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Development of a Simple Rate-Sensitive Model I (Derivation and Verification)

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간단한 전단속도 의존적 모델의 개발 I (유도 및 검증)

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Abstract A rate-sensitive constitutive model was developed, with some simplifying assumptions, in the frame of the standard elastoplastic-viscous relation. The simulations performed using the model were successful for the normally consolidated cohesive soils with the advantage that the parameter values determined at a rate could be used at different rates. Details on the parameters and additional modification of the model are presented in the successive paper.

Key Words : rate-sensitive, model, elastic, plastic, viscous, parameter, constitutive

요 약 본 논문에서는 표준 탄성-소성-점성 관계식에 근거하고, 수식을 간략화하기 위한 몇 가지 가정을 도입하여 전단속도에 능동적으로 대처할 수 있는 비교적 간단한 구성모델을 개발하였다. 개발된 모델은 정규압밀점토의 응력경로를 비교적 성공적으로 묘사하였다. 개발된 모델에서는 하나의 전단속도에서 결정한 모델변수 값들을 다른 전단속도 경우에도 적용할 수 있다는 장점이 있다. 모델변수 및 모델의 추가 간략화는 연속된 논문에서 설명된다.

1. Introduction

The strain rate effects have been proven to be one of the important behaviors of cohesive soils. Such phenomena, as the strain rate at the strain-controlled shearing phase for the common but important triaxial test, the penetration rate of piezocone testing, and the difference of shear strength at various strain rates, come down to the stain rate-dependent stress-strain behavior of soils, specially of cohesive soils.

Several time-dependent models, like the semi-empirical methods, the mathematical theories, and the rheological models have been proposed to adequately simulate the rate-dependent behaviors[1][2][3].

The generalized viscous theory, proposed by Perzyna(1963, 1966) has been successfully used for the time-dependent modeling owing to its generality and practical usefulness[4][5]. On the contrary, the model has the disadvantage that the viscous material parameters should be evaluated, together with the elasto-plastic parameters, each time the strain rate varies.

In this study, a relatively simple constitutive model, which can simulate the rate-dependent behavior of cohesive soils, were developed. The model consists of the generalized Hooke's law and the anisotropic modified Cam-clay model for the elasto-plastic behavior, and the Perzyna's generalized viscous theory and Adachi's model for the rate-sensitive behavior[6][7][8][9]. The mathematical and conceptual derivation and the verification were described in this paper. Then the material parameters and

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the further modification of the model are presented in the successive paper.

2. Concept and Derivation of the Model

2.1 Elastoplastic Modeling

A constitutive equation was developed in the base of the classical elastoplastic-viscous relation scheme. The rate-dependent relation proposed by Adachi et al.(1974, 1982) was incorporated into the developed equation for more practical usage of it in the point of the simpler method of determining the material parameter values at various strain rates[7][8].

The generalized Hooke's law was assumed to be adequate for the elastic behavior since it has clear concept even though it has the disadvantage in that the determination of material parameter values, and is used for the very initial part of the stress-strain behavior, accordingly it has been widely used.

The anisotropic modified Cam-clay model(Dafalias, 1987) was adopted to simulate the plastic stress-strain behavior in the scheme of the associated flow rule[8]. Dafalias(1987) added the anisotropic hardening parameter α to the normal modified Cam-clay model as in eqs. (1) and (2) for the triaxial space.

$$f = p^2 - pp_o + \frac{1}{M}(q^2 - 2\alpha pq + \alpha^2 pp_o) = 0 \quad (1)$$

$$\dot{\alpha} = \langle\bar{\lambda}\rangle \left\{ \frac{1+e_o}{\lambda - \kappa} \left| \frac{\partial f}{\partial p} \right| \frac{c}{p_o} (q - x\alpha p) \right\} \quad (2)$$

where p=mean effective stress, q=deviatoric stress, < >=Macauley bracket, $\bar{\lambda}$=loading index, λ=compression index in e-ln p space, κ=recompression index in e-ln p space, e_o=initial void ratio, c and x=constants. The expression $\langle\bar{\lambda}\rangle(\partial f/\partial p)$ represents the plastic strain rate $\dot{\varepsilon}^p$. Using eqs. (3) and (4), the anisotropic modified Cam-clay model can be expressed as in eqs. (5) and (6).

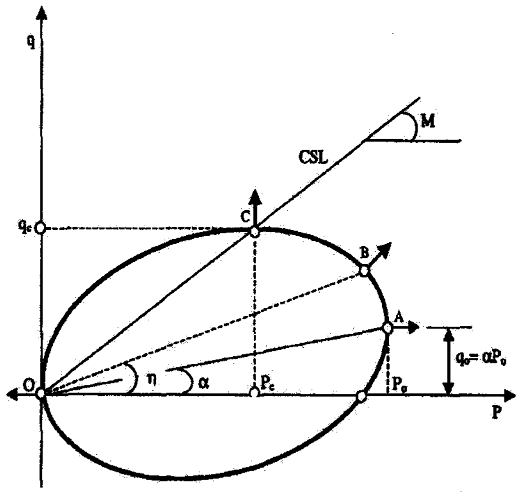
$$q=\{(3/2)s_{ij}s_{ij}\}1/2 \quad (3)$$

$$\alpha=\{(3/2)\alpha_{ij}\alpha_{ij}\}1/2 \quad (4)$$

$$f = p^2 - pp_o + \frac{1}{2M^2}\{ (s_{ij} - p\alpha_{ij})(s_{ij} - p\alpha_{ij}) + (p_o - p)p\alpha_{ij}\alpha_{ij} \} = 0 \quad (5)$$

$$\dot{\alpha} = \langle\bar{\lambda}\rangle \left\{ \frac{1+e_o}{\lambda - \kappa} \left| tr \frac{\partial f}{\partial \sigma_{mn}} \right| \frac{c}{p_o} (s_{ij} - xp\alpha_{ij}) \right\} \quad (6)$$

In the elliptical yield surface in triaxial space(Fig. 1), p_c is the apex of the yield surface and serves as a hardening parameter. The CSL(critical state line) should meet the yield surface at $p_c/2$. For $p \geq p_c/2$, the strain hardening happens, on the other hand, for $p<p_c/2$, the strain softening happens. the yield surface could be used as the perfectly plastic line for strain softening region to prevent tremendous numerical error. From this, we know that the modified Cam-clay model was proposed basically for the normally and isotropically consolidated cohesive soils.



[Fig. 1] Anisotropic Modified Cam-clay Model (Dafalias, 1987) [8]

The yield locus undergoes kinematic hardening in the principal stress space, that means the rotation of the elliptical yield locus in p-q space for which the origin does not change. The shape of the yield locus will be a distorted ellipse when subjected to anisotropic hardening.

One of the strong advantages of the constitutive models, like the modified Cam-clay model in the category of critical state soil mechanics, is that they have relatively few material parameters to be determined compared with their capacity of simulation.

2.2 Viscous and Rate-sensitive Modeling

The generalized viscous relation, eq.(7), proposed by Perzyna(1963, 1966) was used for the simulation of the rate-dependent stress-strain behavior, encouraged by the past successful use of it.

$$\dot{\varepsilon}^{v} = \langle \Phi \rangle \frac{\partial f}{\partial \sigma_{ij}} \tag{7}$$

In eq. (7), the overstress function Φ serves not only the viscous behavior but also the plastic behavior through the coupling effect. Eq. (8) has been widely adopted as the overstress function for cohesive soils[9][10].

$$\Phi = \frac{1}{\hat{V}} \exp\left(J_2/NI_1\right) \Delta\hat{\sigma}^{n} \tag{8}$$

where J_2 is the second invariant of the deviatoric stress tensor, I_1 is the first invariant of the stress tensor, and N is the slope of the critical state line in I_1-J_2 space. $\Delta\hat{\sigma}$ is called normalized overstress and obtained through a closed form solution using the viscous nucleus parameter s_v. $\hat{V}$ has a mathematical form but was tried as a parameter, like n, in this study to avoid complexity.

The generalized viscous theory has so similar format, in which its conceptual and mathematical derivations are described, with the classical plastic theory that both theories might be well formulated with each other.

The rate-sensitive relation as in eq. (9), proposed by Adachi et al.(1974, 1982), was incorporated in the generalized viscous constitutive equation. That results to the convenience in practical use of the constitutive equation in that it is not necessary to determine the parameter values each time rate changes[6][7]. Owing to the incorporation of Adachi's rate-sensitive relation into the constitutive equation, the material parameters determined at a rate(specifically standard rate 0.001%/min.) are expected to be used for the simulation at different rates.

$$\Phi = \frac{1}{\hat{V}} exp\left(\frac{\frac{1}{2}\left[\frac{N\times\frac{1}{3}I_1}{m'} ln\frac{\dot{\epsilon}_{11}^{(1)}}{\dot{\epsilon}_{11}^{(2)}} - \sqrt{2J_2^{(1)}}\right]^2}{NI_1} \right) \times \left(\Delta\hat{\sigma}\right)^{n} \tag{9}$$

where m' is the material parameter determining the slope of the line connecting the maximum deviatoric stresses to the corresponding strain rates. The superscripts (1) and (2) respectively denote something at a rate (1) and different rate (2). The $\dot{\epsilon}_{11}^{(1)}$ means the strain rate (1).

Substituting eq. (5) with associated flow rule, eq. (7) with the rate-sensitive overstress function eq. (9), and the generalized Hooke's law into the formulation eq. (10), a rate-sensitive constitutive equation is obtained in the format of the visco-elasto-plastic relation.

$$\dot{\epsilon} = \dot{\epsilon}^{e} + \dot{\epsilon}^{p} + \dot{\epsilon}^{v} \tag{10}$$

where the superscripts e, p, and v respectively represent elastic, plastic, and viscous parts.

3. Verification of the Model

The experimental results from the Chan typed triaxial testing apparatus were used to be compared with the simulations performed using the developed model stated above. A series of the Ko triaxial compression tests(Ko=0.48) have been performed under a well-calibrated condition[11]. The kaolinite clay specimens with 67.15% liquid limit and 30.75% plastic limit, produced from the slurry consolidation technique, have been tested under the combined condition of the rates(0.005%/min, 0.05%/min, 0.5%/min) and the vertical consolidation stresses(100kPa, 200kPa, 300kPa)

Table 1 shows the material parameter values. M, λ, and κ are the traditional parameters in the critical state soil mechanics. c and x are the anisotropic hardening parameters for lthe anisotropic modified Cam-clay model. s_v to n are for the generalized viscous theory and m' is the Adachi's rate-sensitive parameter. Their identifications are presented briefly in the above section and their values were determined from the standard triaxial test, 0.005%/min in this study, and standard consolidation test. Details on the parameters including parameter sensitivity are described in the successive paper.

In Table 2, e_o means the initial void ratio and r indicates $\frac{\dot{\epsilon}_{11}^{(1)}}{\dot{\epsilon}_{11}^{(2)}}$ in eq. (9). The rate 0.005%/min was taken into account as $\dot{\epsilon}_{11}^{(1)}$ since it is close to the standard rate 0.001%/min even so. Accordingly, the parameter values determined at the rate of 0.005%/min were commonly used for the simulations at all the three rates.

[Table 1] Material Parameter Values

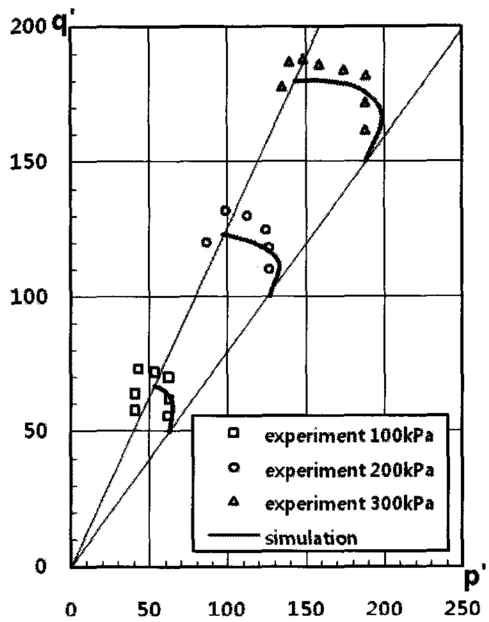
parameter	value
M	1.26
λ	0.15
κ	0.02
c	0.1
x	0.1
s_v	2.00
$\hat{V}$	4×10^8
n	3.00
m'	60

[Table 2] Input Values

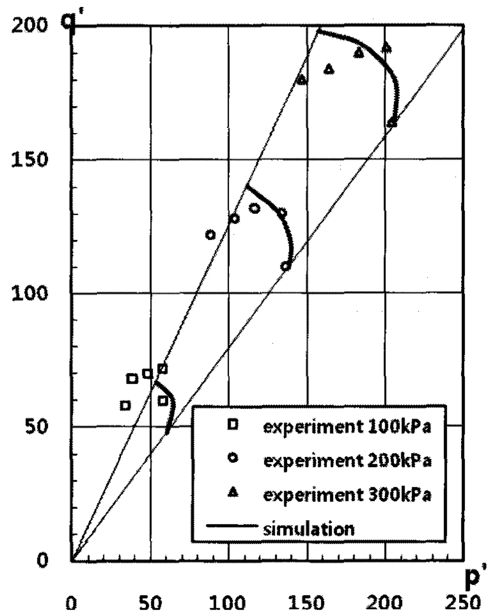
Input	value
e_o	1.3
$r(0.005\%/min)$	1
$r(0.05\%/min)$	0.1
$r(0.5\%/min)$	0.01

Figure 1 presents the comparisons of the simulations conducted using the developed model with the experimental results at various rates and vertical consolidation stresses. The p' and q' respectively denote the effective mean normal stress and the principal stress difference.

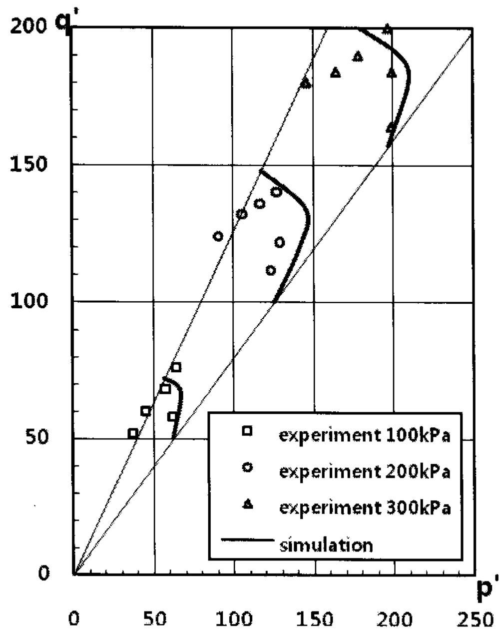
The simulations represent generally good agreement with the experimental data, specially for the case of rate 0.005%/min, at which the material parameters used for all the simulations were evaluated. The simulations at the rates of 0.05%/min and 0.5%/min are thought comprehensively acceptable though they are not as accurate as those at rate 0.005%/min.



(a) rate=0.005%/min.



(b) rate=0.05%/min.



(c) rate=0.5%/min.

[Figure 1] Experimental Results and Simulations at Various Rates

The simulations seem to match with the experimental data for the initial parts of the stress paths; however, the purely elastic response is so short and fast that the discrepancy between the simulations and the experimental results could not be sufficiently appreciated. This is actually true for the most cases of stress paths.

The general shapes of the simulations for the 200kPa and 300kPa show the very typical curves of the stress paths for the normally consolidated soils, namely, the curves gradually increase without a remarkable peak strength. For the 100kPa, the simulations reflect the normally consolidated case just like for cases 200kPa and 300kPa but the experimental results present more remarkable peak strength then decrease. This resulted from the experimental process that the specimens were prepared under slurry consolidation stress 120kPa before the triaxial testing, which means the case of slightly overconsolidation.

The latter parts of the simulations are influenced naturally by the parameter M. The discrepancy between the simulations and the experimental results are mainly due to the reliability and consistency of the experimental data in the critical phase. It is shown in Figure 1 that each stress path has its own M different from the others. The M for 100kPa is higher than the others, and M for 300kPa is lower than the others. The parameter $\hat{V}$ is supposed to be obtained a closed form solution but it was assumed to be a single parameter in this study. This assumption might be judged somewhat proper in this simulations.

4. Conclusions

A rate-sensitive constitutive equation was developed in the frame of the elastoplastic-viscous relation. The equation successfully simulated the stress paths for the normally consolidated cohesive soils with the advantage that the parameter values determined at a rate could be used at different rates.

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<Research Area>
Soil&Foundation, Soft Soils, Ground Exploration&Testing, Numerical Analysis, Constitutive Relations