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P-$\mathcal{I}$-OPEN MAPPINGS, P-$\mathcal{I}$-CONTINUOUS MAPPINGS AND P-$\mathcal{I}$-IRRESOLUTE MAPPINGS

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ABSTRACT. The notions of P-$\mathcal{I}$-open (closed) mappings, P-$\mathcal{I}$-continuous mappings, P-$\mathcal{I}$-neighborhoods, P-$\mathcal{I}$-irresolute mappings and $\mathcal{I}$-irresolute mappings are introduced. Relations between P-$\mathcal{I}$-open (closed) mappings and $\mathcal{I}$-open (closed) mappings are given. Characterizations of P-$\mathcal{I}$-open (closed) mappings are provided. Relations between a P-$\mathcal{I}$-continuous mapping and an $\mathcal{I}$-continuous mapping are discussed, and characterizations of a P-$\mathcal{I}$-continuous mapping are considered. Conditions for a mapping to be an $\mathcal{I}$-irresolute mapping (resp. P-$\mathcal{I}$-irresolute mapping) are provided.

1. INTRODUCTION

In 1990, D. Janković, and T.R. Hamlett have introduced the notion of $\mathcal{I}$-open sets in topological spaces. Since then, several kinds of $\mathcal{I}$-openness, that is, (weakly) semi-$\mathcal{I}$-open set, δ-$\mathcal{I}$-open sets, β-$\mathcal{I}$-open sets, α-$\mathcal{I}$-open sets, b-$\mathcal{I}$-open sets, (weakly) pre-$\mathcal{I}$-open sets, etc. are introduced, and several properties and relations are investigated (see [2, 3, 8, 9, 10, 11, 12, 25, 28]). In [18], Kang and Kim first introduced the notions of pre-local function, semi-local function and α-local function with respect to a topology and an ideal, and investigated several properties. They next introduced the concept of P-$\mathcal{I}$-open set and P-$\mathcal{I}$-closed set in ideal topological spaces, and investigated related properties. They discussed relations between $\mathcal{I}$-open sets and P-$\mathcal{I}$-open sets. Finally they introduced the notion of P-$*$-closure, and investigated many properties related to P-$\mathcal{I}$-open set, pre-local function, semi-local function and α-local function with respect to a topology and an ideal.

In this paper, we deal with P-$\mathcal{I}$-open mappings, P-$\mathcal{I}$-continuous mappings and P-$\mathcal{I}$-irresolute mappings. In section 3, we define the notion of P-$\mathcal{I}$-open (closed) mappings, and give relations between P-$\mathcal{I}$-open (closed) mappings and $\mathcal{I}$-open (closed)

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mappings. We provide characterizations of P-$\mathcal{I}$-open (closed) mappings. In section 4, we define a P-$\mathcal{I}$-continuous mapping and a P-$\mathcal{I}$-neighborhood, and then we investigate relations between a P-$\mathcal{I}$-continuous mapping and an $\mathcal{I}$-continuous mapping. We discuss characterizations of a P-$\mathcal{I}$-continuous mapping. In the final section, we introduce the notions of P-$\mathcal{I}$-irresolute mappings and $\mathcal{I}$-irresolute mappings. We give conditions for a mapping to be an $\mathcal{I}$-irresolute mapping (resp. P-$\mathcal{I}$-irresolute mapping).

2. PRELIMINARIES

Through this paper, (X, τ) and (Y, κ) (simply X and Y) always mean topological spaces. A subset A of X is said to be *semi-open* [19] (respectively, *α-open* [26] and *pre-open* [24]) if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$ (respectively, $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$ and $A \subset \mathrm{Int}(\mathrm{Cl}(A))$). The complement of a pre-open set (respectively, an α-open set and a semi-open set) is called a *pre-closed set* (respectively, an *α-closed set* and a *semi-closed set*). The intersection of all pre-closed sets (respectively, α-closed sets and semi-closed sets) containing A is called the *pre-closure* (respectively, *α-closure* and *semi-closure*) of A, denoted by $p\mathrm{Cl}(A)$ (respectively, $\alpha\mathrm{Cl}(A)$ and $s\mathrm{Cl}(A)$). A subset A is also pre-closed (respectively, α-closed and semi-closed) if and only if $A = p\mathrm{Cl}(A)$ (respectively, $A = \alpha\mathrm{Cl}(A)$ and $A = s\mathrm{Cl}(A)$). We denote the family of all pre-open sets (respectively, α-open sets and semi-open sets) of (X, τ) by τ^p (respectively, τ^α and τ^s).

An *ideal* is defined as a nonempty collection $\mathcal{I}$ of subsets of X satisfying the following two conditions.

(1) If $A \in \mathcal{I}$ and $B \subset A$, then $B \in \mathcal{I}$. (heredity)
(2) If $A \in \mathcal{I}$ and $B \in \mathcal{I}$, then $A \cup B \in \mathcal{I}$. (finite additivity)

An *ideal topological space* is a topological space (X, τ) with an ideal $\mathcal{I}$ on X, and it is denoted by $(X, \tau, \mathcal{I})$. For a subset $A \subset X$, the set

$$A^*(\tau, \mathcal{I}) = \{x \in X : U \cap A \notin \mathcal{I} \text{ for each } U \in \tau(x)\}$$

is called the *local function* of A with respect to τ and $\mathcal{I}$, where

$$\tau(x) = \{U \in \tau : x \in U\}.$$

We will use A^* and/or $A^*(\mathcal{I})$ instead of $A^*(\tau, \mathcal{I})$.

Lemma 2.1 ([16]). *Let (X, τ) be a topological space with ideals $\mathcal{I}$ and $\mathcal{J}$ on X. For subsets A and B of X, we have the following assertions.*

(i) $A \subset B \Rightarrow A^* \subset B^*$.
(ii) $\mathcal{I} \subset \mathcal{J} \Rightarrow A^*(\mathcal{J}) \subset A^*(\mathcal{I})$.
(iii) $A^* = \mathrm{Cl}(A^*) \subset \mathrm{Cl}(A)$ (A^* *is a closed subset of* $\mathrm{Cl}(A)$).
(iv) $(A^*)^* \subset A^*$.
(v) $(A \cup B)^* = A^* \cup B^*$.
(vi) $A^* \setminus B^* = (A \setminus B)^* \setminus B^* \subset (A \setminus B)^*$.
(vii) $U \in \tau \Rightarrow U \cap A^* = U \cap (U \cap A)^* \subset (U \cap A)^*$.
(viii) $B \in \mathcal{I} \Rightarrow (A \cup B)^* = A^* = (A \setminus B)^*$.

Definition 2.2. Let $(X, \tau, \mathcal{I})$ be an ideal topological space. A subset A of X is said to be *$\mathcal{I}$-open* [1] if $A \subset \mathrm{Int}(A^*)$.

The set of all $\mathcal{I}$-open sets in ideal topological space $(X, \tau, \mathcal{I})$ is denoted by $\mathcal{I}O(X, \tau, \mathcal{I})$ or written simply as $\mathcal{I}O(X)$ when there is no chance for confusion.

Definition 2.3 ([18]). Let $(X, \tau, \mathcal{I})$ be an ideal topological space and let A be a subset of X. Then the set

$$A_p^*(\tau, \mathcal{I}) = \{x \in X : U \cap A \notin \mathcal{I} \text{ for each } U \in \tau^p(x)\}$$

is called the *pre-local function* with respect to τ and $\mathcal{I}$, where

$$\tau^p(x) = \{U \in \tau^p : x \in U\}.$$

We will use A_p^* and/or $A_p^*(\mathcal{I})$ instead of $A_p^*(\tau, \mathcal{I})$.

Lemma 2.4 ([18]). *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and let A be a subset of X. Then*

(i) *If $\mathcal{I} = \{\emptyset\}$, then $A_p^* = p\mathrm{Cl}(A)$, $A_s^* = s\mathrm{Cl}(A)$ and $A_\alpha^* = \alpha\mathrm{Cl}(A)$.*
(ii) *If $\mathcal{I} = \mathcal{P}(X)$, then $A_p^* = A_s^* = A_\alpha^* = \emptyset$.*

Lemma 2.5 ([18]). *Let (X, τ) be a topological space with ideals $\mathcal{I}$ and $\mathcal{J}$ on X, and let A, B be subsets of X. Then*

(i) $A \subset B \Rightarrow A_p^* \subset B_p^*$.
(ii) $\mathcal{I} \subset \mathcal{J} \Rightarrow A_p^*(\mathcal{J}) \subset A_p^*(\mathcal{I})$.
(iii) $A_p^* = p\mathrm{Cl}(A_p^*) \subset p\mathrm{Cl}(A)$ (A_p^* *is a pre-closed subset of* $p\mathrm{Cl}(A)$).
(iv) $(A_p^*)_p^* \subset A_p^*$.
(v) $B \in \mathcal{I} \Rightarrow B_p^* = \emptyset$.
(vi) $U \in \tau^\alpha \Rightarrow U \cap A_p^* = U \cap (U \cap A)_p^* \subset (U \cap A)_p^*$.
(vii) $B \in \mathcal{I} \Rightarrow (A \cup B)_p^* = A_p^* = (A \setminus B)_p^*$.
(viii) $A_p^*(\mathcal{I} \cap \mathcal{J}) \supset A_p^*(\mathcal{I}) \cup A_p^*(\mathcal{J})$.

Definition 2.6 ([18]). Let $(X, \tau, \mathcal{I})$ be an ideal topological space. A subset A of X is said to be *P-$\mathcal{I}$-open* if $A \subset p\mathrm{Int}(A_p^*)$. A subset B of X is said to be *P-$\mathcal{I}$-closed* if the complement of B is P-$\mathcal{I}$-open.

The set of all P-$\mathcal{I}$-open sets in $(X, \tau, \mathcal{I})$ is denoted by $P\mathcal{I}O(X, \tau, \mathcal{I})$. Simply $P\mathcal{I}O(X, \tau, \mathcal{I})$ is written as $P\mathcal{I}O(X)$ or $P\mathcal{I}O(X, \tau)$ when there is no chance for confusion.

Definition 2.7 ([1]). A mapping $f : (X, \tau) \to (Y, \kappa, \mathcal{J})$ is said to be *$\mathcal{I}$-open* (resp. *$\mathcal{I}$-closed*) if for each $U \in \tau$ (resp. $U^c \in \tau$), $f(U)$ is an $\mathcal{I}$-open (resp. $\mathcal{I}$-closed) set.

Theorem 2.8 ([18]). *Let $A \in P\mathcal{I}O(X, \tau)$. Then A is $\mathcal{I}$-open.*

Remark 2.9. By Theorem 2.8, we know that P-$\mathcal{I}$-open set implies $\mathcal{I}$-open set. By [1, Remark 2.2], we know that $\mathcal{I}$-open set implies pre-open set. Hence we can deduce that P-$\mathcal{I}$-open set implies pre-open set. The converse is not true, in general.

Theorem 2.10 ([18]). *Let $\{U_i \in P\mathcal{I}O(X) : i \in \Lambda\}$ be a class of P-$\mathcal{I}$-open sets in an ideal topological space $(X, \tau, \mathcal{I})$. Then $\bigcup_{i\in\Lambda} \{U_i \in P\mathcal{I}O(X) : i \in \Lambda\}$ is P-$\mathcal{I}$-open.*

Theorem 2.11 ([18]). *If A is P-$\mathcal{I}$-closed in an ideal topological space $(X, \tau, \mathcal{I})$, then $A \supset (p\mathrm{Int}(A))_p{}^*$.*

Lemma 2.12 ([17]). *Let A be a subset of a topological space (X, τ). Then the following assertions are satisfied.*

(i) $(p\mathrm{Int}(A))^c = p\mathrm{Cl}(A^c)$.
(ii) $(p\mathrm{Cl}(A))^c = p\mathrm{Int}(A^c)$.

3. P-$\mathcal{I}$-OPEN MAPPINGS AND P-$\mathcal{I}$-CLOSED MAPPINGS

Definition 3.1. A mapping $f : (X, \tau) \to (Y, \kappa, \mathcal{J})$ is said to be *P-$\mathcal{I}$-open* (resp. *P-$\mathcal{I}$-closed*) if for each $U \in \tau$ (resp. $U^c \in \tau$), $f(U)$ is a P-$\mathcal{I}$-open set (resp. P-$\mathcal{I}$-closed set).

Example 3.2. Consider a topological space (X, τ) with $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$, and consider an ideal topological space $(Y, \kappa, \mathcal{I})$ where $Y = \{1, 2, 3, 4\}$, $\kappa = \{\emptyset, Y, \{3\}, \{1, 2\}, \{1, 2, 3\}\}$, and $\mathcal{I} = \{\emptyset, \{1\}\}$. Then

$$P\mathcal{I}O(Y, \kappa) = \{\emptyset, \{2\}, \{3\}, \{2, 3\}, \{2, 3, 4\}\}.$$

Let $f : (X, \tau) \to (Y, \kappa, \mathcal{I})$ be a mapping given by $f(a) = 2 = f(b)$ and $f(c) = 3$. Then $f(\{a\}) = \{2\}$, $f(\{b, c\}) = \{2, 3\}$, $f(X) = \{2, 3\}$ and $f(\emptyset) = \emptyset$. Hence f is a P-$\mathcal{I}$-open mapping. Let $g : (X, \tau) \to (Y, \kappa, \mathcal{I})$ be a mapping given by $g(a) = 1 = g(b)$

and $g(c) = 4$. Then $g(\{b,c\}) = \{1,4\} = g(X)$, $g(\{a\}) = \{1\}$ and $g(\emptyset) = \emptyset$. Hence g is a P-$\mathcal{I}$-closed mapping.

Theorem 3.3. *Let $f : (X,\tau) \to (Y,\kappa,\mathcal{I})$ be a P-$\mathcal{I}$-open (resp. P-$\mathcal{I}$-closed) mapping. Then f is an $\mathcal{I}$-open (resp. $\mathcal{I}$-closed) mapping.*

Proof. Suppose that $f : (X,\tau) \to (Y,\kappa,\mathcal{I})$ be a P-$\mathcal{I}$-open (resp. P-$\mathcal{I}$-closed) mapping. Let $G \in \tau$ (resp. $G^c \in \tau$). Then $f(G)$ is a P-$\mathcal{I}$-open set (resp. P-$\mathcal{I}$-closed set) in Y. Since P-$\mathcal{I}$-open (resp. P-$\mathcal{I}$-closed) set is an $\mathcal{I}$-open (resp. $\mathcal{I}$-closed) set by Theorem 2.8, $f(G)$ is an $\mathcal{I}$-open (resp. $\mathcal{I}$-closed) set. Hence f is $\mathcal{I}$-open (resp. $\mathcal{I}$-closed). □

The converse of Theorem 3.3 may not be true as seen in the following example.

Example 3.4. Consider a topological space (X,τ) with $X = \{a,b,c,d\}$ and $\tau = \{\emptyset, X, \{a,b\}, \{a,b,c\}\}$, and consider an ideal topological space $(Y,\kappa,\mathcal{I})$ where $Y = \{1,2,3,4\}$, $\kappa = \{\emptyset, Y, \{3\}, \{1,2\}, \{1,2,3\}\}$, and $\mathcal{I} = \{\emptyset, \{1\}\}$. Then a mapping $f : (X,\tau) \to (Y,\kappa,\mathcal{I})$ given by $f(a) = 1$, $f(b) = 2 = f(c)$, and $f(d) = 3$ is $\mathcal{I}$-open. Since $f(\{a,b\}) = \{1,2\} \not\subset \{2\} = p\mathrm{Int}(\{1,2\}^*_p)$, we know that f is not P-$\mathcal{I}$-open.

Corollary 3.5. *Let $f : (X,\tau) \to (Y,\kappa,\mathcal{I})$ be a P-$\mathcal{I}$-open* (resp. *P-$\mathcal{I}$-closed*) *mapping. Then f is a pre-open* (resp. *pre-closed*) *mapping.*

Proof. Using Theorem 3.3 and Remark 2.9, we know that f is a pre-open (resp. pre-closed) mapping. □

Example 3.6. Consider a topological space (X,τ) with $X = \{1,2,3\}$ and $\tau = \{\emptyset, X, \{1\}, \{2,3\}\}$, and consider an ideal topological space $(Y,\kappa,\mathcal{I})$ where $Y = \{a,b,c,d\}$, $\kappa = \{\emptyset, Y, \{c\}, \{a,b\}, \{a,b,c\}\}$, and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then a mapping $f : (X,\tau) \to (Y,\kappa,\mathcal{I})$ given by $f(1) = b = f(2)$ and $f(3) = c$ is P-$\mathcal{I}$-open. But f is not an open mapping because $f(1) = \{b\} \notin \kappa$ for $\{1\} \in \tau$.

Example 3.7. Consider a topological space (X,τ) with $X = \{1,2,3\}$ and $\tau = \{\emptyset, X, \{1\}, \{2,3\}\}$, and consider an ideal topological space $(Y,\kappa,\mathcal{I})$ where $Y = \{a,b,c,d\}$, $\kappa = \{\emptyset, Y, \{c\}, \{a,b\}, \{a,b,c\}\}$, and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then a mapping $g : (X,\tau) \to (Y,\kappa,\mathcal{I})$ given by $g(1) = c$, $g(2) = a$, and $g(3) = b$ is an open mapping. But g is not a P-$\mathcal{I}$-open mapping since $g(\{2,3\}) = \{a,b\} \not\subset p\mathrm{Int}(\{a,b\}^*_p) = \{b\}$ for $\{2,3\} \in \tau$.

Remark 3.8. We know that the P-$\mathcal{I}$-open mapping and the open mapping are independent notions as seen in Examples 3.6 and 3.7.

Theorem 3.9. *Let $f : (X, \tau) \to (Y, \kappa, \mathcal{J})$ be a mapping. Then the following statements are equivalent.*

(i) *f is a P-$\mathcal{I}$-open mapping.*

(ii) *For each $x \in X$ and each open neighborhood U of x, there exists a P-$\mathcal{I}$-open set $W \subset Y$ containing $f(x)$ such that $W \subset f(U)$*

Proof. (i) $\Rightarrow$ (ii). Suppose that f is a P-$\mathcal{I}$-open mapping. Let $x \in X$. Then for each open set G containing x, $f(x) \in f(G)$. Since f is P-$\mathcal{I}$-open, $f(G)$ is a P-$\mathcal{I}$-open set in Y. Putting $W := f(G)$, we obtain (ii).

(ii) $\Rightarrow$ (i). Let G be an open set in X. Then for any $x \in G$, there exists $W_x \in PIO(Y, \kappa)$ such that $f(x) \in W_x \subset f(G)$. This implies that $f(G) = \bigcup_{x \in G} f(x) \subset \bigcup_{x \in G} W_x \subset f(G)$. Hence $\bigcup_{x \in G} W_x = f(G)$. By Theorem 2.10, $f(G)$ is P-$\mathcal{I}$-open. Therefore f is a P-$\mathcal{I}$-open mapping. □

Theorem 3.10. *Let $f : (X, \tau) \to (Y, \kappa, \mathcal{I})$ be a mapping. Then f is P-$\mathcal{I}$-open if and only if it satisfies the following assertion:*

$$f(\mathrm{Int}(A)) \subset p\mathrm{Int}(f(A)^*_p) \tag{3.1}$$

for all A in (X, τ).

Proof. Suppose that f is a P-$\mathcal{I}$-open mapping. Let A be a subset of X. Then $\mathrm{Int}(A)$ is an open set and $f(\mathrm{Int}(A))$ is a P-$\mathcal{I}$-open set. Hence

$$f(\mathrm{Int}(A)) \subset p\mathrm{Int}(f(\mathrm{Int}(A))^*_p) \subset p\mathrm{Int}(f(A)^*_p).$$

Conversely, suppose that f satisfies (3.1). Let G be an open subset of X. Then $f(G) = f(\mathrm{Int}(G)) \subset p\mathrm{Int}(f(G)^*_p)$. Hence $f(G)$ is a P-$\mathcal{I}$-open set in $(Y, \kappa, \mathcal{I})$. Therefore f is a P-$\mathcal{I}$-open mapping. □

Corollary 3.11. *Let $f : (X, \tau) \to (Y, \kappa, \mathcal{I})$ be a mapping satisfying the inclusion $f(\mathrm{Int}(A)) \subset p\mathrm{Int}(f(A)^*_p)$ for all A in (X, τ). Then f is an $\mathcal{I}$-open mapping.*

Proof. Straightforward. □

If f is an $\mathcal{I}$-open mapping then is Theorem 3.10 true? The answer is negative as seen in the following example.

Example 3.12. Consider a topological space (X, τ) with $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$, and consider an ideal topological space $(Y, \kappa, \mathcal{I})$ where $Y = \{1, 2, 3, 4\}$, $\kappa = \{\emptyset, Y, \{3\}, \{1, 2\}, \{1, 2, 3\}\}$, and $\mathcal{I} = \{\emptyset, \{1\}\}$. Then a mapping $f : (X, \tau) \to (Y, \kappa, \mathcal{I})$ given by $f(a) = 1$, $f(b) = 2 = f(c)$, and $f(d) = 3$ is an $\mathcal{I}$-open mapping. If $A = \{a, b, d\}$, then $f(\text{Int}(A)) = f(\{a, b\}) = \{1, 2\}$ and

$$p\text{Int}(f(A)_p^*) = p\text{Int}(\{1, 2, 3\}_p^*) = p\text{Int}(\{2, 3, 4\}) = \{2, 3, 4\}.$$

Hence we know that $f(\text{Int}(A)) \not\subset p\text{Int}(f(A)_p^*)$.

Theorem 3.13. *Let $f : (X, \tau) \to (Y, \kappa, \mathcal{I})$ be a mapping. Then f is P-$\mathcal{I}$-open if and only if it satisfies the following assertion:*

$$\text{Int}(f^{-1}(B)) \subset f^{-1}(p\text{Int}(B_p^*)) \tag{3.2}$$

for all B in $(Y, \kappa, \mathcal{I})$.

Proof. Suppose that f is P-$\mathcal{I}$-open. Let B be a subset of Y. Then $f^{-1}(B)$ is a subset of (X, τ). Since f is P-$\mathcal{I}$-open, we obtain

$$f(\text{Int}(f^{-1}(B))) \subset p\text{Int}(f(f^{-1}(B))_p^*).$$

It follows that

$$\begin{aligned}\text{Int}(f^{-1}(B)) &\subset f^{-1}(f(\text{Int}(f^{-1}(B))))\\ &\subset f^{-1}(p\text{Int}(f(f^{-1}(B))_p^*))\\ &\subset f^{-1}(p\text{Int}(B_p^*)).\end{aligned}$$

Conversely, suppose that f satisfies (3.2). Let G be an open set in (X, τ). Then $\text{Int}(f^{-1}(f(G))) \subset f^{-1}(p\text{Int}(f(G)_p^*))$ since $f(G)$ is a set in $(Y, \kappa, \mathcal{I})$. Since $G \subset f^{-1}(f(G))$ and $\text{Int}(G) = G$, we have

$$G \subset \text{Int}(f^{-1}(f(G))) \subset f^{-1}(p\text{Int}(f(G)_p^*)).$$

This implies that $f(G) \subset f(f^{-1}(p\text{Int}(f(G)_p^*))) \subset p\text{Int}(f(G)_p^*)$. Hence $f(G)$ is a P-$\mathcal{I}$-open set in $(Y, \kappa, \mathcal{I})$. Therefore f is P-$\mathcal{I}$-open. □

If f is an $\mathcal{I}$-open mapping then does Theorem 3.13 hold? The answer is negative as seen in the following example.

Example 3.14. In Example 3.12, let $B = \{1, 2\}$. Then

$$\text{Int}(f^{-1}(B)) = \text{Int}(\{a, b, c\}) = \{a, b, c\}$$

and $f^{-1}(p\text{Int}(B_p^*)) = f^{-1}(p\text{Int}(\{2\})) = \{b, c\}$. Hence we know that $\text{Int}(f^{-1}(B)) \not\subset f^{-1}(p\text{Int}(B_p^*))$.

Theorem 3.15. *Let $f : (X,\tau) \to (Y,\kappa,\mathcal{I})$ be a mapping. Then f is P-$\mathcal{I}$-closed if and only if it satisfies the following assertion:*

$$pCl(((f(Cl(A))^c)_p^*)^c) \subset f(Cl(A)) \tag{3.3}$$

for A in X.

Proof. Let f be a P-$\mathcal{I}$-closed mapping. Then

$$f(Cl(A))^c \subset pInt((f(Cl(A))^c)_p^*).$$

Hence $pCl(((f(Cl(A))^c)_p^*)^c) \subset f(Cl(A))$.

Conversely, assume that (3.3) is valid and let B be a closed set in X. Then

$$pCl(((f(B)^c)_p^*)^c) = pCl(((f(Cl(B))^c)_p^*)^c) \subset f(Cl(B)) = f(B).$$

This implies that $f(B)^c \subset pInt((f(B)^c)_p^*)$. Hence f is a P-$\mathcal{I}$-closed mapping. □

If f is an $\mathcal{I}$-closed mapping then do f satisfy the following assertion?

$$pCl(((f(Cl(A))^c)_p^*)^c) \subset f(Cl(A))$$

The answer is negative as seen in the following example.

Example 3.16. Let (X,τ) be a topological space with $X = \{a,b,c,d\}$ and $\tau = \{\emptyset, X, \{a,b\}, \{a,b,c\}\}$, and consider an ideal topological space $(Y,\kappa,\mathcal{I})$ where $Y = \{1,2,3,4\}$, $\kappa = \{\emptyset, Y, \{3\}, \{1,2\}, \{1,2,3\}\}$, and $\mathcal{I} = \{\emptyset, \{1\}\}$. Then a mapping $f : (X,\tau) \to (Y,\kappa,\mathcal{I})$ given by $f(a) = 2 = f(b)$, $f(c) = 1$, and $f(d) = 4$ is an $\mathcal{I}$-closed mapping. Let $A = \{d\}$. Then we know that $pCl(((f(Cl(A))^c)_p^*)^c) = \{1\}$ and $f(Cl(A)) = \{4\}$. Hence

$$pCl(((f(Cl(A))^c)_p^*)^c) \not\subset f(Cl(A)).$$

Theorem 3.17. *Let $f : (X,\tau,\mathcal{I}) \to (Y,\kappa,\mathcal{J})$ be P-$\mathcal{I}$-open such that*

$$(\forall A \subset X)(f(A^*) \subset f(A)_p^* \text{ or } f(A^*) \subset f(A)). \tag{3.4}$$

Then the image of each $\mathcal{I}$-open set is P-$\mathcal{I}$-open.

Proof. Suppose that $f : (X,\tau,\mathcal{I}) \to (Y,\kappa,\mathcal{J})$ is a P-$\mathcal{I}$-open mapping. Let A be an $\mathcal{I}$-open set in X. Then $A \subset Int(A^*)$. Since f is a P-$\mathcal{I}$-open mapping, $f(Int(A^*))$ is a P-$\mathcal{I}$-open set in Y. It follows that

$$f(A) \subset f(Int(A^*)) \subset pInt(f(Int(A^*))_p^*) \subset pInt(f(A^*)_p^*).$$

Since $f(A^*) \subset f(A)_p^*$ or $f(A^*) \subset f(A)$, we have

$$f(A) \subset p\mathrm{Int}((f(A)^*_p)^*_p) \subset p\mathrm{Int}(f(A)^*_p),$$

and so $f(A) \subset p\mathrm{Int}(f(A)^*_p)$. □

The converse of Theorem 3.17 is not valid as seen in the following example.

Example 3.18. Consider two ideal topological spaces $(X, \tau, \mathcal{I})$ and $(Y, \kappa, \mathcal{J})$ where $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$, $\mathcal{I} = \{\emptyset, \{a\}\}$, $Y = \{1, 2, 3, 4\}$, $\kappa = \{\emptyset, Y, \{1, 2\}, \{1, 2, 3\}\}$, and $\mathcal{J} = \{\emptyset, \{3\}\}$. Then a mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ given by $f(a) = 1$, $f(b) = 2 = f(c)$ and $f(d) = 4$ is a P-$\mathcal{I}$-open mapping in which the image of each $\mathcal{I}$-open set is a P-$\mathcal{I}$-open set. But if $A = \{b, c\}$ then $f(A^*) = f(X) = \{1, 2, 4\} \not\subset \{2\} = f(A)^*_p = f(A)$.

Corollary 3.19. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ be P-$\mathcal{I}$-open. Assume that every subset A of X satisfies $f(A^*) \subset f(A)^*_p$ or $f(A^*) \subset f(A)$. Then the image of each P-$\mathcal{I}$-open set is P-$\mathcal{I}$-open.*

Proof. We can obtain the result by analogous way to Theorem 3.17. □

We have a question : In Theorem 3.17, if we use the following condition

$$(\forall A \subset X)(f(A^*) \subset f(A)^*) \tag{3.5}$$

instead of the condition (3.4), then does Theorem 3.17 hold?

We provide a partial answer to the above question.

Theorem 3.20. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ be P-$\mathcal{I}$-open such that*

$$\begin{aligned} &(\forall A \subset X)(f(A^*) \subset f(A)^*) \\ &(\forall B \subset Y)((B^*)^*_p \subset B^*_p). \end{aligned} \tag{3.6}$$

Then the image of each $\mathcal{I}$-open set is P-$\mathcal{I}$-open.

Proof. Suppose that $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ is P-$\mathcal{I}$-open. Let A be an $\mathcal{I}$-open set in X. Then $A \subset \mathrm{Int}(A^*)$. Since f is P-$\mathcal{I}$-open, $f(\mathrm{Int}(A^*))$ is a P-$\mathcal{I}$-open set in Y. It follows that

$$\begin{aligned} f(A) &\subset f(\mathrm{Int}(A^*)) \\ &\subset p\mathrm{Int}(f(\mathrm{Int}(A^*))^*_p) \\ &\subset p\mathrm{Int}(f(A^*)^*_p) \end{aligned}$$

$$\subset p\text{Int}((f(A)^*)_p^*)$$
$$\subset p\text{Int}(f(A)_p^*).$$

Hence $f(A)$ is a P-$\mathcal{I}$-open set in Y. □

Theorem 3.21. *Let $f : (X, \tau) \to (Y, \kappa, \mathcal{I})$ be a P-$\mathcal{I}$-open mapping. If $W \subset Y$ and F is a closed set in X containing $f^{-1}(W)$, then there exists a P-$\mathcal{I}$-closed set H in Y containing W such that $f^{-1}(H) \subset F$.*

Proof. Let $f : (X, \tau) \to (Y, \kappa, \mathcal{I})$ be a P-$\mathcal{I}$-open mapping. Suppose that $W \subset Y$ and F is a closed set in X containing $f^{-1}(W)$. Then F^c is open in X and $f(F^c)$ is P-$\mathcal{I}$-open in Y. Putting $H := f(F^c)^c$, we get

$$\begin{aligned} f^{-1}(W) \subset F &\Rightarrow f^{-1}(W^c) \supset F^c \\ &\Rightarrow f(f^{-1}(W^c)) \supset f(F^c) \\ &\Rightarrow W^c \supset f(f^{-1}(W^c)) \supset f(F^c) \\ &\Rightarrow W \subset f(F^c)^c = H, \end{aligned}$$

and $f^{-1}(H) = f^{-1}(f(F^c)^c) \subset (F^c)^c = F$. Hence H is a P-$\mathcal{I}$-closed set containing W and $f^{-1}(H) \subset F$. □

Lemma 3.22. *For any bijective mapping $f : (X, \tau) \to (Y, \kappa, \mathcal{I})$, f is P-$\mathcal{I}$-open if and only if f is P-$\mathcal{I}$-closed.*

Proof. Suppose that f is P-$\mathcal{I}$-open. Let F be closed in X. Then F^c is open in X. This implies that $f(F^c) = f(F)^c$ is P-$\mathcal{I}$-open in Y. Hence $f(F)$ is P-$\mathcal{I}$-closed in Y. Therefore f is a P-$\mathcal{I}$-closed mapping.

Conversely, we can obtain the result by analogous way □

Theorem 3.23. *Let $f : (X, \tau) \to (Y, \kappa, \mathcal{J})$ and $g : (Y, \kappa, \mathcal{J}) \to (Z, \delta, \mathcal{H})$ be two mappings, where $\mathcal{I}, \mathcal{J}, \mathcal{H}$ are ideals on X, Y and Z respectively. Then*

(i) *$g \circ f$ is P-$\mathcal{I}$-open if f is an open mapping and g is a P-$\mathcal{I}$-open mapping.*

(ii) *Assume that $g(V^*) \subset g(V)_p^*$ or $g(V^*) \subset g(V)$ for every subset V of Y. If f is $\mathcal{I}$-open and g is P-$\mathcal{I}$-open, then $g \circ f$ is P-$\mathcal{I}$-open.*

Proof. (i) Straightforward.

(ii) Let $A \subset X$ be an open set. Since f is $\mathcal{I}$-open, $f(A)$ is an $\mathcal{I}$-open set. Since g is P-$\mathcal{I}$-open, it follows from Theorem 3.17 that $g(f(A))$ is a P-$\mathcal{I}$-open set. Hence $g \circ f$ is a P-$\mathcal{I}$-open mapping. □

Corollary 3.24. *Let $f : (X, \tau) \to (Y, \kappa, \mathcal{J})$ and $g : (Y, \kappa, \mathcal{J}) \to (Z, \delta, \mathcal{H})$ be two mappings, where $\mathcal{I}, \mathcal{J}, \mathcal{H}$ are ideals on X, Y and Z respectively. Assume that $g(V^*) \subset g(V)^*_p$ or $g(V^*) \subset g(V)$ for every subset V of Y. If f is P-$\mathcal{I}$-open and g is P-$\mathcal{I}$-open, then $g \circ f$ is P-$\mathcal{I}$-open.*

Proof. Straightforward. □

If f is P-$\mathcal{I}$-open and g is P-$\mathcal{I}$-open then is $g \circ f$ P-$\mathcal{I}$-open? The answer is negative as seen in the following example.

Example 3.25. Consider a topological space

$$(X = \{1, 2, 3, 4\}, \tau = \{\emptyset, X, \{1, 2\}, \{1, 2, 3\}\})$$

and ideal topological spaces $(Y, \kappa, \mathcal{J})$ and $(Z, \delta, \mathcal{H})$ where $Y = \{x, y, z\}$, $\kappa = \{\emptyset, Y, \{x\}\}$, $\mathcal{J} = \{\emptyset, \{y\}\}$, $Z = \{a, b, c, d\}$, $\delta = \{\emptyset, Z, \{c\}, \{a, b\}, \{a, b, c\}\}$, and $\mathcal{H} = \{\emptyset, \{a\}\}$. A mapping $f : (X, \tau) \to (Y, \kappa, \mathcal{J})$ given by $f(1) = x$, $f(2) = y = f(3)$, and $f(4) = z$ is a P-$\mathcal{I}$-open mapping. And a mapping $g : (Y, \kappa, \mathcal{J}) \to (Z, \delta, \mathcal{H})$ given by $g(x) = b$, $g(y) = d$, and $g(z) = c$ is a P-$\mathcal{I}$-open mapping. Let $A = \{1, 2\} \in \tau$. Then $g \circ f(A) = \{b, d\}$ is not a P-$\mathcal{I}$-open set in $(Z, \delta, \mathcal{H})$. Hence $g \circ f$ is not a P-$\mathcal{I}$-open mapping.

Remark 3.26. From Theorem 3.3 and Example 3.25, we know that the answers to the following questions are negative.

(i) If a mapping f is P-$\mathcal{I}$-open and a mapping g is $\mathcal{I}$-open, then is $g \circ f$ P-$\mathcal{I}$-open?

(ii) If a mapping f is $\mathcal{I}$-open and a mapping g is P-$\mathcal{I}$-open, then is $g \circ f$ P-$\mathcal{I}$-open?

(iii) If a mapping f is $\mathcal{I}$-open and a mapping g is $\mathcal{I}$-open, then is $g \circ f$ P-$\mathcal{I}$-open?

If a mapping f is P-$\mathcal{I}$-open and a mapping g is open, then is $g \circ f$ P-$\mathcal{I}$-open? The answer is negative as seen in the following example.

Example 3.27. Consider the example as presented in Example 3.25. A mapping $f : (X, \tau) \to (Y, \kappa, \mathcal{J})$ given by $f(1) = x$, $f(2) = y = f(3)$, and $f(4) = z$ is a P-$\mathcal{I}$-open mapping. And a mapping $g : (Y, \kappa, \mathcal{J}) \to (Z, \delta, \mathcal{H})$ given by $g(x) = c$, $g(y) = a$, and $g(z) = b$ is an open mapping. Let $A = \{1, 2\} \in \tau$. Then $g \circ f(A) = \{a, c\}$ is not a P-$\mathcal{I}$-open set in $(Z, \delta, \mathcal{H})$. Hence $g \circ f$ is not a P-$\mathcal{I}$-open mapping.

Let $f : (X, \tau) \to (Y, \kappa, \mathcal{J})$ and $g : (Y, \kappa, \mathcal{J}) \to (Z, \delta, \mathcal{H})$ be two mappings. We have two questions as follow.

(i) If $g \circ f$ is P-$\mathcal{I}$-open and g is P-$\mathcal{I}$-open, then is f an open mapping?

(ii) If $g \circ f$ is P-$\mathcal{I}$-open and f is open, then is g a P-$\mathcal{I}$-open mapping?

The answers to these questions are negative as seen in the following two examples.

Example 3.28. Let $X = \{1, 2, 3, 4\}$, $\tau = \{\emptyset, X, \{1, 2\}, \{1, 2, 3\}\}$. Let $Y = \{x, y, z\}$ $\kappa = \{\emptyset, Y, \{x\}\}$, $\mathcal{J} = \{\emptyset, \{y\}\}$ and let $Z = \{a, b, c, d\}$, $\delta = \{\emptyset, Z, \{c\}, \{a, b\}, \{a, b, c\}\}$, $\mathcal{H} = \{\emptyset, \{a\}\}$. Consider mappings $f : (X, \tau) \to (Y, \kappa, \mathcal{J})$ given by $f(1) = x = f(2)$, $f(3) = z = f(4)$ and $g : (Y, \kappa, \mathcal{J}) \to (Z, \delta, \mathcal{H})$ given by $g(x) = b$, $g(y) = d$ and $g(z) = c$. Then $g \circ f$ and g are P-$\mathcal{I}$-open. But f is not an open mapping because $f(A) = \{x, z\} \notin \kappa$ for $A = \{1, 2, 3\} \in \tau$.

Example 3.29. Let $X = \{1, 2, 3, 4\}$, $\tau = \{\emptyset, X, \{1, 2\}, \{1, 2, 3\}\}$. Let $Y = \{x, y, z\}$ $\kappa = \{\emptyset, Y, \{x\}, \{x, y\}\}$, $\mathcal{J} = \{\emptyset, \{y\}\}$ and let $Z = \{a, b, c, d\}$, $\delta = \{\emptyset, Z, \{c\}, \{a, b\}, \{a, b, c\}\}$, $\mathcal{H} = \{\emptyset, \{a\}\}$. Consider mappings $f : (X, \tau) \to (Y, \kappa, \mathcal{J})$ given by $f(1) = f(2) = f(3) = x$, $f(4) = y$ and $g : (Y, \kappa, \mathcal{J}) \to (Z, \delta, \mathcal{H})$ given by $g(x) = b$, $g(y) = c$, $g(z) = a$. Then $g \circ f$ is P-$\mathcal{I}$-open and f is open. But g is not a P-$\mathcal{I}$-open mapping because $g(Y) = \{a, b, c\}$ is not a P-$\mathcal{I}$-open set in Z.

4. P-$\mathcal{I}$-CONTINUOUS MAPPINGS

Definition 4.1. A mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa)$ is said to be P-$\mathcal{I}$-continuous if $f^{-1}(V) \in P\mathcal{I}O(X, \tau, \mathcal{I})$ for all $V \in \kappa$.

Example 4.2. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$ and an ideal $\mathcal{I} = \{\emptyset, \{c\}\}$. Then we know that

$$P\mathcal{I}O(X, \tau, \mathcal{I}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}.$$

Let $Y = \{1, 2, 3, 4, 5\}$ with topology $\kappa = \{\emptyset, Y, \{2\}, \{3\}, \{2, 3\}\}$. Then a mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa)$ given by $f(a) = 2$, $f(b) = 3$, and $f(c) = 5 = f(d)$ is a P-$\mathcal{I}$-continuous mapping.

Theorem 4.3. *If a mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa)$ is P-$\mathcal{I}$-continuous, then it is $\mathcal{I}$-continuous.*

Proof. It follows from Theorem 2.8. □

Corollary 4.4. *If a mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa)$ is P-$\mathcal{I}$-continuous, then it is pre-continuous.*

Proof. It follows from Remark 2.9. □

Is any $\mathcal{I}$-continuous mapping a P-$\mathcal{I}$-continuous mapping? The answer to this question is negative as seen in the following example.

Example 4.5. Consider an ideal topological space $(X,\tau,\mathcal{I})$ where $X=\{a,b,c,d\}$, $\tau=\{\emptyset,X,\{c\},\{a,b\},\{a,b,c\}\}$ and $\mathcal{I}=\{\emptyset,\{a\}\}$. Then

$$PIO(X,\tau,\mathcal{I})=\{\emptyset,\{b\},\{c\},\{b,c\},\{b,c,d\}\},$$

$$IO(X,\tau,\mathcal{I})=\{\emptyset,\{b\},\{c\},\{a,b\},\{b,c\},\{a,b,c\},\{b,c,d\},X\}.$$

Let (Y,κ) be a topological space where $Y=\{1,2,3,4\}$ and

$$\kappa=\{\emptyset,Y,\{1\},\{2\},\{1,2\},\{1,2,3\}\}.$$

Consider a mapping $f:(X,\tau,\mathcal{I})\to(Y,\kappa)$ given by $f(a)=3=f(d)$, $f(b)=1$ and $f(c)=2$. Then $f^{-1}(\{1\})=\{b\}$, $f^{-1}(\{2\})=\{c\}$, $f^{-1}(\{1,2\})=\{b,c\}$ and $f^{-1}(\{1,2,3\})=X=f^{-1}(Y)$. Hence f is $\mathcal{I}$-continuous. But f is not P-$\mathcal{I}$-continuous because $f^{-1}(\{1,2,3\})=X$ is not P-$\mathcal{I}$-open.

Is any P-$\mathcal{I}$-continuous mapping a continuous mapping and vice versa? The following examples show that the answer to this question is negative.

Example 4.6. Let $(X,\tau,\mathcal{I})$ be an ideal topological space with $X=\{a,b,c,d\}$, $\tau=\{\emptyset,X,\{a,b\},\{a,b,c\}\}$, and $\mathcal{I}=\{\emptyset,\{a\}\}$. Consider a topological space (Y,κ) with $Y=\{1,2,3\}$ and $\kappa=\{\emptyset,Y,\{1\},\{2\},\{1,2\}\}$. Let $f:(X,\tau,\mathcal{I})\to(Y,\kappa)$ be defined by $f(a)=f(b)=f(c)=1$ and $f(d)=3$. Then $f^{-1}(\{1\})=\{a,b,c\}=f^{-1}(\{1,2\})$, $f^{-1}(\{2\})=\emptyset$ and $f^{-1}(Y)=X$. Hence f is continuous. But f is not P-$\mathcal{I}$-continuous because $f^{-1}(Y)=X$ is not P-$\mathcal{I}$-open.

Example 4.7. Consider an ideal topological space $(X,\tau,\mathcal{I})$ with $X=\{a,b,c\}$, $\tau=\{\emptyset,X,\{a\}\}$, and $\mathcal{I}=\{\emptyset,\{b\}\}$. Let $Y=\{1,2,3,4\}$ with topology $\kappa=\{\emptyset, Y, \{1,2\}, \{1,2,3\}\}$. Define a mapping $g:(X,\tau,\mathcal{I})\to(Y,\kappa)$ by $g(a)=1$, $g(b)=2$ and $g(c)=4$. Then $g^{-1}(\{1,2\})=\{a,b\}=f^{-1}(\{1,2,3\})$ and $f^{-1}(Y)=X$. Hence f is P-$\mathcal{I}$-continuous. However, f is not continuous because $f^{-1}(\{1,2\})=\{a,b\}$ is not open.

Definition 4.8. Let $(X,\tau,\mathcal{I})$ be an ideal topological space. A subset S of X is called a *P-$\mathcal{I}$-neighborhood* of x if S is a superset of a P-$\mathcal{I}$-open set G containing x.

Example 4.9. Let $X=\{a,b,c,d\}$ with a topology $\tau=\{\emptyset,X,\{a,b\},\{a,b,c\}\}$ and an ideal $\mathcal{I}=\{\emptyset,\{c\}\}$. Then

$$PIO(X,\tau,\mathcal{I}) = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{a,d\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, X\},$$

and the set $S = \{a, c, d\}$ is a P-$\mathcal{I}$-neighborhood of a because there exists a P-$\mathcal{I}$-open set $\{a, d\}$ such that $a \in \{a, d\} \subset S$. But S is not a P-$\mathcal{I}$-neighborhood of c.

Theorem 4.10. *For a mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa)$, the following statements are equivalent.*

(i) *f is P-$\mathcal{I}$-continuous.*
(ii) *For each $x \in X$ and each $V \in \kappa$ containing $f(x)$, there exists*

$$W \in PIO(X,\tau,\mathcal{I})$$

containing x such that $f(W) \subset V$.
(iii) *For each $x \in X$ and each $V \in \kappa$ containing $f(x)$, $f^{-1}(V)^*_p$ is a P-$\mathcal{I}$-neighborhood of x.*
(iv) *For each $x \in X$ and each $V \in \kappa$ containing $f(x)$, $f^{-1}(V)^*_p$ is a pre-neighborhood of x.*

Proof. (i) $\Rightarrow$ (ii) Let $x \in X$ and $V \in \kappa$ containing $f(x)$. Since f is P-$\mathcal{I}$-continuous, $f^{-1}(V)$ is a P-$\mathcal{I}$-open set. Putting $W := f^{-1}(V)$, we have $f(W) \subset V$.

(ii) $\Rightarrow$ (i) Let A be an open set in Y. If $f^{-1}(A) = \emptyset$ then $f^{-1}(A)$ is clearly P-$\mathcal{I}$-open. Assume that $f^{-1}(A) \neq \emptyset$. Let $x \in f^{-1}(A)$. Then $f(x) \in A$, which implies that there exist P-$\mathcal{I}$-open W containing x such that $f(W) \subset A$. Thus $W \subset f^{-1}(f(W)) \subset f^{-1}(A)$. Since W is P-$\mathcal{I}$-open, $x \in W \subset p\mathrm{Int}(W^*_p) \subset p\mathrm{Int}(f^{-1}(A)^*_p)$ and so $f^{-1}(A) \subset p\mathrm{Int}(f^{-1}(A)^*_p)$. Hence $f^{-1}(A)$ is a P-$\mathcal{I}$-open set and so f is P-$\mathcal{I}$-continuous.

(ii) $\Rightarrow$ (iii) Let $x \in X$ and $V \in \kappa$ containing $f(x)$. Then there exist P-$\mathcal{I}$-open W containing x such that $f(W) \subset V$. It follows that $W \subset f^{-1}(f(W)) \subset f^{-1}(V)$. Since W is P-$\mathcal{I}$-open,

$$x \in W \subset p\mathrm{Int}(W^*_p) \subset p\mathrm{Int}(f^{-1}(V)^*_p) \subset f^{-1}(V)^*_p.$$

Hence $f^{-1}(V)^*_p$ is a P-$\mathcal{I}$-neighborhood of x.

(iii) $\Rightarrow$ (iv) By Remark 2.9, it is straightforward.

(iv) $\Rightarrow$ (i) Let A be an open set in Y. If $f^{-1}(A) = \emptyset$ then $f^{-1}(A)$ is clearly P-$\mathcal{I}$-open. Assume that $f^{-1}(A) \neq \emptyset$ and let $x \in f^{-1}(A)$. Then $f(x) \in A$. Since $f^{-1}(A)^*_p$ is a pre-neighborhood of x, there exists a pre-open set H such that $x \in H \subset f^{-1}(A)^*_p$. Since H is pre-open, $x \in H = p\mathrm{Int}(H) \subset p\mathrm{Int}(f^{-1}(A)^*_p)$ and so

$f^{-1}(A) \subset p\text{Int}(f^{-1}(A)^*_p)$. Hence $f^{-1}(A)$ is a P-$\mathcal{I}$-open set. Therefore f is P-$\mathcal{I}$-continuous. □

Theorem 4.11. *For a mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa)$, the following statements are equivalent.*

(i) *f is P-$\mathcal{I}$-continuous.*
(ii) *The inverse image of each closed set in Y is P-$\mathcal{I}$-closed.*
(iii) *For each subset A of Y, $f^{-1}(\text{Int}(A)) \subset p\text{Int}(f^{-1}(A)^*_p)$.*

Proof. (i) ⇒ (ii) Let F be a closed subset of X. Then F^c is open in Y. Since f is P-$\mathcal{I}$-continuous, $f^{-1}(F^c) = (f^{-1}(F))^c$ is P-$\mathcal{I}$-open. Hence $f^{-1}(F)$ is P-$\mathcal{I}$-closed.

(ii) ⇒ (i) Let G be an open set in (Y, κ). Then G^c is closed. By (ii), $f^{-1}(G^c) = (f^{-1}(G))^c$ is P-$\mathcal{I}$-closed. Hence $f^{-1}(G)$ is P-$\mathcal{I}$-open, and so f is P-$\mathcal{I}$-continuous.

(i) ⇒ (iii) Suppose that f is P-$\mathcal{I}$-continuous. Let A be a subset of Y. Then $f^{-1}(\text{Int}(A))$ is P-$\mathcal{I}$-open. It follows that

$$f^{-1}(\text{Int}(A)) \subset p\text{Int}(f^{-1}(\text{Int}(A))^*_p) \subset p\text{Int}(f^{-1}(A)^*_p).$$

(iii) ⇒ (i) Let A be an open set in (Y, κ). Then $f^{-1}(A) = f^{-1}(\text{Int}(A)) \subset p\text{Int}(f^{-1}(A)^*_p)$ by (iii). Hence $f^{-1}(A)$ is P-$\mathcal{I}$-open. Therefore f is P-$\mathcal{I}$-continuous. □

Proposition 4.12. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then the following statements are equivalent.*

(i) *$X = X^*_p$.*
(ii) *$\tau^p \cap \mathcal{I} = \{\emptyset\}$. ($\tau^p$ is a set of all pre-open sets in (X, τ)).*
(iii) *If $A \in \mathcal{I}$, then $p\text{Int}(A) = \emptyset$.*

Proof. (i) ⇒ (ii) Suppose that $\tau^p \cap \mathcal{I} \neq \{\emptyset\}$. Then there exists $G(\neq \emptyset) \in \tau^p \cap \mathcal{I}$. Let $a \in G$, i.e., $a \notin X \setminus G$. Then $G \in \tau^p(a)$ and $X \cap G = G \in \mathcal{I}$. Thus $a \notin X^*_p$ and so $X^*_p \subset X \setminus G$. Since $G \neq \emptyset$, $X^*_p \neq X$. This is a contradiction. Hence $\tau^p \cap \mathcal{I} = \{\emptyset\}$.

(ii) ⇒ (iii) Let $A \in \mathcal{I}$. If $A = \emptyset$ then clearly $p\text{Int}(A) = \emptyset$. Assume that A is not empty. Then for every $H \in \tau^p \setminus \{\emptyset\}$, we have $H \notin \mathcal{I}$ by (ii) and so, $H \not\subset A$. Hence $p\text{Int}(A) = \emptyset$.

(iii) ⇒ (i) Let $x \in X$. If there exist $G_x \in \tau^p(x)$ such that $G_x \cap X \in \mathcal{I}$, then $G_x = p\text{Int}(G_x) = p\text{Int}(G_x \cap X) = \emptyset$ by (iii). It is a contradiction. Hence $G_x \cap X \notin \mathcal{I}$ for every $G_x \in \tau^p(x)$ and so $x \in X^*_p$. This means that $X = X^*_p$. □

Theorem 4.13. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. If $U \subset U_p^*$ for every pre-open U, then $X = X_p^*$.*

Proof. Since X is always pre-open, $X \subset X_p^*$ by the hypothesis. In general, $X_p^* \subset X$. Hence $X = X_p^*$ □

The converse of Theorem4.13 may not be true as seen in the following example.

Example 4.14. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$, ideal $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\tau^p = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\},$ $\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. We knows that $X = X_p^*$ but there exist a pre-open set $\{a, c\}$ such that $\{a, c\} \not\subset \{a, c\}_p^* = \{a\}$.

Theorem 4.15. *If $f : (X, \tau, \mathcal{I}) \to (Y, \kappa)$ is P-$\mathcal{I}$-continuous, then $X = X_p^*$.*

Proof. Suppose that $f : (X, \tau, \mathcal{I}) \to (Y, \kappa)$ is P-$\mathcal{I}$-continuous. Since Y is an open set in (Y, κ) and f is P-$\mathcal{I}$-continuous, $f^{-1}(Y) = X$ is a P-$\mathcal{I}$-open set and thus $X \subset p\text{Int}(X_p^*) \subset X_p^*$. Hence $X = X_p^*$ because $X_p^* \subset X$ in general. □

The converse of Theorem 4.15 may not be true as seen in the following example.

Example 4.16. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$, ideal $\mathcal{I} = \{\emptyset, \{c\}\}$. Let $Y = \{1, 2, 3\}$ with a topology $\kappa = \{\emptyset, Y, \{1\}, \{1, 2\}\}$, ideal $\mathcal{J} = \{\emptyset, \{2\}\}$. Consider a mapping $f : X \to Y$ defined by $f(a) = 2 = f(b)$, $f(c) = 1$, $f(d) = 3$. Then $X = X_p^*$ but f is not P-$\mathcal{I}$-continuous.

Remark 4.17. By Proposition 4.12 and Theorem 4.15, we can deduce that if $f : (X, \tau, \mathcal{I}) \to (Y, \kappa)$ is P-$\mathcal{I}$-continuous, then the following statements are valid.

(i) $X = X_p^*$.
(ii) $\tau^p \cap \mathcal{I} = \{\emptyset\}$, ($\tau^p$ is a set of all pre-open sets in (X, τ)).
(iii) If $A \in \mathcal{I}$, then $p\text{Int}(A) = \emptyset$.

5. P-$\mathcal{I}$-IRRESOLUTE MAPPINGS

Definition 5.1. A mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ is said to be *P-$\mathcal{I}$-irresolute* if $f^{-1}(V) \in P\mathcal{I}O(X, \tau, \mathcal{I})$ for all $V \in P\mathcal{I}O(Y, \kappa, \mathcal{J})$.

Definition 5.2. A mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ is said to be *$\mathcal{I}$-irresolute* if $f^{-1}(V) \in \mathcal{I}O(X, \tau, \mathcal{I})$ for all $V \in \mathcal{I}O(Y, \kappa, \mathcal{J})$.

Example 5.3. Let $(X, \tau, \mathcal{I})$ be an ideal topological space with $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}\}$, $\mathcal{I} = \{\emptyset, \{a\}\}$, and let $(Y, \kappa, \mathcal{J})$ be an ideal topological space with $Y = \{1, 2, 3, 4\}$, $\kappa = \{\emptyset, Y, \{1, 2\}, \{1, 2, 3\}\}$ and $\mathcal{J} = \{\emptyset, \{2\}\}$. Then

$\mathcal{I}O(X,\tau,\mathcal{I}) = \{\emptyset, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}, X\}$,
$P\mathcal{I}O(X,\tau,\mathcal{I}) = \{\emptyset, \{b\}, \{c\}, \{b,c\}, \{b,c,d\}\}$,
$\mathcal{I}O(Y,\kappa,\mathcal{J}) = \{\emptyset, \{1\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, Y\}$,
$P\mathcal{I}O(Y,\kappa,\mathcal{J}) = \{\emptyset, \{1\}, \{1,3\}, \{1,4\}, \{1,3,4\}\}$.

(a) A mapping $f : (X,\tau,\mathcal{I}) \to (Y,\kappa,\mathcal{J})$ given by $f(a) = 2$, $f(b) = 1$, $f(c) = 4 = f(d)$ is both P-$\mathcal{I}$-irresolute and $\mathcal{I}$-irresolute.

(b) A mapping $g : (X,\tau,\mathcal{I}) \to (Y,\kappa,\mathcal{J})$ given by $g(a) = 2 = g(d)$, $g(b) = 1$, $g(c) = 3$ P-$\mathcal{I}$-irresolute which is not $\mathcal{I}$-irresolute.

(c)A mapping $h : (X,\tau,\mathcal{I}) \to (Y,\kappa,\mathcal{J})$ given by $h(a) = 3$, $h(b) = 1$, $h(c) = 2 = h(d)$ $\mathcal{I}$-irresolute which is not P-$\mathcal{I}$-irresolute.

(d) A mapping $i : (X,\tau,\mathcal{I}) \to (Y,\kappa,\mathcal{J})$ given by $i(a) = 1$, $i(b) = 2 = i(c)$, $i(d) = 3$ is neither $\mathcal{I}$-irresolute nor P-$\mathcal{I}$-irresolute.

The above example shows that an $\mathcal{I}$-irresolute mapping and a P-$\mathcal{I}$-irresolute mapping are independent.

Theorem 5.4. *If a mapping $f : (X,\tau,\mathcal{I}) \to (Y,\kappa)$ satisfy the following conditions,*

- *f is P-$\mathcal{I}$-continuous.*
- *$f^{-1}(V^*) \subset f^{-1}(V)$ or $f^{-1}(V^*) \subset f^{-1}(V)^*_p$ for each $V \subset Y$.*

then f is both an $\mathcal{I}$-irresolute mapping and a P-$\mathcal{I}$-irresolute mapping.

Proof. Assume that f satisfy two conditions. It is sufficient to show that the inverse image of $\mathcal{I}$-open set is P-$\mathcal{I}$-open set because every P-$\mathcal{I}$-open set is an $\mathcal{I}$-open set by Theorem 2.8. Let A be an $\mathcal{I}$-open set. Then $A \subset \text{Int}(A^*)$. Since f is P-$\mathcal{I}$-continuous, $f^{-1}(\text{Int}(A^*))$ is P-$\mathcal{I}$-open and hence $f^{-1}(\text{Int}(A^*)) \subset p\text{Int}(f^{-1}(\text{Int}(A^*))^*_p)$. It follows from the second condition that

$$\begin{aligned} f^{-1}(A) &\subset p\text{Int}(f^{-1}(\text{Int}(A^*))^*_p) \\ &\subset p\text{Int}(f^{-1}(A^*)^*_p) \\ &\subset p\text{Int}(f^{-1}(A)^*_p). \end{aligned}$$

Hence $f^{-1}(A)$ is P-$\mathcal{I}$-open. Since every P-$\mathcal{I}$-open set is an $\mathcal{I}$-open set by Theorem 2.8, f is both an $\mathcal{I}$-irresolute mapping and a P-$\mathcal{I}$-irresolute mapping. □

The following example shows that a P-$\mathcal{I}$-continuous mapping is neither an $\mathcal{I}$-irresolute mapping nor a P-$\mathcal{I}$-irresolute mapping.

Example 5.5. Consider two ideal topological spaces $(X,\tau,\mathcal{I})$ and $(Y,\kappa,\mathcal{J})$ where $X = \{a,b,c,d\}$, $\tau = \{\emptyset, X, \{a,b\}, \{a,b,c\}\}$, $\mathcal{I} = \{\emptyset, \{c\}\}$, $Y = \{1,2,3,4\}$, $\kappa = \{\emptyset, Y, \{3\}, \{1,2\}, \{1,2,3\}\}$, and $\mathcal{J} = \{\emptyset, \{1\}\}$. Define a mapping $f : (X,\tau,\mathcal{I}) \to$

$(Y,\kappa,\mathcal{J})$ by $f(a)=3$, $f(b)=1$, $f(c)=4$ and $f(d)=2$. Then f is a P-$\mathcal{I}$-continuous mapping. Note that $A=\{2\}$ is both an $\mathcal{I}$-open set and a P-$\mathcal{I}$-open set in $(Y,\kappa,\mathcal{J})$. But $f^{-1}(A)=\{d\}$ is neither an $\mathcal{I}$-open set nor a P-$\mathcal{I}$-open set. Hence f is neither an $\mathcal{I}$-irresolute mapping nor a $\mathcal{I}$-irresolute mapping.

Theorem 5.6. *If a mapping $f:(X,\tau,\mathcal{I})\to(Y,\kappa,\mathcal{J})$ satisfy the following conditions,*

- *f is $\mathcal{I}$-continuous.*
- *$f^{-1}(V^*)\subset f^{-1}(V)$ or $f^{-1}(V^*)\subset f^{-1}(V)^*$ for each $V\subset Y$.*

then f is an $\mathcal{I}$-irresolute mapping.

Proof. Assume that f satisfy two given conditions. Let A be an $\mathcal{I}$-open set. Then $A\subset \mathrm{Int}(A^*)$. Since f is $\mathcal{I}$-continuous, $f^{-1}(\mathrm{Int}(A^*))$ is $\mathcal{I}$-open. It follows that

$$\begin{aligned} f^{-1}(A) &\subset f^{-1}(\mathrm{Int}(A^*)) \\ &\subset \mathrm{Int}(f^{-1}(\mathrm{Int}(A^*))^*) \\ &\subset \mathrm{Int}(f^{-1}(A^*)^*) \\ &\subset \mathrm{Int}(f^{-1}(A)^*) \end{aligned}$$

so that $f^{-1}(A)$ is $\mathcal{I}$-open. Therefore f is an $\mathcal{I}$-irresolute mapping. □

The following example shows that although a mapping f satisfy two conditions of Theorem 5.6, f may not be a P-$\mathcal{I}$-irresolute mapping.

Example 5.7. Let $X=\{a,b,c,d\}$, $\tau=\{\emptyset,X,\{a,b\},\{a,b,c\}\}$ and $\mathcal{I}=\{\emptyset,\{c\}\}$. Let $Y=\{1,2,3\}$, $\kappa=\{\emptyset,Y,\{1\},\{1,2\}\}$, and $\mathcal{J}=\{\emptyset,\{2\}\}$. A mapping $f:(X,\tau,\mathcal{I})\to(Y,\kappa,\mathcal{J})$ given by $f(a)=f(c)=1$, $f(b)=2$, and $f(d)=3$ is $\mathcal{I}$-irresolute and satisfy the condition

$$f^{-1}(V^*)\subset f^{-1}(V) \text{ or } f^{-1}(V^*)\subset f^{-1}(V)^* \text{ for each } V\subset Y.$$

But f is not P-$\mathcal{I}$-irresolute because $f^{-1}(\{1\})=\{a,c\}\notin PIO(Y,\kappa,\mathcal{J})$.

Theorem 5.8. *Let $f:(X,\tau,\mathcal{I})\to(Y,\kappa,\mathcal{J})$ be a mapping. If*

$$f^{-1}(A_p^*)\subset p\mathrm{Int}(f^{-1}(A)_p^*)$$

for each $A\subset Y$, then f is a P-$\mathcal{I}$-irresolute mapping.

Proof. Let A be a P-$\mathcal{I}$-open set. Then $A\subset p\mathrm{Int}(A_p^*)$ which implies that

$$f^{-1}(A)\subset f^{-1}(p\mathrm{Int}(A_p^*))\subset f^{-1}(A_p^*)\subset p\mathrm{Int}(f^{-1}(A)_p^*).$$

Hence f is a P-$\mathcal{I}$-irresolute mapping. □

The converse of above theorem may not be true as seen in the following example.

Example 5.9. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Let $Y = \{1, 2, 3\}$, $\kappa = \{\emptyset, Y, \{1\}, \{1, 2\}\}$, and $\mathcal{J} = \{\emptyset, \{2\}\}$. A mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ given by $f(a) = 1$, $f(b) = 2 = f(c)$ and $f(d) = 3$ is P-$\mathcal{I}$-irresolute. For a set $A = \{1\}$, we have

$$f^{-1}(A_p^*) = X \not\subset p\text{Int}(f^{-1}(A)_p^*) = \{a\}.$$

If $f^{-1}(A_p^*) \subset p\text{Int}(f^{-1}(A)_p^*)$ for each $A \subset Y$, then is f a $\mathcal{I}$-irresolute mapping? The answer is negative as seen in the following example.

Example 5.10. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Let $Y = \{1, 2, 3, 4\}$, $\kappa = \{\emptyset, Y, \{1, 2\}, \{1, 2, 3\}\}$, and $\mathcal{J} = \{\emptyset, \{2\}\}$. A mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ given by $f(a) = 2 = f(d)$, $f(b) = 1$, $f(c) = 3$, is satisfied $f^{-1}(A_p^*) \subset p\text{Int}((f^{-1}(A))_p^*)$ for each $A \subset Y$. But f is not a $\mathcal{I}$-irresolute mapping because $f^{-1}(\{1, 2\}) = \{a, b, d\} \notin \mathcal{IO}(X, \tau, \mathcal{I})$ for $\{1, 2\} \in \mathcal{IO}(Y, \kappa, \mathcal{J})$.

Theorem 5.11. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ be a mapping. If*

$$f^{-1}(A^*) \subset \text{Int}(f^{-1}(A)^*)$$

for each $A \subset Y$, then f is an $\mathcal{I}$-irresolute mapping.

Proof. Let A be an $\mathcal{I}$-open set. Then $A \subset \text{Int}(A^*)$ which implies that

$$f^{-1}(A) \subset f^{-1}(\text{Int}(A^*)) \subset f^{-1}(A^*) \subset \text{Int}(f^{-1}(A)^*).$$

Hence f is an $\mathcal{I}$-irresolute mapping. □

The converse of above theorem may not be true as seen in the following example.

Example 5.12. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Let $Y = \{1, 2, 3\}$, $\kappa = \{\emptyset, Y, \{1\}, \{1, 2\}\}$, and $\mathcal{J} = \{\emptyset, \{2\}\}$. A mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ given by $f(a) = 1 = f(c)$, $f(b) = 2$ and $f(d) = 3$ is $\mathcal{I}$-irresolute. For a set $A = \{3\}$, we obtain

$$f^{-1}(A_p^*) = \{d\} \not\subset p\text{Int}(f^{-1}(A)^*) = \emptyset.$$

If $f^{-1}(A^*) \subset \text{Int}(f^{-1}(A)^*)$ for each $A \subset Y$, then is f a P-$\mathcal{I}$-irresolute mapping? The answer is negative as seen in the following example.

Example 5.13. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Let $Y = \{1, 2, 3, 4\}$, $\kappa = \{\emptyset, Y, \{3\}, \{1, 2\}, \{1, 2, 3\}\}$, and $\mathcal{J} = \{\emptyset, \{1\}, \{3\}, \{1, 3\}\}$. A mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ given by $f(a) = 2 = f(c)$, $f(b) = 1$, $f(d) = 3$, is satisfied $f^{-1}(A^*) \subset \mathrm{Int}(f^{-1}(A)^*)$ for each $A \subset Y$. But f is not a P-$\mathcal{I}$-irresolute mapping because $f^{-1}(\{2\}) = \{a, c\} \notin P\mathcal{I}O(X, \tau, \mathcal{I})$ for $\{2\} \in P\mathcal{I}O(Y, \kappa, \mathcal{J})$.

Lemma 5.14 ([18]). *Let A be a subset in an ideal topological space $(X, \tau, \mathcal{I})$. Then* $p\mathrm{Int}(A_p^*) \subset \mathrm{Int}(A^*)$.

Lemma 5.15 ([18]). *For any subset A of an ideal topological space $(X, \tau, \mathcal{I})$, we have*

(i) $A_p^* \subset A^*$.
(ii) $A_p^* \subset p\mathrm{Cl}(A)$.

Corollary 5.16. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ be a mapping. If*

$$f^{-1}(A^*) \subset p\mathrm{Int}(f^{-1}(A)_p^*)$$

for each $A \subset Y$, then f is both an $\mathcal{I}$-irresolute mapping and a P-$\mathcal{I}$-irresolute mapping.

Proof. Since $A_p^* \subset A^*$ by Lemma 5.15(i), $f^{-1}(A_p^*) \subset f^{-1}(A^*)$. It follows that $f^{-1}(A_p^*) \subset f^{-1}(A^*) \subset p\mathrm{Int}(f^{-1}(A)_p^*) \subset \mathrm{Int}(f^{-1}(A)^*)$ by the hypothesis and Lemma 5.14. Thus f is both P-$\mathcal{I}$-irresolute and $\mathcal{I}$-irresolute by Theorem 5.8 and Theorem 5.11. □

Theorem 5.17. *For two mappings $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$ and $g : (Y, \kappa, \mathcal{J}) \to (Z, \delta, \mathcal{H})$, the following statements are valid.*

(i) *If f is P-$\mathcal{I}$-irresolute and g is P-$\mathcal{I}$-irresolute, then $g \circ f$ is P-$\mathcal{I}$-irresolute.*
(ii) *If f is P-$\mathcal{I}$-irresolute and g is P-$\mathcal{I}$-continuous, then $g \circ f$ is P-$\mathcal{I}$-continuous.*
(iii) *If f is $\mathcal{I}$-irresolute and g is $\mathcal{I}$-irresolute, then $g \circ f$ is $\mathcal{I}$-irresolute.*
(iv) *If f is $\mathcal{I}$-irresolute and g is P-$\mathcal{I}$-continuous, then $g \circ f$ is $\mathcal{I}$-continuous.*
(v) *If f is $\mathcal{I}$-irresolute and g is $\mathcal{I}$-continuous, then $g \circ f$ is $\mathcal{I}$-continuous.*

Proof. Straightforward. □

Theorem 5.18. *Let mapping $f : (X, \tau, \mathcal{I}) \to (Y, \kappa, \mathcal{J})$, $g : (Y, \kappa, \mathcal{J}) \to (Z, \delta, \mathcal{H})$. If g is an injective mapping then the followings are valid.*

(i) *If g is P-$\mathcal{I}$-open and $g \circ f$ is P-$\mathcal{I}$-irresolute, then f is P-$\mathcal{I}$-continuous.*
(ii) *If g is $\mathcal{I}$-open and $g \circ f$ is $\mathcal{I}$-irresolute, then f is $\mathcal{I}$-continuous.*
(iii) *If g is open and $g \circ f$ is P-$\mathcal{I}$-continuous, then f is P-$\mathcal{I}$-continuous.*
(iv) *If g is open and $g \circ f$ is $\mathcal{I}$-continuous, then f is $\mathcal{I}$-continuous.*

Proof. Straightforward. □

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