}{2\sigma+1}}$$

$$+ \frac{l(1-R)d}{R} + ia\left(\frac{cd^2}{2\sigma^2 ia^2}\right)^{\frac{\sigma}{2\sigma+1}}\left(\frac{R^2/d - 2R/r + 1/r}{R^2}\right)^{\frac{\sigma}{2\sigma+1}}$$

$$+ imR^{\rho} - (iaS_0^{-\sigma} + imR_0^{\rho}) \qquad (8)$$

s.t. $S \leq S_0$, and $d/r < R_0 \leq R \leq 1$, in (2)

Proposition 2 : The following is true :

a : $TPC(R)$ is monotonically decreasing convex in R.

b : $TIC(R)$ and $TUC(R)$ are decreasing convex (concave) in R in the range of R satisfying $R^2(3-2R)/d$ >(<) $((2R-1)(3-2R) + (2(\sigma+1)/(2\sigma+1))(1-R)^2)/r$, given $d/r < R_0 \leq R \leq 1$.

c : $TLC(R)$ is monotonically decreasing convex in R.

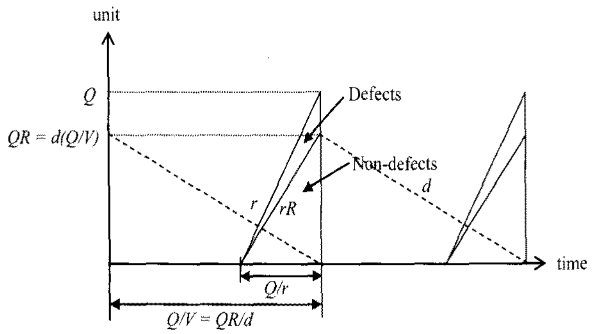
d : $TRC(R)$ is monotonically increasing convex in R, since $\rho > 1$.

Proof : Differentiating each cost component in (8) with simple algebra proves part (a-d). Since it is straightforward, we omit the details. Thus, the proof is complete. □

From Proposition 2 for simultaneous optimization, it is interesting to discover that the tradeoff in optimal solutions with respect to R exists between process reliability improvement investment opportunity cost and all the other cost components (i.e., production, inventory, failure, and setup cost reduction investment opportunity costs). And an increase in R decreases total production, inventory and setup cost reduction investment opportunity costs as well as failure cost. In a later section, we will numerically show these behaviors.

3. Imperfect-quality Inventory Model for Discrete Shipping

The discrete shipping model differs from the previous continuous shipping model in that the shipping of an entire lot to meet the demands occurs discretely at the end of each production (run) cycle (see [Figure 2] for inventory behavior and refer to the same notation used before). It is worthwhile to examine this environment, since it is typically found in dyadic supply chains or two-stage production and inventory systems. Further, the upstream supply chain in the EOQ model reflects a situation where a firm receives



[Figure 2] Discrete shipping inventory model for an imperfect process an entire lot at once each cycle before being continuously shipped to buyers. Thus, the maximum inventory becomes simply the production lot size (Q) and only non-defective units (RQ) are shipped at once at the end of each cycle ($Q/V = QR/d$) to satisfy its cycle demand. Thus the average inventory per cycle becomes $(Q^2/r)/2$, consisting of cycle inventory of non-defects $((Q^2 R/r)/2)$ and defects $((Q^2(1-R)/r)/2)$. So the average inventory per unit time becomes $[(Q^2/r)/2](V/Q) = (Q/2)d/(rR)$.

Similarly as before, this model below simultaneously finds the optimal operations batching quantity (Q^*, not an extended EPQ), reduced setup cost (S^*), and improved process reliability level (R^*) which minimize total costs (TC), consisting of total production (TPC), inventory (TIC) (i.e., setup (TSC) and inventory holding (THC)), failure (TLC), and investment for setup cost reduction (TUC) and process reliability improvement (TRC).

$$\text{Min } TC(Q,\ S,\ R) = TPC + TSC + THC + TLC + TUC + TRC$$

$$= cV + \frac{SV}{Q} + ic\frac{Q}{2}\frac{V}{r} + l(1-R)V + iK_S + iK_R$$

$$= \frac{(c+l(1-R))d}{R} + \frac{Sd}{RQ} + \frac{icdQ}{2Rr} + iaS^{-\sigma}$$

$$+ imR^{\rho} - (iaS_0^{-\sigma} + imR_0^{\rho}) \quad (9)$$

s.t. $S \le S_0$ and $d/r < R_0 \le R \le 1$ (10)

Proposition 3 : The following is true :

a : $TC(Q, S, R)$ is strictly convex and therefore has a unique global minimum.

b : $$Q(S,\ R) = \sqrt{\frac{2Sr}{ic}}, \quad (11)$$

$$TIC(S,\ R) = \sqrt{\frac{2icSd^2}{R^2 r}}, \text{ and} \quad (12)$$

$$TC(S,\ R) = \frac{(c+l(1-R))d}{R} + \sqrt{\frac{2icSd^2}{R^2 r}}$$

$$+ iaS^{-\sigma} + imR^{\rho} - (iaS_0^{-\sigma} + imR_0^{\rho}) \quad (13)$$

c : $Q(S, R)$ is monotonically increasing concave in S, whereas not affected by R. $TIC(S,R)$ is monotonically increasing concave in S, whereas monotonically decreasing convex in R.

d : $$S(R) = \left(\frac{2\sigma^2 ia^2 r}{cd^2}\right)^{\frac{1}{2\sigma+1}} R^{\frac{2}{2\sigma+1}} \text{ and} \quad (14)$$

$$Q(R) = \left(\frac{2r}{ic}\right)^{1/2} \left(\frac{2\sigma^2 ia^2 r}{cd^2}\right)^{\frac{1}{2(2\sigma+1)}} R^{\frac{1}{2\sigma+1}}. \quad (15)$$

e : $S(R)$ is monotonically increasing concave in R. $Q(R)$ is monotonically increasing concave in R.

Proof : See Appendix B.

Observe from Proposition 3(b-c) that $Q(S, R)$ and $TIC(S, R)$ in the discrete shipping model with $R = 1$ are not identical to the EPQ solutions for continuous shipping. Interestingly, $Q(S, R)$ for partial integration of considering only one investment option increases in S as in the continuous shipping model, but it is not affected by R given a fixed S, different from the behavior in continuous shipping. It is because the lot size $Q(S, R)$ depends on production rate (r) only, neither demands (d) nor process reliability level (R), as seen from the inventory behavior in [Figure 2]. But note that $TIC(S, R)$ is affected by all those factors due to the tradeoff between total setup and holding costs. In the simultaneous optimization of considering both investment options, however, be aware that lot size $Q(R)$ is only increasing in R, due to an increase in $S(R)$

in R, different from the finding in the continuous shipping, as shown in Proposition 3(e). These different behaviors are illustrated numerically in the next section.

Now in solving the model, since $TC'_Q(Q, S, R) = TC'_S(Q, S, R) = TC'_R(Q, S, R) = 0$ in (B1-B3) in Appendix B are also not solvable simultaneously as in continuous shipping but $TC(Q,S,R)$ is strictly convex, we obtain the global minimum solutions by solving $TC(R)$ below reduced from $TC(Q, S, R)$ by substituting (14) into (13).

$$\text{Min}\, TC(R) = TPC(R) + TIC(R) + TLC(R) + TUC(R) + TRC(R)$$

$$= \frac{cd}{R} + \left(\frac{2icd^2}{r}\right)^{1/2}\left(\frac{2\sigma^2 ia^2 r}{cd^2}\right)^{\frac{1}{2(2\sigma+1)}} R^{-\frac{2\sigma}{2\sigma+1}} + \frac{l(1-R)d}{R}$$

$$+ ia\left(\frac{2\sigma^2 ia^2 r}{cd^2}\right)^{-\frac{\sigma}{2\sigma+1}} R^{\frac{2\sigma}{2\sigma+1}} + imR^{\rho} - (iaS_0^{-\sigma} + imR_0^{\rho}) \quad (16)$$

s.t. $S \le S_0$, and $d/r < R_0 \le R \le 1$, in (10)

Proposition 4 : The following is true :

a : $TPC(R)$ is monotonically decreasing convex in R.

b : $TIC(R)$ and $TUC(R)$ are monotonically decreasing convex in R.

c : $TLC(R)$ is monotonically decreasing convex in R.

d : $TRC(R)$ is monotonically increasing convex in R.

Proof : Differentiating each cost component in (16) with simple algebra proves part (a-d). Since it is straightforward, we omit the details. Thus, the proof is complete. □

From Proposition 4 for simultaneous optimization, we can see that again as in the continuous shipping model, the tradeoff in optimal solutions with respect to R exists between process reliability improvement investment opportunity cost and all the other cost components. And an increase in R decreases total production, inventory and setup cost reduction investment opportunity costs as well as failure cost.

4. Numerical Example Solutions and Managerial Implications

This section illustrates solution differences between the continuous and discrete shipping models for an imperfect process with both investment options (full integration, i.e., simultaneous optimization), using the data set below. We also show the results of these models with no investment option (basic model) and with only one option (partial integration of either setup cost reduction or process reliability improvement investment) for a comparative purpose.

Data set.

d = demand rate per unit time = 15,000 units/year

r = production rate per unit time ($r > d/R_0 \ge d/R$) = 40,000 units/year

c = production (and inspection) cost per unit = \$150/unit

i = inventory holding cost rate fraction or a firm's cost of capital = 0.3/unit/year

l = failure cost of defects per unit = \$50/unit

K_R = process reliability improvement investment \$ per unit time = $mR^{\rho}-mR_0^{\rho}$ = $4{,}000{,}000(R^{4.0}-0.8^{4.0})$

K_S = setup cost reduction investment \$ per unit time = $aS^{-\sigma}-aS_0^{-\sigma}$ = $1{,}000{,}000(S^{-0.1}-2{,}000^{-0.1})$

<Table 1> summarizes the optimal solutions to (8) and (16) for continuous and discrete ship-

ping, respectively, along with the basic and partial model solutions and extreme point solutions for the cases with ($S = S_0$, $R = 1$) and ($S = S^o$, $R = 1$), where S^o = optimal solution to S given $R = 1$, obtained by using a nonlinear programming solver LINGO. As obvious from the result, the fully integrated, simultaneous optimization models realize the most savings in both continuous and discrete shipping environment. Thus, it is important to jointly consider setup cost reduction and process reliability improvement investment decisions for an integrated lean six sigma practice today.

In more detail, regarding total cost components, for the cases with the reduced setup cost (S^o or S^*) as a function of R (see three columns with the reduced setup in <Table 1>), observe that $TPC(R)$, $TIC(R)$, $TLC(R)$ and $TUC(R)$ de-

<Table 1> Numerical solutions (S^o, R^o = Optimal solutions of S and R, given $R = R_0$ or 1 and $S = S_0$, respectively)

(a) Continuous shipping model

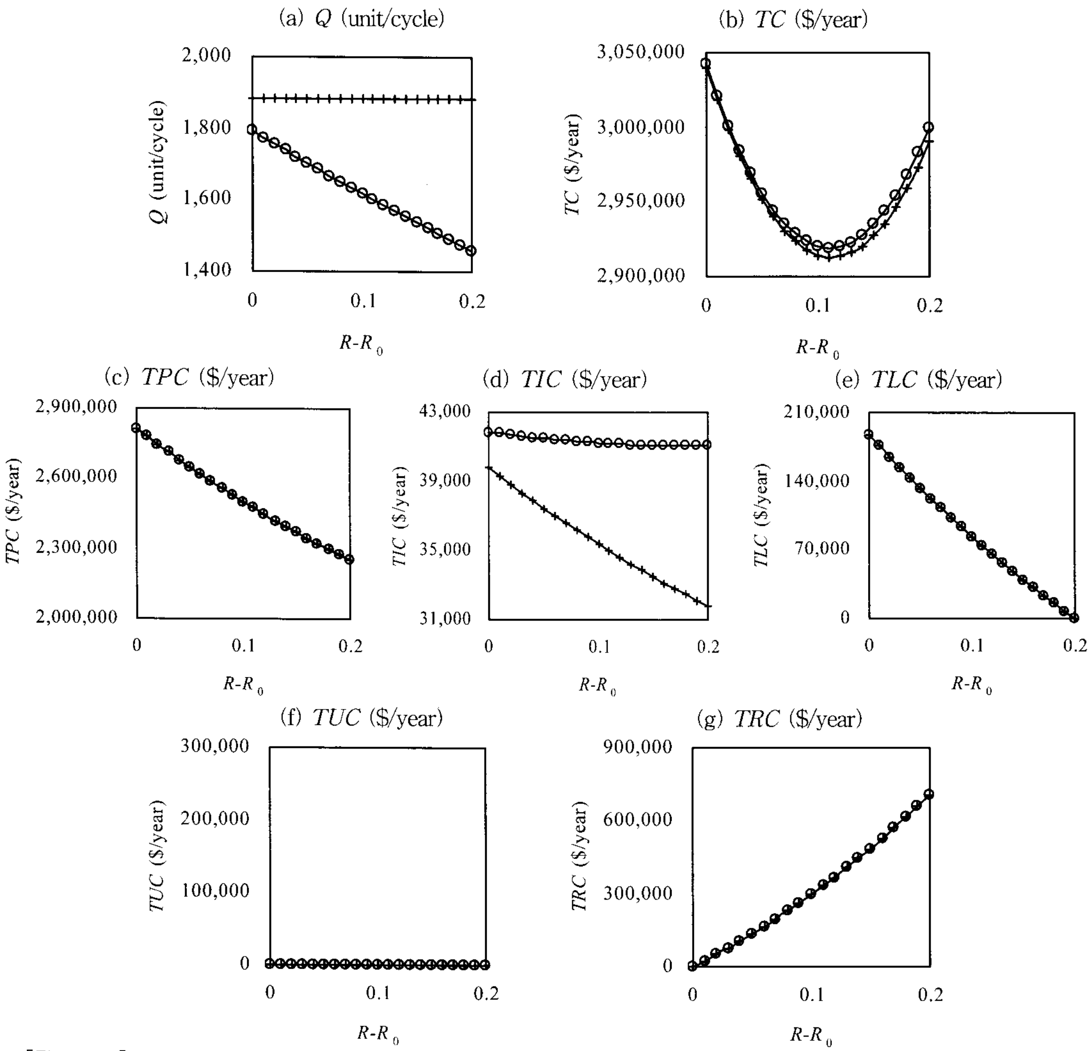
Models	Basic	Partial Integration		Full	Extreme Points	
Decision variables and costs	Current setup and current reliability (S_0, R_0)	Reduced setup and current reliability (S^o, R_0)	Current setup and improved reliability (S_0, R^o)	Reduced setup and improved reliability (S^*, R^*)	Current setup and perfect reliability (S_0, 1)	Reduced setup and perfect reliability (S^o, 1)
S($/cycle)	2,000	1,027	2,000	1,054	2,000	1,059
R(%)	80.00	80.00	91.04	91.04	100.00	100.00
Q(unit/cycle)	1,792	1,285	1,600	1,161	1,461	1,063
TC($/year)	3,041,842	3,039,653	2,919,281	2,917,270	2,999,559	2,997,581
TPC($/year)	2,812,500	2,812,500	2,471,364	2,471,473	2,250,000	2,250,000
TIC($/year)	41,842	29,990	41,198	29,913	41,079	29,898
TLC($/year)	187,500	187,500	73,788	73,824	0	0
TUC($/year)	0	9,663	0	9,276	0	9,203
TRC($/year)	0	0	332,931	332,786	708,480	708,480

(b) Discrete shipping model

Models	Basic	Partial Integration		Full	Extreme Points	
Decision variables and costs	Current setup and current reliability (S_0, R_0)	Reduced setup and current reliability (S^o, R_0)	Current setup and improved reliability (S_0, R^o)	Reduced setup and improved reliability (S^*, R^*)	Current setup and perfect reliability (S_0, 1)	Reduced setup and perfect reliability (S^o, 1)
S($/cycle)	2,000	1,118	2,000	1,391	2,000	1,622
R(%)	80.00	80.00	91.22	91.19	100.00	100.00
Q(unit/cycle)	1,886	1,410	1,886	1,572	1,886	1,698
TC ($/year)	3,039,775	3,038,139	2,913,001	2,912,396	2,990,300	2,990,105
TPC($/year)	2,812,500	2,812,500	2,466,552	2,467,413	2,250,000	2,250,000
TIC($/year)	39,775	29,738	34,882	29,096	31,820	28,652
TLC($/year)	187,500	187,500	72,184	72,471	0	0
TUC($/year)	0	8,402	0	5,193	0	2,973
TRC($/year)	0	0	339,383	338,224	708,480	708,480

crease in R, while $TRC(R)$ increases in R, as analytically shown in Proposition 2 and 4 for both shipping models. Further, in terms of the difference between the two shipping models, it is interesting to see that the continuous shipping environment requires a smaller lot size with more setup cost reduction than the discrete shipping case due to a higher burden of holding inventories as shown by the magnitude of $TIC(R)$ and $TUC(R)$ in <Table 1> and also in [Figure 1] and [Figure 2]. Furthermore, we can discover that the impact of process reliability improvement on setup cost is less in continuous shipping than discrete shipping as shown by the magnitude of a reduced setup cost and an improved reliability level in <Table 1> and also easily seen by Proposition 1 and 3.

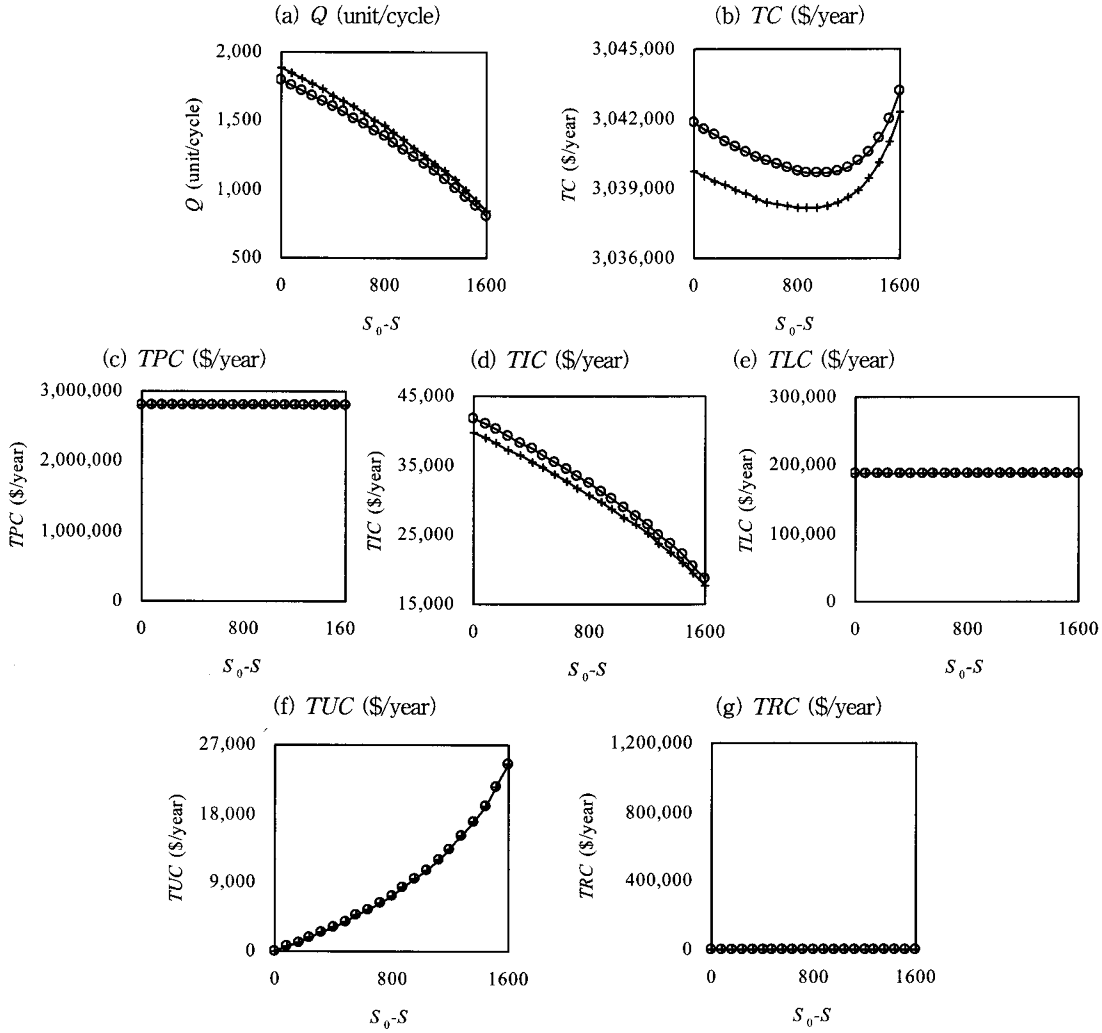
Now let's further look at the effect of process



[Figure 3] The effect of process reliability improvement (R-R_0) on Q, TC and TC components
(○ : continuous shipping model, + : discrete shipping model)

reliability improvement and setup cost reduction each, given the other fixed, on lot size Q, total cost TC and its components. These results, illustrated graphically in [Figure 3] and [Figure 4] using the same data set, can also be considered those of the sensitivity analyses for the partial integration models with only one investment option.

First, for the *effect of process reliability improvement* (R-R_0) given a fixed current setup cost ($S = S_0 = 2{,}000$ and thus $TUC = 0$ in [Figure 3](f)), we can discover an interesting fact that the lot size Q (i.e., $Q(S, R)$) in [Figure 3](a) decreases in R (or R-R_0) in continuous shipping, while it stays the same in R in discrete shipping, as analytically shown in Proposition 1(b-c) and 3(b-c), respectively (see also those three columns with the current setup (S_0) in <Table 1>).



[Figure 4] The effect of setup cost reduction (S_0-S) on Q, TC and TC components
(○ : continuous shipping model, + : discrete shipping model)

This difference is attributed to their different inventory behaviors shown in [Figure 1] and [Figure 2]. Thus, a manager needs to carefully choose the right inventory model by understanding the environment. Regarding total cost and its components, however, in both shipping cases, as the process reliability improvement level of R-R_0 increases with an increase in *TRC*, other cost components such as *TPC*, *TIC* and *TLC* decrease (see [Figure 3](c-e, g)). In this tradeoff relationship between *TPC*, *TIC* and *TLC* together and *TRC*, the total cost *TC* is strictly convex and thus has the unique optimal solution in the process reliability level of R (see [Figure 3](b)). It should also be noted, however, that the impact of process reliability investment is less in continuous shipping than discrete shipping. Consequently, a manager should fully understand the tradeoff relationship given his or her shipping environment to make an intelligent investment decision on process reliability improvement.

Second, for the *effect of setup cost reduction* (S_0-S) given a fixed current process reliability level ($R = R_0 = 0.8$ and thus $TRC = 0$ in [Figure 4](g)), we can find an interesting aspect that the lot size $Q(S, R)$ in [Figure 4](a) shows similar behaviors in both shipping, different from the case for the process reliability improvement. That is, $Q(S, R)$ decreases with a decrease in S (or an increase in S_0-S) in both shipping, as also analytically shown in Proposition 1(b-c) and 3(b-c). Also, total cost and its components show similar behaviors in both shipping, although again different from the effect of process reliability improvement. In detail, as the level of setup cost reduction (S_0-S) increases with an increase in *TUC*, only *TIC* decreases, while both *TPC* and *TLC* stay the same (see [Figure 4](c-f)). In this tradeoff relationship between *TIC* and *TUC*, the total cost *TC* is strictly convex with the unique optimal solution in the setup cost of S (see [Figure 4](b)). It should also be noted, however, that the continuous shipping environment requires a smaller lot size with more setup cost reduction than the discrete shipping. Thus, it is important for a manager to be aware of his or her shipping environment.

5. Concluding Remarks

This study has investigated the problem of making a manufacturer's cost-minimizing decision of determining the optimal production lot size and investment levels of setup cost reduction and process reliability improvement, for a stable and non-deteriorating imperfect production process. We examined a previously untapped discrete shipping environment in a dyadic supply chain as well as continuous shipping. We developed optimal solution approaches based on nonlinear programming and differential calculus. The results showed an importance of jointly considering both setup cost reduction and process reliability improvement investment decisions for an integrated lean six sigma practice. Moreover, the different behaviors of key decision variables in different shipping environments called for a manager's careful attention in making intelligent decisions on lot sizing and setup cost reduction and process reliability improvement investment.

In sum, this study is significant in that it not only develops a practical model of jointly determining lot size and investment levels for an important lean six sigma practice, but also examines a previously unexplored discrete shipping environment in dyadic supplier-buyer supply chains or two-stage production and inventory systems as well as a continuous shipping context in extended EOQ or EPQ models with imperfect production processes.

Despite its significance in both practice and academia, however, there are some limitations in this study, which deserve further investigation through future research in richer environment. First, it may be desirable to incorporate other important factors such as defect inspection and disposition options including rework and salvage to a second market (e.g., by extending Yoo [44]). Second, related to setup reduction, it may be interesting to further explore the issues of setup time reduction, effective capacity, and flexibility (e.g., by extending Vőrős [40]). Third, it may be also desirable to examine interactions among those mentioned variables with a variable demand rate in a more complex supply chain context (e.g., by extending Lee et al. [22] ; Kim and Lee [15]). Finally, it will be interesting to treat the production rate as a variable by incorporating the concept of takt time along with cycle time in a lean/JIT context [10, 36, 41] in a supply chain environment with discrete shipping.

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〈Appendix〉

A. Proof of Proposition 1

In part (a), for TC to be strictly convex to have a unique global minimum, its extremum conditions should be satisfied, i.e., $TC_j = 0$ for j for the first-order necessary conditions (FONC) and the principal minors of the Hessian $|H| =: |H_i| > 0$ for i, where $|H| = |TC''_{jk}|$ for j, k for the second-order sufficient conditions (SOSC) [6]. From (1), FONC :

$$TC'_Q = -\frac{Sd}{RQ^2} + \frac{icd}{2}\left(\frac{R^2}{d} - \frac{2R}{r} + \frac{1}{r}\right)\frac{1}{R}, \quad (A1)$$

$$TC'_S = \frac{d}{RQ} - \sigma iaS^{-(\sigma+1)}, \text{ and} \quad (A2)$$

$$TC'_R = -\left((c+l) + \frac{S}{Q} + \frac{icQ}{2r}\right)\frac{d}{R^2} + \rho imR^{\rho-1} + \frac{icQ}{2} \quad (A3)$$

And SOSC :

$$|H_1| = TC''_{QQ} = \frac{2Sd}{RQ^3} > 0,$$

$$|H_2| = TC''_{QQ}TC''_{SS} - TC''_{QS}{}^2$$

$$= \left(\frac{2Sd}{RQ^3}\right)\left(\sigma(\sigma+1)iaS^{-(\sigma+2)}\right) - \left(\frac{d}{RQ^2}\right)^2$$

$$= \frac{(2\sigma+1)d^2}{R^2Q^4} > 0$$

from (A2), and

$$|H_3| = TC''_{QQ}TC''_{SS}TC''_{RR} - TC''_{QQ}TC''_{SR}{}^2 - TC''_{SS}TC''_{QR}{}^2 - TC''_{RR}TC''_{QS}{}^2 + 2TC''_{QS}TC''_{QR}TC''_{SR}$$

$$= TC''_{RR}\frac{(2\sigma+1)d^2}{R^2Q^4} + \left(\frac{d^2}{R^3Q^3}TC''_{QR} - \frac{2Sd^3}{R^5Q^5}\right) + TC''_{QR}\left(\frac{d^2}{R^3Q^3} - \sigma(\sigma+1)iaS^{-(\sigma+2)}TC''_{QR}\right),$$

where $TC''_{RR} = \left((c+l) + \frac{S}{Q} + \frac{icQ}{2r}\right)\frac{2d}{R^3} + (\rho-1)\rho imR^{\rho-2}$ and $TC''_{QR} = \frac{Sd}{R^2Q^2} + \frac{icd}{2}\left(\frac{1}{d} - \frac{1}{rR^2}\right)$.

Due to the mathematical difficulty of proving $|H_3| > 0$ using differential calculus and algebra, we develop a nonlinear programming model below for its proof.

Min $|H_3|$ (A4)

s.t.

$S \le S_0$ and $d/r < R_0 \le R \le 1$, in (2)

$R, S, Q, a, c, l, d, i, m, r, \delta, \sigma > 0$

$\rho > 1$

The solution of (A4) using a nonlinear programming problem solver LINGO yields that $|H_3| = 0$, thereby satisfying $|H_3| \ge 0$, which supports the existence of only one global optimal solution of $TC(Q, S, R)$ in (1) without local optima, given the constraints in (2). This proves part (a). From (A1), we obtain (3). By substituting (3) into the total inventory costs (TIC) in (1), we obtain (4). Then, substituting (4) into (1) yields (5). This proves part (b). Differentiating (3-4) with respect to S and R with simple algebra proves part (c). Now differentiating (5) with respect to S gives

$$TC'_S(S,R) = \frac{1}{2}\sqrt{\frac{2icd^2(R^2/d - 2R/r + 1/r)}{R^2S}} - \sigma iaS^{-(\sigma+1)} \quad (A5)$$

Solving (A5) = 0 for S yields (6). Substituting (6) into (3) yields (7). This proves part (d). And differentiating (6~7) with respect to R with simple algebra proves part (e), so we omit the details. Thus, the proof is complete. □

B. Proof of Proposition 3

From (9), FONC :

$$TC'_Q = -\frac{Sd}{RQ^2} + \frac{icd}{2Rr} = 0 \quad (B1)$$

$$TC'_S = \frac{d}{RQ} - \sigma iaS^{-(\sigma+1)} = 0 \text{, and} \quad (B2)$$

$$TC'_R = -\left((c+l)d + \frac{Sd}{Q} + \frac{icdQ}{2r}\right)\frac{1}{R^2} + \rho imR^{\rho-1} = 0. \quad (B3)$$

And SOSC :

$$|H_1| = TC''_{QQ} = \frac{2Sd}{RQ^3} > 0,$$

$$|H_2| = TC''_{QQ} TC''_{SS} - TC''_{QS}{}^2$$

$$= \left(\frac{2Sd}{RQ^3}\right)\sigma(\sigma+1)iaS^{-(\sigma+2)} - \left(\frac{d}{RQ^2}\right)^2 = \frac{(2\sigma+1)d^2}{R^2Q^4} > 0$$

from (B2), and

$$|H_3| = TC''_{QQ} TC''_{SS} TC''_{RR} - TC''_{QQ} TC''_{SR}{}^2 - TC''_{SS} TC''_{QS}{}^2 + 2TC''_{QS} TC''_{QR} TC''_{SR}$$

$$= \left(2\left((c+l)d + \frac{Sd}{Q} + \frac{icdQ}{2r}\right)\frac{1}{R^3} + \rho(\rho-1)imR^{\rho-2}\right)\frac{(2\sigma+1)d^2}{R^2Q^4}$$

$$-\frac{d}{R^2Q}\left(\left(-\frac{d}{R^2Q}\right)\left(\frac{Sd}{R^2Q^2} - \frac{icd}{2R^2r}\right) - \frac{2Sd}{RQ^3}\left(-\frac{d}{R^2Q}\right)\right)$$

$$+\left(\frac{Sd}{R^2Q^2} - \frac{icd}{2R^2r}\right)\left(\left(-\frac{d}{R^2Q}\right)\left(-\frac{d}{R^2Q}\right)\right.$$

$$\left. - \sigma(\sigma+1)iaS^{-(\sigma+2)}\left(\frac{Sd}{R^2Q^2} - \frac{icd}{2R^2r}\right)\right)$$

$$= (\rho+1)\left((c+l)d + \frac{icdQ}{2r}\right)\frac{(2\sigma+1)d^2}{R^5Q^4}$$

$$+\frac{((\rho+1)(2\sigma+1)-2)Sd^3}{R^5Q^5} > 0$$

from (B1-B3) with $\rho > 1$.

This proves part (a). From (B1), we obtain (11). By substituting (11) into the total inventory costs (TIC) in (9), we obtain (12). Then, substituting (12) into (9) yields (13). This proves part (b). Differentiating (11-12) with respect to S and R proves part (c). Now by differentiating (13) with respect to S, we get

$$TC'_S(S, R) = \sqrt{\frac{icd^2}{2R^2Sr}} - \sigma iaS^{-(\sigma+1)} \quad (B4)$$

By solving (B4) = 0 for S, we obtain (14). Substituting (14) into (11) yields (15). This proves part (d). And differentiating (14-15) with respect to R proves part (e). Thus, the proof is complete. □