한국학교수학회논문집 제 12 권, 제 4 호
Journal of the Korean School Mathematics Society
Volume 12, Number 4, 493-508, December 2009

How to Investigate Students' Zone of Proximal Development (ZPD)

Kim, Dong-Joong[1)]

This study investigates aspects of the zone of proximal development (ZPD), the distance between the actual development and the potential development. Out of 18 university students taking a geometry course, two students with the same actual developmental level in the van Hiele model in the pre-test and post-test were interviewed for measuring their potential developmental level. Based on the communicational approach to cognition, the characteristics of the two interviewees' discourse on 3D reflective symmetry were identified. There were considerable differences between the two interviewees in terms of their potential developmental level. Methodological implications for how to investigate students' ZPD in mathematics education research were addressed.

Key Words : ZPD, The communicational approach to cognition, Discourse analysis, Qualitative research, Research methodology

INTRODUCTION

If a person was left on his own from the moment he was born and survived, will he have a language? Can he use the same concepts that we consider as characteristics of human thinking? If we consider the relation between thinking and speaking in the sense of Piaget, the person can have a language because speech for communicating with others and logical thinking are developed after internal thinking. However, if we consider the same situation from Vygotsky's point of view, that person will not develop a language because there is no communication with others. If this is true, how significant a role does language play in internal developmental processes? As Vygotsky points out, if higher mental processes are tightly related to tool-mediated activity, how deeply is language as a tool related to student learning?

In the sense of Piaget, students construct meaning in a new situation from the basis of the cognitive resources that they bring to the task (Hoyles & Healy, 1997). Then,

1) Indiana State University (Dong-Joong.Kim@indstate.edu)

knowledge can be constructed by a pupil's own activity (Mammana & Villani, 1998a). In this process, what kinds of tools are needed to develop student understanding? Furthermore, where does interaction between teacher and student occur within this process? Lastly, what roles do students' interactions with different contexts play in the development of understanding? As Cohen and Ball (1999) point out, we must consider the effects of interactions between teacher, student, mathematics, and the contexts on student understanding. Therefore, how do we deal with those many factors that are critical to student understanding? If we consider only students'cognitive aspects in their learning, we are like blind men who believe an elephant is a pillar after feeling only one leg. Moreover, how can we include not only those epistemological issues but also the pedagogical, the technological, and the socio-political issues surrounding student understanding (Mammana & Villani, 1998b)?

THEORETICAL BACKGROUND

The communicational approach of assessing understanding (Sfard, 2002; Sfard & Kieran, 2001) is a way of including the importance of language as a tool in student understanding, because the everyday discourses that students participate in and are surrounded by can embed the issues mentioned above within language. Discourses here mean the acts of verbal and non-verbal communication with others or with oneself. Language and discourse are both tools and products of cognitive, social, and cultural practice (Vygotsky, 1978). Thinking can be regarded as a special case of communicative activity (Ben-Yehuda et al., 2005). Mathematical activity can be seen as a form of communication. Thus, mathematics is a form of discourse. As Rogoff (1990) points out, young children come to their conceptual development as a result of social interactions with more discursively advanced others. Thus, mathematics learning is the development of discourse as it evolves with those who are more knowledgeable about mathematics than the learner.

To examine the communicational approach as a way of including the importance of a language and other many factors in students' understanding, I will consider the zone of proximal development (ZPD) of two students. Vygotsky (1978) points out two levels of development: the actual development and the potential development. The level of potential development is determined through problem solving under more knowledgeable guidance or in collaboration with more capable peers, while the actual developmental level is determined through independent problem solving. The ZPD is the distance between the actual development and the potential development. To look at the actual developmental level in the ZPD, van Hiele's levels are considered. van Hiele's levels that describe growth in student thinking have a five step hierarchical series: visual, descriptive, abstract, formal deduction, and mathematical levels (Clements & Battista, 1992). The categorization of the van Hiele level for reflective symmetry of 2D was used in this study. The interviews took place to examine the student's potential developmental level of reflective symmetry in the two dimensional representation of a cube

One of the most useful pieces of information for both teachers and students is the ZPD because knowing the difference can help teachers create the best scaffolded learning environment that help students learn most efficiently (Cazden, 2001). In addition, the ZPD can play a significant role in the analysis of the internal developmental processes which are stimulated by teaching and which are needed for subsequent learning (Vygotsky, 1978, p. 131). Therefore, I will look at the potential developmental differences of two students who have the same actual developmental level in the van Hiele model. Those distinctions can explain implicitly not only the importance of visualization but also the limitations of the van Hiele theory in the sense of Piaget. Those limitations can clarify the importance of a language and other important factors outside of the cognitive aspects in students' learning.

RESEARCH QUESTION

My interest in characterizing students' ZPD led to the following research question: What differences in potential development exist for students who have the same actual developmental level in the van Hiele model?

METHODOLOGY

1. Participants

I collected data from 18 university students currently taking a geometry course. All questions in the pre-test and post-test are about reflective symmetry in 2D and the interview questions are about the same subject in 3D. Two students who were determined to have the same actual developmental level in the van Hiele model in the pre-test and post-test are interviewed for measuring their potential developmental level. Two interviews were audio-taped and transcribed for further analysis.

2. Material

The task for this research consists of three problem sets: the pre-test, post-test, and interview questions. The purposes of pre-test and post-test are to determine the student's van Hiele level of reasoning of reflective symmetry in order to select two interviewees who have the same actual developmental level in the van Hiele model. The pre-test consists of two problems in 2D reflective symmetry (Appendix A). The pre-test was given after introducing the basic concepts in 2D reflective symmetry. These two problems are embedded in the actual test with the problems on rotational symmetry

After learning reflective symmetry in 2D for a week, students took the post-test. The post-test consists of six problems in 2D reflective symmetry (Appendix B). Based on the categorization of the van Hiele level for reflective symmetry of 2D, the 6-problem set was made. In order to solve problem 1 correctly, students would need to reason at van Hiele's level 1 because it requires them to recognize reflective symmetry as a type of balance in a circular, rectangular, hexagonal, and balanced shape. They would need van Hiele's level 2 in problem 2 because they need to know that there are differences in angles or segments here but not here. They also would require van Hiele's level 2 in problem 3 because it involves understanding equi-distance and perpendicularity while the segment of the symmetric figure and its reflective image meet the symmetry line at a the same angle. In problem 4, they need to actually draw the reflections themselves thereby requiring van Hiele's level 2. They also need to draw reflective symmetry for a given design in problem 5. Although problems 2, 3, 4, and 5 are at van Hiele's level 2, their difficulty is different. In problem 5, the segments of the triangle are intentionally sloping and the circle does not possess reflective symmetry along any segment of the triangle. In the last problem, they need to define reflective symmetry as a part of the "abstract" level. To interpret the qualitative aspect in students' level of actual development, the question, "what do you think reflective symmetry is?" was asked at the end of problem solving.

The interview question consists of the 4-problem set (See Appendix C). The purpose of the interview question is to look into the student's potential developmental level of reflective symmetry in the two dimensional representation of a cube because the two dimensional representation of three dimensions is important and related to other fields (Mammana & Villani, 1998b; Cooper, 1992). In addition, I will look into the differences between two students who have the same actual developmental level in the van Hiele model.

3. Procedures

As it was pointed out, the pre-test was embedded in the actual classroom test that was directly related to students' grades at the beginning of learning about reflective symmetry. After finishing a one-week lecture on reflective symmetry, students took the post-test. Since the instructor was out of class, I taught part of the class and then the post-test was given afterward. If students participate in the study, they can get one bonus point per question on his/her homework grade, for a maximum of 6 bonus homework points. Out of 18 students in their independent problem solving in pre-test and post-test, two students who are determined to have the same actual developmental level are selected for the interview.

In the pre-test, Mary found all four lines of symmetry in a 12-gon polygon, as did John (See Appendix A). However, they have a similar misconception that there is a line of symmetry in a parallelogram. Figure 1 shows the results of Mary and John in finding all lines of symmetry in parallelogram.

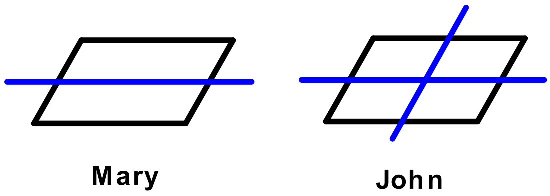


Figure 1. The results of pre-test

In the post-test a week later, they have the same results except for their definitions. They were correct on problems 1 through 5. Moreover, they explained the equidistance property in reflective symmetry when they justified their answer in the problem 3-c. Although their definitions are slightly different, their basic principle is similar. They defined reflective symmetry as follows:

Mary: The mirror image of a design over a particular line.
John: Being able to take an image fold it down the center and have both halvesline up exactly. No rough edges.

Here, Mary has a static image while John considers it as a dynamic concept of doing. However, they have the same conception of dividing the halves over a line. Based on the results of the pre- and post- tests, I can assume that they are in the same actual developmental level in van Hiele model that is, they are both at the analytic level in reflective symmetry in 2D. They may be in the transitional phase to the abstract level because they have a sense of mathematical definition.

Mary and John were interviewed for around twenty minutes based on the interview questions (Appendix C). The interviews took place in a university office. All interviews were audio-taped and transcribed for further analysis. To look into their ZPD, some prompts were used for each student. For instance, when Mary had difficulty in the second cube problem, the prompt of "why don't you use the two properties that you just used in the last one?"was given to her. When John had no idea about the fourth cube problem, the prompt of "why don't you try the second option that is looking at the plane?" was used.

4. Methods of analysis

Data were analyzed based on three distinctive features of mathematical discourses: mathematical uses of words, discursive routines, and endorsed narratives, to scrutinize the differences in Mary's and John's potential developments (Ben-Yehuda et al., 2005). Uses of words imply how the participants use key words of colloquial and mathematical discourse regarding reflective symmetry. Discursive routines are the patterns of repetitive actions in students' discourses. Endorsed narratives are propositions that are accepted as facts in mathematics. Based on these three distinctive aspects, the degree of objectification in each student's discourse was also discussed.

FINDINGS

1. Key words and their use

In the first cube problem, Mary's key words were the perpendicular line and the same distance while John's key words were visually, mentally, half and half, and equidistance. In the second cube problem, Mary used equidistance, a perpendicular line, and angle while John expressed visualize, the reflecting line, and reflex. In the third problem, Mary's key words were not clear because there were a few prompts for her potential development. However, John's key words were the reflecting plane and vertices. In summary, Mary's key words are perpendicular, equidistance, and angle while John's key words are visually, mentally, half and half, vertices, and the reflecting plane. Mary had difficulty in understanding angle with the two properties of equidistance and perpendicularity in 3D. In the uses of key words, she continued to focus on angle. It's seems that she could use equidistance and perpendicularity in only 2D despite solving the first cube problem. She didn't use her key words with visualization in 3D (e.g., "I have difficulty understanding that the perpendicular line like how to draw without my compass on two fixed points under drawing it"). Although John knew equidistance and perpendicularity in 2D (e.g., He explained two properties when he drew the image in the post-test), he did not used perpendicularity and angle as his key words. Instead, he used "visually and mentally"and focused the points and the reflecting plane in 3D. John also changed his key words from the line of symmetry or the reflecting line to the reflecting plane.

2. Routines

When John and Mary tried the cube problems, they showed differences in their flexibility. Although Mary had difficulty in understanding angle (e.g., "I have a difficulty like from angle to angle not like a line"), she continued to focus on angle in 3D reflective symmetry. Although the way in looking at the points was suggested during the interview, she sticks to the way in considering an angle. However, John demon strated flexibility. After solving the third problem by considering the point in the cube, he was asked "can you think of it in a different way?" He explained the way in looking at the reflecting plane from a different point (e.g., "Kind of have it in a different way and image it"). Although there is no evidence for Mary's corrigibility, John showed an ability of retracing his way of thinking and correcting it. In the second problem, while he explained cutting it on the diagonal, he changed his mind and corrected his wrong answer and justified the answer.

Mary did not use visualization while John focused on visualization. She often explained a difficulty without her compass, while John clarified his visualization of what he could not see (e.g., "I kind of think this is the side I see and the side I don't see" and "I am not gonna be able to see it from side"). In addition, John showed the transition from syntactic to objectified mode. His first scanned process may be called syntactic mode, as it requires only knowing and using a problem expression itself. Then he reified the problem with the purpose of finding a reflectional image in the task, called objectified mode. In contrast, Mary used her scanned process syntactically. The following Episode 1 exemplifies John's objectified use in finding a reflectional image in the interview question 3.

Episode 1. John's objectified use in routines

Speaker	**What was said**
1. John	Umm-umm [……..] I guess you can think of it as looking at the reflecting plane from this point. So, I am looking at it over here and this is kind of forming like these two lines, a kind of form you got to try to image it like forming, you know, top of a triangle, so to speak. That kind of put on the perspective like that might be a right answer. So I guess I still even open for a right answer or not
2. Interviewer	So, why do you put the plane in your part?
3. John	Umm, just get kind of a different perspective, I guess.
4. Interviewer	Okay, I am just curious because in this problem, you just look at…like this [laughing]
5. John	Yes, ye, kind of try to see the…[laughing]
6. Interviewer	I am just wondering why you just put the figure in a different way.
7. John	I don't know. I guess because this is kind of a trouble with that now and trying to think of maybe scene this is like, I don't know, kind of have it in a different way and image it.

3. Endorsed narratives

Mary tried to understand equidistance, perpendicularity, and angle in 3D based on the same properties on 2D. However, it seems that she just memorized and followed a way to find an image of reflective symmetry in 2D (e.g., "That would be just right like educated steps basically"). However, John showed a sense of thinking reflective symmetry visually and mentally in 3D based on his understanding on 2D instead of memorizing and following the steps. In a sense, John showed a way of substantiating in translating his understanding in 2D into reflective symmetric problems in 3D. At first, Mary did not know that if a point would be on the reflecting plane, the point would stay the same under a reflective symmetry over that plane. However, she used that concept in the next problem. So, this narrative is Mary's potential development while John already knew that narrative.

CONCLUSIONS

We have looked into Mary's and John's differences in their key words, routines, and endorsed narratives. Those differences can also be summarized from the perspective of objectification. When Mary tried to apply the concept of reflective symmetry from 2D to 3D, she focused on the educated steps in 2D. However, John's application in reflective symmetry from 2D to 3D is related to a mental visualization. John had a flexible way of thinking and visualization as an essence of reflective symmetry in 3D while Mary concentrated on the educated steps based on angle with equidistance and perpendicularity. Therefore, John's conception in 3D reflective symmetry is more objectified than Mary's because John was able to apply to 3D.

John's flexibility from on syntactic to objectified modein his routines indicates a degree of objectification while Mary sticks to the educated steps in her routines. The syntactic mode allows for very little interpretation of visualizations and no predictions, whereas the objectified modes provide flexibility regarding both. While there is no evidence in Mary's routines, John also showed his corrigibility as one of the important aspects of objectification. In endorsed narratives, John showed a sense of thinking reflective symmetry visually and mentally in 3D while Mary memorized and followed the narratives that she learned to find an image of reflective symmetry in 2D. Clearly, the way in interpreting the essential concepts of reflective symmetry is more objectified than memorization.

DISCUSSION

Although Mary and John had the same van Hiele actual developmental level, they showed differences in their key words, routines, endorsed narratives, and objectification. Much more importantly, they clarified their different abilities in the transitional phase from reflective symmetry in 2D to one in 3D. van Hiele pointed out visualization as the first level in geometric reasoning. However, it seems that visualization is more than the first level in geometric reasoning. Actually, John showed us another importance of visualization in the transitional phase from reflective symmetry in 2D to one in 3D. As Arcavi (2003) points out, visualization can make students foresee the unseen at the service of problem solving. Moreover, as Brownell (1945) pointed out the essential meanings of arithmetic, visualization may be not part of the several levels but the essential component in geometric reasoning

The van Hiele model can provide some overall global view of the thinking process in geometry. However, those assessments can hide heterogeneous phenomena under homogeneous levels (Ben-Yehuda et al., 2005). Moreover, dynamic approaches that can assess potential level of thinking may be more illuminating than the more typical static approach (Clements, & Battista, 1992). In addition, when students are thinking, meta-discursive rules guide them to do it in certain patterns. For instance, John's mathematical discourse and his regulation as a meta-discursive rule behind his discourse are intertwined processes in mathematical thinking, as shown in the previous routine analysis. Therefore, mathematical discourses are seen as composed of a number of object-level and meta-level activities (Sfard, 2001).

Obviously, scrutinizing epistemological analyses should be based on object-level and meta-level activities in mathematical thinking. For instance, as for object-level activities, the van Hiele model can be considered. If learning mathematics is becoming more skilful in mathematical discourse, however, the meta-discursive rules that regulate students' mathematical discourse in certain patterns deserve particular attention simultaneously. While the mediating tools that students use deal with the object-level activities in mathematical discourse, the meta-discursive rules guide the meta-level factors of mathematical discourse (Sfard, 2001). For instance, symbolic tools used by students are the mediating tools. The informal tools that students make by themselves are another example. The certain mathematical ways of defining and proving, and non-reflective observed meta-rules that regulate student discourse are the meta-discursive rules (Sfard, 2001). Student preference to a specific representation out of several representations is another example of the meta-discursive rules. A certain way of monitoring and adjusting what one is doing, some repetitive patterns and strategies with which students react to requests, questions, and problems, noticed in their responses, can be other mathematical meta-discursive rules.

To more fully understand the nature of student mathematical thinking and investigate students' ZPD, meta-discursive rules need to be revealed because thinking is interwoven with meta-discursive rules as the guiding forces for mathematical thinking. Therefore, not only object-level activities but also meta-discursive rules need to be investigated in a methodology because of their interdependence. When examining discourse analysis, mathematical meta-discursive rules may be found as tacit forces for learning. Identifying meta-discursive rules may shed light on certain educational principles which have been implicit and overlooked in a sense. Therefore, to reveal student meta-discursive rules and have knowledge of students' ZPD, the methodology in research should be based on qualitative appreciation of discourse analysis.

REFERENCES

Arcavi. A. (2003). The role of visual representations in the learning of mathematics. Educational Studies in Mathematics, 52, 215 – 338.

Ben-Yehuda, M., Lavy, I., Linchevski, L., Sfard, A. (2005). Doing wrong with words or what ars students' access to arithmetical discourses. Journal for Research in Mathematics Education, 36(3), 176-247.

Brownell, W. A. (1945). When is arithmetic meaningful? Journal of Educational Research, 38(7), 481-498.

Cazden, C. (2001). Classroom discourse and student learning. In Classroom Discourse (pp. 60-80). Portsmouth, NH: Heinemann.

Clements, D. H. & Battista, M. T. (1992). Geometry and spatial reasoning. In D. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning (pp. 420-464). Macmillan, New York: NCTM.

Cohen, D. K., & Ball, D. L. (1999). Instruction, capacity, and improvement (CPRE Research Report No. RR-043). Philadelphia: University of Pennsylvania, Consortium for Policy Research in Education. Available: http://www.cpre.org/Publications/rr43.pdf [April 30, 2004]

Cooper, M. (1992). Three-dimensional symmetry. Educational Studies in Mathematics, 23, 179-202

Gutierrez, A., Jaime, A., & Fortuny, J.M. (1991). An alternative paradigm to evaluate the acquisition of the van Hiele levels. Journal for Research in Mathematics Education, 22, 237-251.

Hoyles C. & Healy L. (1997). Unfolding Meanings for Reflective Symmetry, International Journal of Computers in Mathematical Learning, 2, 27-59.

Mammana, C. & Villani, V. (Eds.) (1998a). The evolution of geometry education since 1900. Perspectives on the Teaching of Geometry for the 21st Century (pp. 193 – 235). The Netherlands: Kluwer Academic Publishers.

Mammana, C. & Villani, V. (Eds.) (1998b). Changes and trends in geometry curricula. Perspectives on the Teaching of Geometry for the 21st Century (pp. 235 – 263). The Netherlands: Kluwer Academic Publishers.

Rogoff, B. (1990). Apprenticeship in thinking. Oxford, England: Oxford University Press.

Sfard, A. (2002). Learning discourse: Communicational framework for investigating mathematics learning. Paper presented at AERA Symposium Conceptualizing mathematics learning: theory and practice in interaction, New Orleans, April 2002.

Sfard, A. & Kieran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. Mind, Culture, and Activity, 8(1), 42-76.

Vygotsky, L.S. (1978). Mind in society. In M. Cole, V. John-Steiner, S. Scribner, & U.E. Souberman (Eds.) Cambridge, MA: Harvard University Press.

How to investigate students' zone of proximal development (ZPD)

학생들의 근접발달영역(ZPD)에 대한 탐구

김동중[2]

초록

본 연구는 실제적 발단과 잠재적 발달간의 거리, 즉 근접발달영역의 특징들을 조사하는 것이다. 선시험과 후시험이 18명의 대학생들을 대상으로 실시되었으며 반힐레 수준 이론을 통해 실제적 발달이 같은 두 학생이 잠재적 발달 조사를 위해 선발되었다. 인지－의사소통 이론을 바탕으로 삼차원 면대칭에 대한 두 학생의 담화 특징들을 확인하였다. 잠재적 발달 조사결과 두 학생사이에 상당한 차이가 있었다. 수학교육연구에서 학생들의 근접발달영역을 조사하기위한 연구방법론적 시사점을 제안한다.

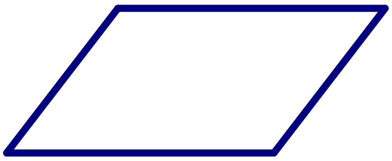
주요용어 : 근접발달영역, 인지－의사소통 이론, 담화 분석, 질적 연구, 연구방법론

2) 인디애나 주립대학 (Dong-Joong.Kim@indstate.edu)

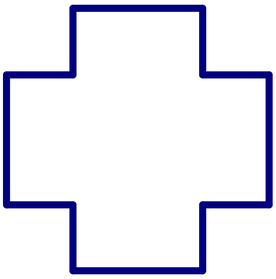
Appendix A. Pre-test

Draw all symmetry lines on the figure.

1.



2.

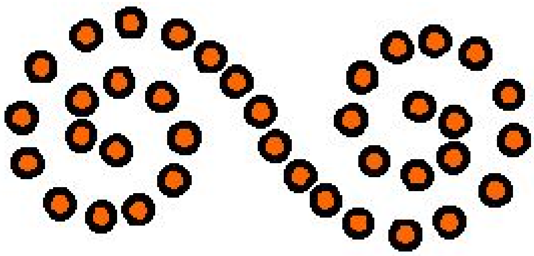


Appendix B. Post test

Problem 1. Which of the following does not have reflective symmetry?

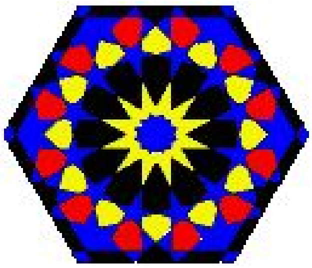
A.

B.

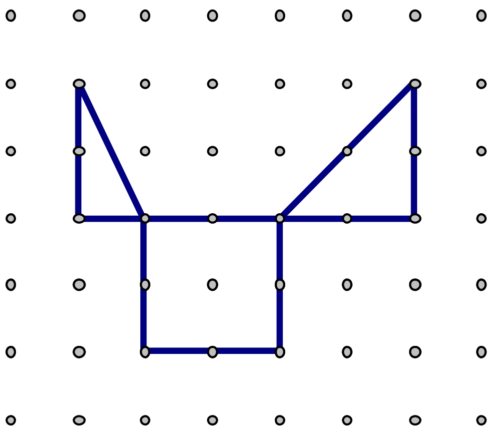


C.

D.

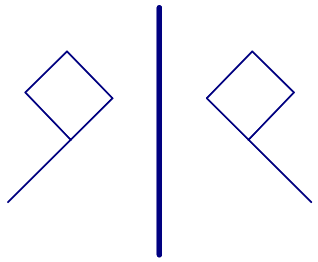


Problem 2. Does the following design have reflective symmetry?

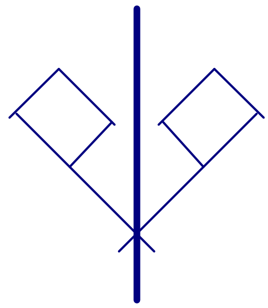


Problem 3. For each, is the bold segment a line of reflection? If not, provide a short explanation.

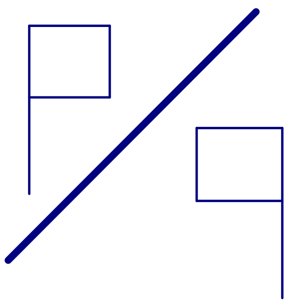
A.



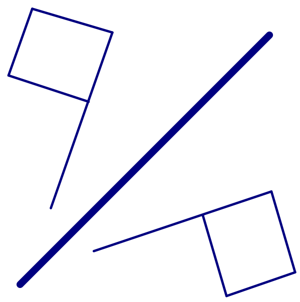
B.



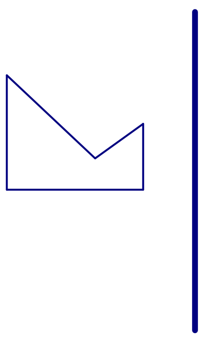
C.



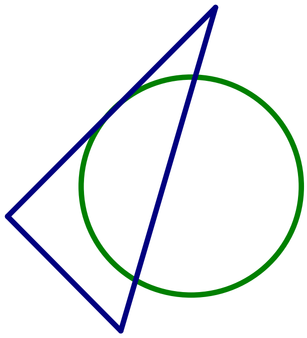
D.



Problem 4. Sketch the image of the below polygon under reflective symmetry over the bold segment.



Problem 5. Add to the following design so that it has reflective symmetry. Draw a line of symmetry.



Problem 6. What do you think reflective symmetry is?

Appendix C: 3D Problems and John's and Mary's Answers

Problems	John's answers	Mary's answers