DOI : 10.3745/JIPS.2009.5.1.001

A New Variational Level Set Evolving Algorithm for Image Segmentation

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Abstract: Level set methods are the numerical techniques for tracking interfaces and shapes. They have been successfully used in image segmentation. A new variational level set evolving algorithm without re-initialization is presented in this paper. It consists of an internal energy term that penalizes deviations of the level set function from a signed distance function, and an external energy term that drives the motion of the zero level set toward the desired image feature. This algorithm can be easily implemented using a simple finite difference scheme. Meanwhile, not only can the initial contour can be shown anywhere in the image, but the interior contours can also be automatically detected.

Keywords: *Level Set Methods, Evolving Algorithm, without Re-initialization, Image Segmentation*

1. Introduction

Level set methods were first introduced by Osher and Sethian [1] to capture moving fronts. Active contours were introduced in order to segment objects in images using dynamic curves. Level set methods provide mathematical and computational tools for the tracking of evolving interfaces with sharp corners and cusps, and topological changes. They efficiently compute optimal robot paths around obstacles, and extract clinically useful features from the noisy output of images.

In traditional level set methods, re-initialization, a technique for periodically re-initializing the level set function to a signed distance function, has been used as a numerical algorithm for maintaining stable curve evolution. However, many proposed re-initialization schemes have the undesirable side effect of moving the zero level set away from its original location. As such, there are certain drawbacks associated with re-initialization [2].

In this paper, we present a new variational level set evolving algorithm without re-initialization [3]. It consists of an internal energy term that penalizes deviations of the level set function from a signed distance function, and an external energy term that drives the motion of the zero level set toward the desired image feature [4].

This algorithm can be computed more efficiently and implemented using only a very simple finite difference scheme. Meanwhile, the initial contour can be anywhere in the image and a larger time step can be used to speed up the evolution.

2. Background

The basic idea of the level set method is to represent contours as the zero level set of an implicit function defined in a higher dimension, usually referred to as the level set function, and to evolve the level set function according to a partial differential equation (*PDE*). In typical *PDE* methods, images are assumed to be continuous functions sampled on a grid.

Geometric active contour models are typically derived using the Euler-Lagrange equation. This type of variational method is known as the variational level set method [5, 6].

Energy functions are directly formulated in the level set domain. Chan and Vese [5] proposed an active contour model using a variational level set formulation.

2.1 Level Set Formulation

Active contours implemented via level set methods can be formulated as the zero level set of a time dependent function φ that evolves according to the evolution equation:

$$\frac{\partial \phi}{\partial t} + F|\nabla \phi| = 0 \tag{1}$$

which is called level set equation [1]. The function F is called the speed function. For image segmentation, the function F depends on the image data and the level set function φ [7].

2.2 Re-initialization

In many situations, the level set function will develop steep or flat gradients leading to problems in numerical approximations. It is then necessary to reshape the level set function to a more useful form, while keeping the zero location unchanged. One way to do this is to perform what is called distance re-initialization [8] by evolving the following *PDE* to a steady state:

$$\frac{\partial \phi}{\partial t} = sign(\phi_0)(1 - |\nabla \phi|) \tag{2}$$

where φ_0 is the function to be re-initialized, and sign(φ) is the sign function.

Manuscript received December 19, 2008; revised February 27, 2009; accepted February 27, 2009.
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If φ_0 is not smooth or φ_0 is much steeper on one side of the interface than on the other, the zero level set of the resulting function φ can be moved incorrectly from that of the original function. Moreover, when the level set function is far away from a signed distance function, these methods may not be able to re-initialize the level function to a signed distance function.

Thus it constitutes a very important step in finding a new variational level set evolving algorithm.

3. Formulation of the Proposed Algorithm

The task of image segmentation is to find a collection of non-overlapping sub-regions of a given image.

Let $\Omega \subset R^2$ be the image domain, and $u_0: \Omega \rightarrow R$ be a given image. The Mumford-Shah algorithm [9] formulated image segmentation as a problem of seeking an optimal contour C that divides the image domain into disjoint subregions, and an optimal function u that fits the original image u_0 and that is smooth within each of the subregions; assume that the energy functional ε is an integral operator on u over Ω_0:

$$\varepsilon(u,\Omega_0)=\int_{\Omega_0} F(u(x))dx \qquad (3)$$

In the Chan-Vese algorithm [5], the constants c1 and c2 are introduced to fit the image intensities in the regions inside(C) and outside(C), respectively.

Energy is defined as:

$$\varepsilon(\phi,c1,c2;u_0)=u\int_\Omega \delta(\phi)|\nabla\phi|dx+\lambda 1\int_\Omega |u_0-c1|^2 H(\phi)dx+ \lambda 2\int_\Omega |u_0-c2|^2(1-H(\phi))dx \qquad (4)$$

where H is the Heaviside function, and λ1 and λ2 are the two constants.

Energy can be represented by a level set formulation, from which an implicit active contour model will be obtained to handle the topological changes automatically.

If we regularize the δ function and the H by two suitable smooth functions δ_ε and H_ε, then formally, the Euler-Lagrange equations can be written as (5) with the natural boundary condition:

$$\partial_\phi \varepsilon=-\delta_\varepsilon(\phi)\left[u\nabla\cdot\frac{\nabla_\phi}{|\nabla_\phi|}-\upsilon-\lambda 1(u0-c1)^2+\lambda 2(u0-c2)^2\right]=0 \qquad (5)$$

We define c1 and c2 as:

$$c1(\phi)=\frac{\int_\Omega u_0(x)H_\varepsilon(\phi(x))dx}{\int_\Omega H_\varepsilon(\phi(x))dx} \qquad (6)$$

$$c2(\phi)=\frac{\int_\Omega u_0(x)(1-H_\varepsilon(\phi(x)))dx}{\int_\Omega (1-H_\varepsilon(\phi(x)))dx} \qquad (7)$$

A common approach to solving the minimization problem consists in performing a gradient descent on the regularized Euler-Lagrange equation (5). Solving the following time dependent equation (5) to steady state:

$$\frac{\partial\phi}{\partial t}=-\partial_\phi\varepsilon=\delta_\varepsilon(\phi)\left[u\nabla\cdot\frac{\nabla_\phi}{|\nabla_\phi|}-\upsilon-\lambda 1(u0-c1)^2+\lambda 2(u0-c2)^2\right] \qquad (8)$$

where u and υ are non-negative constants, and c1 and c2 are defined in (6) and (7).

The Heaviside function H(x) is defined as:

$$H_\varepsilon(x)=\begin{cases}\frac{1}{2}\left(1+\frac{2}{\pi}\arctan\left(\frac{x}{\pi}\right)\right) & |x|\le\varepsilon \\ 0 & |x|>\varepsilon\end{cases} \qquad (9)$$

and we obtain the following:

$$\delta_\varepsilon(x)=H_\varepsilon{}'(x) \qquad (10)$$

Then we discretize the equation (8) in time as:

$$\phi_{i,j}^{n+1}=\tau G\left(\phi_{i,j}^{n}\right)+\phi_{i,j}^{n} \qquad (11)$$

where $G(\varphi_{i,j}^n)$ is the approximation of the term on the right-hand side of (8), and τ is the time step for evolution.

4. Segmentation Algorithm

Supposing φ_0 as the initial function, let region Ω_0 be a subset in the image domain Ω, and $\partial\Omega_0$ be all the points on the boundaries.

Initialize φ as a binary function as:

$$\phi_0(x,y)=\begin{cases}-c_0 & (x,y)\in\Omega_0-\partial\Omega_0 \\ 0 & (x,y)\in\partial\Omega_0 \\ c_0 & \Omega-\Omega_0\end{cases} \qquad (12)$$

We propose a fast one-pass segmentation algorithm [10, 11] which is built upon flipping the values of φ at each grid point/pixel from positive to negative or vice versa according to the rule R, and which contains 3 main steps:

1. Initialize $\varphi_0: \Omega \rightarrow \{-1,1\}$;
2. Advance: for each grid point: set $\varphi^{n+1}(x)=-\varphi^n(x)$ if $R(\varphi^{n+1}, \varphi^n, x)=1$;
3. Repeat until $\varphi^{n+1}\equiv \varphi^n$;

$R(\varphi^{n+1}, \varphi^n)$ can be interpreted as the logical evaluation of the following inequality:

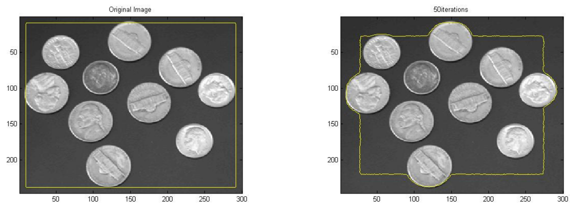
$$\varepsilon(\phi^{n+1},c1,c2;u_0)\le\varepsilon(\phi^n,c1,c2;u_0) \qquad (13)$$

5. Implementation and Results

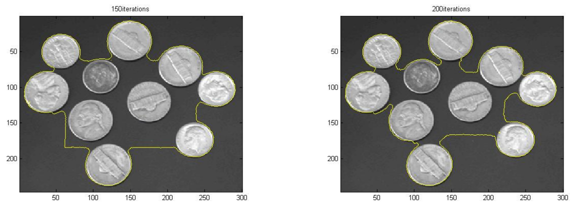
5.1 Results of Our Algorithm

We set $c0=1$, $\lambda 1=\lambda 2=5$, $u=0.04$, $\upsilon=4.0$, and time step $\tau=5.0$. To obtain the results of this work, we implemented our algorithm in MATLAB 7.0, and tested our algorithm on a Coins.png image.

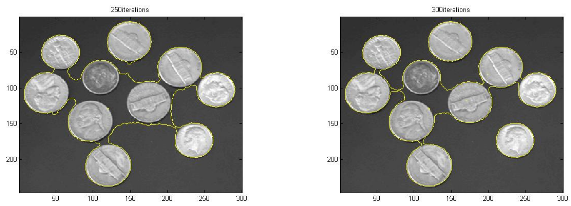
Fig. 1 shows the image results of each step of the segmentation algorithm.



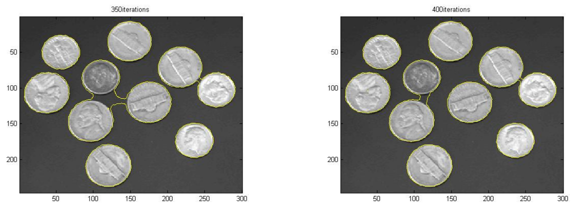
(1) Original Image (2) 50 Iterations



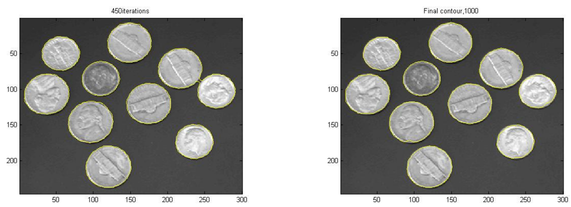
(3) 150 Iterations (4) 200 Iterations



(5) 250 Iterations (6) 300 Iterations



(7) 350 Iterations (8) 400 Iterations



(9) 450 Iterations (10) 1000 Iterations

Fig. 1. The results of the segmentation of the coins image with our algorithm

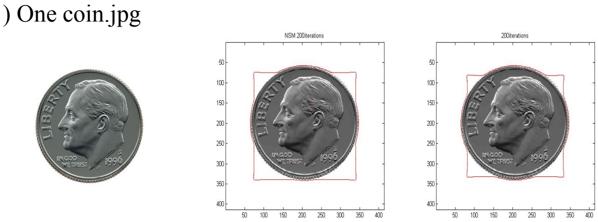
5.2 Comparison with a Novel Snake Model

The results of the automatic segmentation of the coins (shown in Fig.1) clearly demonstrate the obvious advantage of our algorithm. Next, the focus is on the comparison of our algorithm using a novel snake model (NSM) in [12].

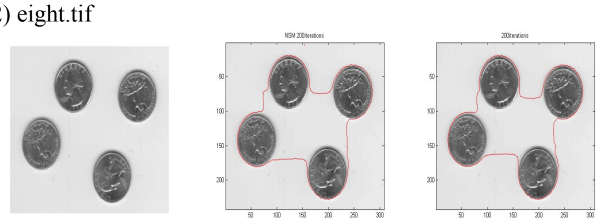
In the NSM algorithm which is based upon but is more advantageous than the algorithm which was introduced by C. Li, et al. [7], the key observation is that only the signs of the level set function matter in the energy functional. This can easily be seen from the model defined in equation (4), in which one sees that the energy is a function of $H(-\varphi)$. With our algorithm, according to (13), the sign of $\varphi^n(x)$ is flipped only if the energy (4) doesn't increase. This ensures the stability of the algorithm.

Fig.2 shows the results of our algorithm (in the third column) and the NSM algorithm [12] using the same initial contours. The CPU times for these images are listed in Table 1, which were recorded from our experiments with Matlab code run on a TOSHIBA Satellite100, and with Centrino Duo 1.7GHz, 1GB RAM, and Matlab 7.0 on Windows XP. In the experiments with the images shown in Fig.2 and Table 1, it is faster than the NSM algorithm [12].

(1) One coin.jpg



(2) eight.tif



(3) coins.png

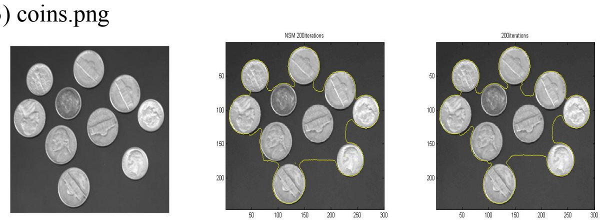


Fig. 2. Comparison of our algorithm with the NSM algorithm (First column: Original image in different formats; Second column: 200 Iterations of the NSM algorithm; Third column: 200 Iterations of our algorithm)

Table 1 CPU time (in seconds for 200 iterations) for the NSM algorithm and our algorithm for the images in Fig.2

	One coin.jpg	Eight.tif	Coins.png
NSM Algorithm [12]	96.312	38.874	37.60
Our Algorithm	48.956	19.887	18.64

6. Conclusion

In this paper we proposed a new variational level set evolving algorithm without re-initialization for the image segmentation. This algorithm can be easily implemented using a simple finite difference scheme. Meanwhile, not only can the initial curve be shown anywhere in the image, but the interior contours can also be automatically and quickly detected. The applicability of the proposed algorithm is manifold during the required image processing, such as Image Processing or Biomedical Engineering.

Finally, it is further suggested that the proposed algorithm be extended to color images and the video framework.

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