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REAL n-DIMENSIONAL QR-SUBMANIFOLDS OF MAXIMAL QR-DIMENSION IMMERSED IN $QP^{(n+p)/4}$

Hyang Sook Kim, Jung-Hwan Kwon, and Jin Suk Pak

Abstract. The purpose of this paper is to study n-dimensional QR-submanifolds of $(p-1)$ QR-dimension immersed in a quaternionic projective space $QP^{(n+p)/4}$ of constant Q-sectional curvature 4 and especially to determine such submanifolds under the additional condition concerning with shape operator.

1. Introduction

Let M be an n-dimensional QR-submanifold of $(p-1)$ QR-dimension isometrically immersed in a quaternionic space form $\overline{M}^{(n+p)/4}(c)$. Denoting by $\{F,G,H\}$ the quaternionic structure of $\overline{M}^{(n+p)/4}(c)$, it follows by definition (cf. [2, 6]) that there exists a $(p-1)$-dimensional subbundle ν of the normal bundle $TM^{\perp}$ such that

$$(1.1)\qquad \begin{cases} F\nu_x \subset \nu_x, \quad G\nu_x \subset \nu_x, \quad H\nu_x \subset \nu_x, \\ F\nu_x^{\perp} \subset T_xM, \quad G\nu_x^{\perp} \subset T_xM, \quad H\nu_x^{\perp} \subset T_xM \end{cases}$$

for each $x \in M$, where $\nu^{\perp}$ denotes the complementary orthogonal subbundle to ν in $TM^{\perp}$. Thus there is a naturally distinguished unit normal vector field ξ to M such that $\nu_x^{\perp} = \mathrm{Span}\{\xi\}$ for each $x \in M$ and the vector fields U, V, W defined by

$$(1.2)\qquad U = -F\xi, \quad V = -G\xi, \quad W = -H\xi$$

are tangent to M. On the other hand, each tangent space T_xM is decomposed as

$$T_xM = D_x \oplus D_x^{\perp},$$

where D_x is the maximal quaternionic invariant subspace of T_xM defined by

$$D_x = T_xM \cap FT_xM \cap GT_xM \cap HT_xM$$

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and $D_x^\perp$ its orthogonal complement in T_xM. But, as already shown in [2, 6], $D_x^\perp = \mathrm{Span}\{U, V, W\}$ and so $D : x \mapsto D_x$ defines an $(n-3)$-dimensional distribution on M. Furthermore it is clear that

$$FT_xM,\ GT_xM,\ HT_xM\ \subset T_xM \oplus \mathrm{Span}\{\xi\}$$

and consequently, for any tangent vector X to M, we have the following decomposition in tangential and normal components

$$\begin{aligned} FX &= \phi X + u(X)\xi, \quad GX = \psi X + v(X)\xi, \\ HX &= \theta X + w(X)\xi. \end{aligned} \tag{1.3}$$

By means of the hermitian property of $\{F, G, H\}$ it can be easily shown that ϕ, ψ and θ are skew-symmetric endomorphisms acting on T_xM.

Under those circumstance in our previous paper [6], we studied n-dimensional QR-submanifolds of $(p-1)$ QR-dimension immersed in a quaternionic projective space $QP^{(n+p)/4}$ and proved

Theorem K-P. *Let M be an n-dimensional QR-submanifold of $(p-1)$ QR-dimension in a quaternionic projective space $QP^{(n+p)/4}(4)$ and let the normal vector field ξ be parallel with respect to the normal connection. If*

$$A_1\phi = \phi A_1, \quad A_1\psi = \psi A_1, \quad A_1\theta = \theta A_1$$

on M, then $\pi^{-1}(M)$ is locally a product of $M_1 \times M_2$ where M_1 and M_2 belong to some $(4n_1+3)$- and $(4n_2+3)$-dimensional spheres and A_1 denotes the shape operator corresponding to ξ (π is the Hopf fibration $S^{n+p+3}(1) \to QP^{(n+p)/4}$).

In this paper we shall prove the following theorems as improvements of Theorem K-P :

Theorem 1. *Let M be an n-dimensional QR-submanifold of $(p-1)$ QR-dimension in a quaternionic projective space $QP^{(n+p)/4}(4)$ and let the normal vector field ξ be parallel with respect to the normal connection. If*

$$\overset{\circ}{\mathcal{L}}_U\, A_1 = 0 \quad \overset{\circ}{\mathcal{L}}_V\, A_1 = 0, \quad \overset{\circ}{\mathcal{L}}_W\, A_1 = 0$$

on M, where $\overset{\circ}{\mathcal{L}}_U$, $\overset{\circ}{\mathcal{L}}_V$ and $\overset{\circ}{\mathcal{L}}_W$ denote the operators defined by (4.1) and A_1 the shape operator corresponding to ξ, then $\pi^{-1}(M)$ is locally a product of $M_1 \times M_2$ where M_1 and M_2 belong to some $(4n_1+3)$- and $(4n_2+3)$-dimensional spheres.

Theorem 2. *Let M be an $n(>3)$-dimensional QR-submanifold of $(p-1)$ QR-dimension in a quaternionic projective space $QP^{(n+p)/4}(4)$ and let the normal vector field ξ be parallel with respect to the normal connection. Suppose that*

$$\overset{\circ}{\mathcal{L}}_U\, h_1 = 0, \quad \overset{\circ}{\mathcal{L}}_V\, h_1 = 0, \quad \overset{\circ}{\mathcal{L}}_W\, h_1 = 0$$

on M, where $\overset{\circ}{\mathcal{L}}_U$, $\overset{\circ}{\mathcal{L}}_V$ and $\overset{\circ}{\mathcal{L}}_W$ denote the operators defined by (5.1), respectively and $h_1(X,Y) = g(A_1X, Y)$, A_1 being the shape operator corresponding to ξ. If the function λ appeared in (3.10) is not zero at some point of M and if U, V

and W are the eigenvectors of A_1, then $\pi^{-1}(M)$ is locally a product of $M_1 \times M_2$ where M_1 and M_2 belong to some $(4n_1+3)$- and $(4n_2+3)$-dimensional spheres.

2. Fundamental formulas for QR-submanifolds

Let $\overline{M}^{(n+p)/4}(c)$ be a real $(n+p)$-dimensional quaternionic space form, that is, quaternionic Kähler manifold with constant Q-sectional curvature c (cf. [4, 5]). Then, by definition, there is a 3-dimensional vector bundle V consisting with tensor fields of type (1,1) over $\overline{M}$ satisfying the following conditions (a), (b) and (c):

(a) In any coordinate neighborhood $\overline{\mathcal{U}}$, there is a local basis $\{F,\ G,\ H\}$ of V such that

$$\begin{cases} F^2 = -I,\ G^2 = -I,\ H^2 = -I, \\ FG = -GF = H,\ GH = -HG = F,\ HF = -FH = G. \end{cases} \tag{2.1}$$

(b) There is a Riemannian metric g which is hermite with respect to all of F, G and H.

(c) For the Riemannian connection $\overline{\nabla}$ with respect to g

$$\begin{pmatrix} \overline{\nabla}F \\ \overline{\nabla}G \\ \overline{\nabla}H \end{pmatrix} = \begin{pmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{pmatrix} \begin{pmatrix} F \\ G \\ H \end{pmatrix}, \tag{2.2}$$

where p, q and r are local 1-forms defined in $\overline{\mathcal{U}}$. Such a local basis $\{F, G, H\}$ is called a *canonical local basis* of the bundle V in $\overline{\mathcal{U}}$ (cf. [4, 5]).

For canonical local bases $\{F_1, G_1, H_1\}$ and $\{F_2, G_2, H_2\}$ of V in coordinate neighborhoods $\overline{\mathcal{U}_1}$ and $\overline{\mathcal{U}_2}$, it follows that in $\overline{\mathcal{U}_1} \cap \overline{\mathcal{U}_2}$

$$\begin{pmatrix} F_2 \\ G_2 \\ H_2 \end{pmatrix} = (s_{xy}) \begin{pmatrix} F_1 \\ G_1 \\ H_1 \end{pmatrix} \qquad (x,\ y = 1,\ 2,\ 3) \tag{2.3}$$

with differentiable functions s_{xy}, where the matrix $S = (s_{xy})$ is contained in $SO(3)$ as a consequence of (2.1). As is well known (cf. [4, 5]), every quaternionic Kähler manifold is orientable.

From now on we consider a real n-dimensional QR-submanifold M of $(p-1)$ QR-dimension immersed in $QP^{(n+p)/4}(c)$ and use the same notations as in Section 1. we now take a local orthonormal basis $\{\xi_\alpha; \alpha = 1, \ldots, p\}$ $(\xi_1 = \xi)$ of normal vectors to M consider the following decomposition in tangential and normal components:

$$\begin{aligned} &F\xi_\alpha = -U_\alpha + P_1\xi_\alpha, \quad G\xi_\alpha = -V_\alpha + P_2\xi_\alpha, \\ &H\xi_\alpha = -W_\alpha + P_3\xi_\alpha \end{aligned} \tag{2.4}$$

$(\alpha = 1, \ldots, p)$. Then P_1, P_2 and P_3 are skew-symmetric endomorphisms acting on $T_xM^\perp$. Moreover, by means of (1.3), $(2.1)_{(b)}$ and (2.4) imply

$$(2.5)\qquad \begin{cases} g(X, \phi U_\alpha) = -u(X)g(\xi_1, P_1\xi_\alpha), \\ g(X, \psi V_\alpha) = -v(X)g(\xi_1, P_2\xi_\alpha), \quad \alpha = 1, \ldots, p, \\ g(X, \theta W_\alpha) = -w(X)g(\xi_1, P_3\xi_\alpha), \end{cases}$$

$$(2.6)\qquad \begin{cases} g(U_\alpha, U_\beta) = \delta_{\alpha\beta} - g(P_1\xi_\alpha, P_1\xi_\beta), \\ g(V_\alpha, V_\beta) = \delta_{\alpha\beta} - g(P_2\xi_\alpha, P_2\xi_\beta), \quad \alpha, \beta = 1, \ldots, p. \\ g(W_\alpha, W_\beta) = \delta_{\alpha\beta} - g(P_3\xi_\alpha, P_3\xi_\beta), \end{cases}$$

Also, from $g(FX, \xi_\alpha) = -g(X, F\xi_\alpha)$, $g(GX, \xi_\alpha) = -g(X, G\xi_\alpha)$ and $g(HX, \xi_\alpha) = -g(X, H\xi_\alpha)$, it follows that

$$g(X, U_\alpha) = u(X)\delta_{1\alpha}, \quad g(X, V_\alpha) = v(X)\delta_{1\alpha}, \quad g(X, W_\alpha) = w(X)\delta_{1\alpha}$$

and hence

$$(2.7)\qquad \begin{aligned} &g(U_1, X) = u(X),\ g(V_1, X) = v(X),\ g(W_1, X) = w(X), \\ &U_\alpha = 0,\ V_\alpha = 0,\ W_\alpha = 0,\ \alpha = 2, \ldots, p. \end{aligned}$$

On the other hand, comparing (1.2) and (2.4) with $\alpha = 1$, we have $U_1 = U$, $V_1 = V$, $W_1 = W$, which and (2.7) imply

$$(2.8)\qquad g(U, X) = u(X), \quad g(V, X) = v(X) \quad g(W, X) = w(X).$$

In the sequel we shall use the notations U, V, W instead of U_1, V_1, W_1.

Next, applying F to the first equation of (1.3) and using (2.4), (2.7) and (2.8), we have

$$\phi^2 X = -X + u(X)U, \quad u(X)P_1\xi = -u(\phi X)\xi.$$

Similarly we have

$$(2.9)\qquad \begin{aligned} &\phi^2 X = -X + u(X)U,\ \psi^2 X = -X + v(X)V, \\ &\theta^2 X = -X + w(X)W, \end{aligned}$$

$$(2.10)\qquad \begin{aligned} &u(X)P_1\xi = -u(\phi X)\xi,\ v(X)P_2\xi = -v(\psi X)\xi, \\ &w(X)P_3\xi = -w(\theta X)\xi, \end{aligned}$$

from which, taking account of the skew-symmetry of P_1, P_2 and P_3 and using (2.5), we also have

$$(2.11)\qquad \begin{cases} u(\phi X) = 0, \quad v(\psi X) = 0, \quad w(\theta X) = 0, \\ \phi U = 0, \quad \psi V = 0, \quad \theta W = 0, \\ P_1\xi = 0, \quad P_2\xi = 0, \quad P_3\xi = 0. \end{cases}$$

So (2.4) can be rewritten in the form

$$(2.12)\qquad \begin{aligned} &F\xi = -U, \quad G\xi = -V, \quad H\xi = -W, \\ &F\xi_\alpha = P_1\xi_\alpha, \quad G\xi_\alpha = P_2\xi_\alpha, \quad H\xi_\alpha = P_3\xi_\alpha, \end{aligned}$$

where $\alpha = 2, \dots, p$.

Applying G and H to the first equation of (1.3) and using (1.3), (2.1) and (2.12), we have

$$\theta X + w(X)\xi = -\psi(\phi X) - v(\phi X)\xi + u(X)V,$$
$$\psi X + v(X)\xi = \theta(\phi X) + w(\phi X)\xi - u(X)W,$$

and consequently

$$\psi(\phi X) = -\theta X + u(X)V, \quad v(\phi X) = -w(X),$$
$$\theta(\phi X) = \psi X + u(X)W, \quad w(\phi X) = v(X). \tag{2.13}$$

From the other equations of (1.3) we have by quite similar method

$$\phi(\psi X) = \theta X + v(X)U, \quad u(\psi X) = w(X),$$
$$\theta(\psi X) = -\phi X + v(X)W, \quad w(\psi X) = -u(X), \tag{2.14}$$

$$\phi(\theta X) = -\psi X + w(X)U, \quad u(\theta X) = -v(X),$$
$$\psi(\theta X) = \phi X + w(X)V, \quad v(\theta X) = u(X). \tag{2.15}$$

From the first three equations of (2.12), we also have

$$\begin{cases} \psi U = -W, \quad v(U) = 0, \quad \theta U = V, \quad w(U) = 0, \\ \phi V = W, \quad u(V) = 0, \quad \theta V = -U, \quad w(V) = 0, \\ \phi W = -V, \quad u(W) = 0, \quad \psi W = U, \quad v(W) = 0. \end{cases} \tag{2.16}$$

On the other hand, we may put

(2.17)

$$P_1\xi_\alpha = \sum_{\beta=2}^{p} P_{1\alpha\beta}\xi_\beta, \quad P_2\xi_\alpha = \sum_{\beta=2}^{p} P_{2\alpha\beta}\xi_\beta, \quad P_3\xi_\alpha = \sum_{\beta=2}^{p} P_{3\alpha\beta}\xi_\beta, \quad (\alpha = 2, \dots, p)$$

from which, substituting in the last three equations of (2.12) and using the hermitian property of $\{F, G, H\}$, we have

(2.18)

$$\sum_\gamma P_{1\alpha\gamma}P_{1\gamma\beta} = -\delta_{\alpha\beta}, \quad \sum_\gamma P_{2\alpha\gamma}P_{2\gamma\beta} = -\delta_{\alpha\beta}, \quad \sum_\gamma P_{3\alpha\gamma}P_{3\gamma\beta} = -\delta_{\alpha\beta}.$$

Also, from (2.1), (2.12) and (2.17), we have

$$\begin{cases} \sum_\beta P_{1\alpha\beta}P_{2\beta\gamma} = -P_{3\alpha\gamma}, \quad \sum_\beta P_{1\alpha\beta}P_{3\beta\gamma} = P_{2\alpha\gamma}, \\ \sum_\beta P_{2\alpha\beta}P_{3\beta\gamma} = -P_{1\alpha\gamma}, \quad \sum_\beta P_{2\alpha\beta}P_{1\beta\gamma} = P_{3\alpha\gamma}, \\ \sum_\beta P_{3\alpha\beta}P_{1\beta\gamma} = -P_{2\alpha\gamma}, \quad \sum_\beta P_{3\alpha\beta}P_{2\beta\gamma} = P_{1\alpha\gamma}. \end{cases} \tag{2.19}$$

The equations (2.6)-(2.11) and (2.13)-(2.16) tell us that M admits the so-called almost contact 3-structure (for definition, see [5, 6, 12]) and consequently $n = 4m + 3$ for some integer m.

Now let ∇ be the Levi-Civita connection on M and let $\nabla^\perp$ the normal connection induced from $\overline{\nabla}$ in the normal bundle $TM^\perp$ of M. Then Gauss and Weingarten formulae are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y), \tag{2.20}$$

$$\overline{\nabla}_X \xi_\alpha = -A_\alpha X + \nabla^\perp_X \xi_\alpha, \quad \alpha = 1, \dots, p \tag{2.21}$$

for X, Y tangent to M. Here h denotes the second fundamental form and A_α the shape operator corresponding to ξ_α. They are related by $h(X,Y) = \sum_{\alpha=1}^p g(A_\alpha X, Y)\xi_\alpha$. Furthermore, put

$$\nabla^\perp_X \xi_\alpha = \sum_{\beta=1}^p s_{\alpha\beta}(X)\xi_\beta, \tag{2.22}$$

where $(s_{\alpha\beta})$ is the skew-symmetric matrix of connection forms of $\nabla^\perp$. Finally, if the ambient manifold $\overline{M}$ is of constant Q-sectional curvature c, the equations of Gauss, Codazzi and Ricci are respectively given as follows:

$$\begin{aligned} R(X,Y)Z = \frac{c}{4}\{&g(Y,Z)X - g(X,Z)Y \\ &+ g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y - 2g(\phi X, Y)\phi Z \\ &+ g(\psi Y, Z)\psi X - g(\psi X, Z)\psi Y - 2g(\psi X, Y)\psi Z \\ &+ g(\theta Y, Z)\theta X - g(\theta X, Z)\theta Y - 2g(\theta X, Y)\theta Z\} \\ &+ \sum_\alpha g(A_\alpha Y, Z)A_\alpha X - \sum_\alpha g(A_\alpha X, Z)A_\alpha Y, \end{aligned} \tag{2.23}$$

$$\begin{aligned} &g((\nabla_X A_1)Y - (\nabla_Y A_1)X, Z) \\ &= \frac{c}{4}\{g(\phi Y, Z)u(X) - g(\phi X, Z)u(Y) - 2g(\phi X, Y)u(Z) \\ &\quad + g(\psi Y, Z)v(X) - g(\psi X, Z)v(Y) - 2g(\psi X, Y)v(Z) \\ &\quad + g(\theta Y, Z)w(X) - g(\theta X, Z)w(Y) - 2g(\theta X, Y)w(Z)\} \\ &\quad + \sum_\beta \{g(A_\beta X, Z)s_{\beta 1}(Y) - g(A_\beta Y, Z)s_{\beta 1}(X)\}, \end{aligned} \tag{2.24}$$

$$g(\overline{R}(X,Y)\xi_\alpha, \xi_\beta) = g(R^\perp(X,Y)\xi_\alpha, \xi_\beta) + g([A_\beta, A_\alpha]X, Y) \tag{2.25}$$

for X, Y, Z tangent to M, where $\overline{R}$ and R denote the Riemannian curvature tensor of $\overline{M}$ and M, respectively and $R^\perp$ is the curvature tensor of the normal connection $\nabla^\perp$.

3. Some properties of the shape operator A_1

In this section, we assume that the ambient manifold $\overline{M}$ is a quaternionic space form $\overline{M}^{(n+p)/4}(c)$ of constant Q-sectional curvature c.

Differentiating the first equation of (1.3) covariantly and using (1.3), (2.2), (2.4), (2.7), (2.20) and (2.21), we have

$$(3.1)\qquad \begin{aligned}(\nabla_Y\phi)X &= r(Y)\psi X - q(Y)\theta X + u(X)A_1Y - g(A_1Y,X)U,\\ (\nabla_Y u)X &= r(Y)v(X) - q(Y)w(X) + g(\phi A_1Y,X).\end{aligned}$$

From the other equations of (1.3) we also have

$$(3.2)\qquad \begin{aligned}(\nabla_Y\psi)X &= -r(Y)\phi X + p(Y)\theta X + v(X)A_1Y - g(A_1Y,X)V,\\ (\nabla_Y v)X &= -r(Y)u(X) + p(Y)w(X) + g(\psi A_1Y,X),\end{aligned}$$

$$(3.3)\qquad \begin{aligned}(\nabla_Y\theta)X &= q(Y)\phi X - p(Y)\psi X + w(X)A_1Y - g(A_1Y,X)W,\\ (\nabla_Y w)X &= q(Y)u(X) - p(Y)v(X) + g(\theta A_1Y,X).\end{aligned}$$

Next, differentiating the first equation of (2.12) covariantly and comparing the tangential and normal parts, we have

$$(3.4)\qquad \begin{cases}\nabla_Y U &= r(Y)V - q(Y)W + \phi A_1 Y,\\ g(A_\alpha U,Y) &= -\sum_{\beta=2}^{p} s_{1\beta}(Y)P_{1\beta\alpha}, \quad \alpha = 2,\dots,p.\end{cases}$$

From the other equations of (2.12), we have similarly

$$(3.5)\qquad \begin{cases}\nabla_Y V &= -r(Y)U + p(Y)W + \psi A_1 Y,\\ g(A_\alpha V,Y) &= -\sum_{\beta=2}^{p} s_{1\beta}(Y)P_{2\beta\alpha}, \quad \alpha = 2,\dots,p,\end{cases}$$

$$(3.6)\qquad \begin{cases}\nabla_Y W &= q(Y)U - p(Y)V + \theta A_1 Y,\\ g(A_\alpha W,Y) &= -\sum_{\beta=2}^{p} s_{1\beta}(Y)P_{3\beta\alpha}, \quad \alpha = 2,\dots,p.\end{cases}$$

In what follows *we assume that the distinguished normal vector field ξ is parallel with respect to the normal connection, that is, $\nabla^{\perp}_X\xi = 0$.* Hence it follows from (2.22) that $s_{\beta 1} = 0$, $\beta = 2,\dots,p$, and consequently

$$A_\alpha U = 0,\ A_\alpha V = 0,\ A_\alpha W = 0, \qquad \alpha = 2,\dots,p$$

because of (3.4)-(3.6).

We now put

$$T := \nabla_U U + \nabla_V V + \nabla_W W + (\mathrm{div}U)U + (\mathrm{div}V)V + (\mathrm{div}W)W$$

and take an orthonormal basis

$$\{e_a,\ e_{a^*},\ e_{a^{**}},\ e_{a^{***}},\ e_{4m+1} = U,\ e_{4m+2} = V,\ e_{4m+3} = W\}_{a=1,\dots,m}$$

of tangent vectors to M such that

$$e_{a^*} := \phi e_a, \quad e_{a^{**}} := \psi e_a, \quad e_{a^{***}} := \theta e_a, \quad a = 1,\dots,m.$$

Then it follows from (2.6), (2.7), (2.10) and (2.12)-(2.15) that

$$(3.7)\qquad T = \phi A_1 U + \psi A_1 V + \theta A_1 W,$$

$$(3.8)\qquad g(T,U) = g(T,V) = g(T,W) = 0,$$

$$(3.9)\qquad g(T,T) = \mu - \lambda^2 + 2\{g(\phi A_1 V, A_1 W)$$

$$+ g(\psi A_1 W, A_1 U) + g(\theta A_1 U, A_1 V)\},$$

where

(3.10) $\mu = u(A_1{}^2 U)+v(A_1{}^2 V)+w(A_1{}^2 W),\ \lambda = u(A_1 U)+v(A_1 V)+w(A_1 W).$

Moreover it is clear that

$$\text{(3.11)}\qquad \begin{cases} \phi T &= -A_1 U + \lambda U - \psi A_1 W + \theta A_1 V, \\ \psi T &= -A_1 V + \lambda V - \theta A_1 U + \phi A_1 W, \\ \theta T &= -A_1 W + \lambda W - \phi A_1 V + \psi A_1 U. \end{cases}$$

For later use we compute

$$\begin{aligned} &\operatorname{div} T \\ &= g(U, \nabla_U T) + g(V, \nabla_V T) + g(W, \nabla_W T) \\ &\quad + \sum_{a=1}^{m} \{g(e_a, \nabla_{e_a} T) + g(e_{a^*}, \nabla_{e_{a^*}} T) + g(e_{a^{**}}, \nabla_{e_{a^{**}}} T) + g(e_{a^{***}}, \nabla_{e_{a^{***}}} T)\}. \end{aligned}$$

Differentiating (3.7) covariantly and using (2.22)-(2.25), we have

$$\text{(3.12)}\qquad \begin{aligned} \nabla_X T = {} & \lambda A_1 X - g(A_1{}^2 U, X)U - g(A_1{}^2 V, X)V - g(A_1{}^2 W, X)W \\ & + \phi A_1 \phi A_1 X + \psi A_1 \psi A_1 X + \theta A_1 \theta A_1 X \\ & + \phi(\nabla_X A_1)U + \psi(\nabla_X A_1)V + \theta(\nabla_X A_1)W, \end{aligned}$$

from which, taking account of (2.6)-(2.8), (2.10) and (2.12)-(2.15),

$$\begin{aligned} &\operatorname{div} T \\ &= \lambda \operatorname{tr} A_1 - \mu + \sum_{i=1}^{n} \{g(\phi A_1 \phi A_1 e_i, e_i) + g(\psi A_1 \psi A_1 e_i, e_i) + g(\theta A_1 \theta A_1 e_i, e_i)\} \\ &\quad - \sum_{a=1}^{m} \{g((\nabla_{e_a} A_1)e_{a^*} - (\nabla_{e_{a^*}} A_1)e_a + (\nabla_{e_{a^{**}}} A_1)e_{a^{***}} - (\nabla_{e_{a^{***}}} A_1)e_{a^{**}}, U) \\ &\quad + g((\nabla_{e_a} A_1)e_{a^{**}} - (\nabla_{e_{a^{**}}} A_1)e_a + (\nabla_{e_{a^{***}}} A_1)e_{a^*} - (\nabla_{e_{a^*}} A_1)e_{a^{***}}, V) \\ &\quad + g((\nabla_{e_a} A_1)e_{a^{***}} - (\nabla_{e_{a^{***}}} A_1)e_a + (\nabla_{e_{a^*}} A_1)e_{a^{**}} - (\nabla_{e_{a^{**}}} A_1)e_{a^*}, W)\} \\ &\quad - g((\nabla_V A_1)W - (\nabla_W A_1)V, U) - g((\nabla_W A_1)U - (\nabla_U A_1)W, V) \\ &\quad - g((\nabla_U A_1)V - (\nabla_V A_1)U, W), \end{aligned}$$

or equivalently

(3.13)

$$\begin{aligned} \operatorname{div} T = {} & \lambda \operatorname{tr} A_1 - \mu + \frac{3(n-3)}{4} c \\ & + \sum_{i=1}^{n} \{g(\phi A_1 \phi A_1 e_i, e_i) + g(\psi A_1 \psi A_1 e_i, e_i) + g(\theta A_1 \theta A_1 e_i, e_i)\} \end{aligned}$$

because of (2.24) with $s_{\beta 1} = 0$.

On the other hand, by using (2.6)-(2.8), (2.10) and (2.12)-(2.15) we can easily show that

(3.14)
$$\|\phi A_1 - A_1\phi\|^2 + \|\psi A_1 - A_1\psi\|^2 + \|\theta A_1 - A_1\theta\|^2$$
$$= 6\,\mathrm{tr}A_1{}^2 - 2\mu + 2\sum_{i=1}^{n}\{g(\phi A_1\phi A_1 e_i, e_i) + g(\psi A_1\psi A_1 e_i, e_i) + g(\theta A_1\theta A_1 e_i, e_i)\},$$

from which, combining with (3.13),

$$\begin{aligned}\text{div } T = &\frac{1}{2}\{\|\phi A_1 - A_1\phi\|^2 + \|\psi A_1 - A_1\psi\|^2 + \|\theta A_1 - A_1\theta\|^2\}\\ &+ \frac{3(n-3)}{4}c - 3\ trA_1{}^2 + \lambda\ trA_1.\end{aligned} \tag{3.15}$$

Remark. Let M be an $n(>3)$-dimensional QR-submanifold of $(p-1)$ QR-dimension in a quaternionic space form $\overline{M}^{(n+p)/4}(c)$ $(c<0)$ and let the normal vector field ξ be parallel with respect to the normal connection. If the commutativity conditions

$$\phi A_1 = A_1\phi, \quad \psi A_1 = A_1\psi, \quad \theta A_1 = A_1\theta$$

hold on M, then the vector field $T = 0$ and consequently it follows from (3.15) that the function $\lambda\ trA_1$ cannot be zero at any point in M. Hence M cannot be minimal.

4. Proof of Theorem 1

We define

$$\begin{pmatrix}\overset{\circ}{\mathcal{L}}_U X\\ \overset{\circ}{\mathcal{L}}_V X\\ \overset{\circ}{\mathcal{L}}_W X\end{pmatrix} = \begin{pmatrix}\mathcal{L}_U X\\ \mathcal{L}_V X\\ \mathcal{L}_W X\end{pmatrix} + \begin{pmatrix}0 & r(X) & -q(X)\\ -r(X) & 0 & p(X)\\ q(X) & -p(X) & 0\end{pmatrix}\begin{pmatrix}U\\ V\\ W\end{pmatrix}, \tag{4.1}$$

where $\mathcal{L}_U$ denotes the Lie differentiation with respect to U.

If we put

$$\begin{cases}(\overset{\circ}{\mathcal{L}}_U A_1)X := \overset{\circ}{\mathcal{L}}_U (A_1X) - A_1\overset{\circ}{\mathcal{L}}_U X,\\ (\overset{\circ}{\mathcal{L}}_V A_1)X := \overset{\circ}{\mathcal{L}}_V (A_1X) - A_1\overset{\circ}{\mathcal{L}}_V X,\\ (\overset{\circ}{\mathcal{L}}_W A_1)X := \overset{\circ}{\mathcal{L}}_W (A_1X) - A_1\overset{\circ}{\mathcal{L}}_W X,\end{cases}$$

then it follows from (3.4)-(3.6) that

$$\begin{cases}(\overset{\circ}{\mathcal{L}}_U A_1)X = (\nabla_U A_1)X - \phi A_1{}^2X + A_1\phi A_1X,\\ (\overset{\circ}{\mathcal{L}}_V A_1)X = (\nabla_V A_1)X - \psi A_1{}^2X + A_1\psi A_1X,\\ (\overset{\circ}{\mathcal{L}}_W A_1)X = (\nabla_W A_1)X - \theta A_1{}^2X + A_1\theta A_1X.\end{cases} \tag{4.2}$$

From now on we assume that

$$\overset{\circ}{\mathcal{L}}_U A_1 = 0, \quad \overset{\circ}{\mathcal{L}}_V A_1 = 0, \quad \overset{\circ}{\mathcal{L}}_W A_1 = 0. \tag{4.3}$$

Then (4.2) and (4.3) imply

$$
\text{(4.4)} \qquad \begin{cases} (\nabla_U A_1)X &= \phi {A_1}^2 X - A_1\phi A_1 X, \\ (\nabla_V A_1)X &= \psi {A_1}^2 X - A_1\psi A_1 X, \\ (\nabla_W A_1)X &= \theta {A_1}^2 X - A_1\theta A_1 X, \end{cases}
$$

from which, taking account of (2.24) with $s_{1\beta} = 0$ and using (2.11) and (2.16), we also have

$$
\text{(4.5)} \qquad \begin{aligned} (\nabla_X A_1)U &= \frac{c}{4}\{-\phi X + v(X)W - w(X)V\} + \phi {A_1}^2 X - A_1\phi A_1 X, \\ (\nabla_X A_1)V &= \frac{c}{4}\{-\psi X + w(X)U - u(X)W\} + \psi {A_1}^2 X - A_1\psi A_1 X, \\ (\nabla_X A_1)W &= \frac{c}{4}\{-\theta X + u(X)V - v(X)U\} + \theta {A_1}^2 X - A_1\theta A_1 X. \end{aligned}
$$

Moreover, (2.9), (2.16) and (4.5) give

$$
\begin{aligned} &\phi(\nabla_X A_1)U + \psi(\nabla_X A_1)V + \theta(\nabla_X A_1)W \\ &= \frac{3}{4}c\{X - u(X)U - v(X)V - w(X)W\} - 3{A_1}^2X + u({A_1}^2X)U \\ &\quad + v({A_1}^2X)V + w({A_1}^2X)W - \phi A_1\phi A_1 X - \psi A_1\psi A_1 X - \theta A_1\theta A_1 X, \end{aligned}
$$

which together with (3.12) yields

$$
\nabla_X T = \lambda A_1 X + \frac{3}{4}c\{X - u(X)U - v(X)V - w(X)W\} - 3{A_1}^2X.
$$

Hence div $T = \frac{3(n-3)}{4}c - 3\ \mathrm{tr}{A_1}^2 + \lambda\ \mathrm{tr}A_1$, which and (3.15) imply

$$
\phi A_1 = A_1\phi, \quad \psi A_1 = A_1\psi, \quad \theta A_1 = A_1\theta.
$$

Thus we complete the proof of Theorem 1. □

Corollary 1. *Let M be a real hypersurface of a quaternionic projective space $QP^{(n+p)/4}(4)$. If*

$$
\overset{\circ}{\mathcal{L}}_U\, A_1 = 0 \quad \overset{\circ}{\mathcal{L}}_V\, A_1 = 0, \quad \overset{\circ}{\mathcal{L}}_W\, A_1 = 0
$$

on M, where $\overset{\circ}{\mathcal{L}}_U$, $\overset{\circ}{\mathcal{L}}_V$ and $\overset{\circ}{\mathcal{L}}_W$ denote the operators defined by (4.1) and A_1 the shape operator corresponding to a unit vector field ξ normal to M, then $\pi^{-1}(M)$ is locally a product of $M_1 \times M_2$ where M_1 and M_2 belong to some $(4n_1+3)$- and $(4n_2+3)$-dimensional spheres.

5. Proof of Theorem 2

In this section we want to prove Theorem 2 stated in section 1. First of all we define

$$
\text{(5.1)} \qquad \begin{cases} (\overset{\circ}{\mathcal{L}}_U\, h_1)(X,Y) := Uh_1(X,Y) - h_1(\overset{\circ}{\mathcal{L}}_U\, X, Y) - h_1(X, \overset{\circ}{\mathcal{L}}_U\, Y), \\ (\overset{\circ}{\mathcal{L}}_V\, h_1)(X,Y) := Vh_1(X,Y) - h_1(\overset{\circ}{\mathcal{L}}_V\, X, Y) - h_1(X, \overset{\circ}{\mathcal{L}}_V\, Y), \\ (\overset{\circ}{\mathcal{L}}_W\, h_1)(X,Y) := Wh_1(X,Y) - h_1(\overset{\circ}{\mathcal{L}}_W\, X, Y) - h_1(X, \overset{\circ}{\mathcal{L}}_W\, Y), \end{cases}
$$

where $h_1(X,Y) := g(A_1X,Y)$. In fact, using the definition of $\overset{\circ}{\mathcal{L}}_U\, X$, $\overset{\circ}{\mathcal{L}}_V\, X$ and $\overset{\circ}{\mathcal{L}}_W\, X$, we have

$$
(5.1)' \quad
\begin{aligned}
(\overset{\circ}{\mathcal{L}}_U\, h_1)(X,Y) &= (\mathcal{L}_U h_1)(X,Y) - r(X)h_1(V,Y)\\
&\quad + q(X)h_1(W,Y) - r(Y)h_1(X,V) + q(Y)h_1(X,W),\\
(\overset{\circ}{\mathcal{L}}_V\, h_1)(X,Y) &= (\mathcal{L}_V h_1)(X,Y) + r(X)h_1(U,Y)\\
&\quad - p(X)h_1(W,Y) + r(Y)h_1(X,U) - p(Y)h_1(X,W),\\
(\overset{\circ}{\mathcal{L}}_W\, h_1)(X,Y) &= (\mathcal{L}_W h_1)(X,Y) - q(X)h_1(U,Y)\\
&\quad + p(X)h_1(V,Y) - q(Y)h_1(X,U) + p(Y)h_1(X,V).
\end{aligned}
$$

From now on we assume that

$$
(5.2) \qquad \overset{\circ}{\mathcal{L}}_U\, h_1 = 0, \quad \overset{\circ}{\mathcal{L}}_V\, h_1 = 0, \quad \overset{\circ}{\mathcal{L}}_W\, h_1 = 0.
$$

By means of $(5.1)'$ we can easily obtain that

$$
(5.3) \qquad
\begin{cases}
(\nabla_X A_1)U = -\frac{c}{4}\{\phi X + w(X)V - v(X)W\}, \ \nabla_U A_1 = 0,\\
(\nabla_X A_1)V = -\frac{c}{4}\{\psi X + u(X)W - w(X)U\}, \ \nabla_V A_1 = 0,\\
(\nabla_X A_1)W = -\frac{c}{4}\{\theta X + v(X)U - u(X)V\}, \ \nabla_W A_1 = 0
\end{cases}
$$

and consequently (3.12) reduces to

$$
(5.4) \quad
\begin{aligned}
\nabla_X T = \frac{3}{4}c\{X - u(X)U - v(X)V - w(X)W\} + \lambda A_1 X\\
- g(A_1{}^2U,X)U - g(A_1{}^2V,X)V - g(A_1{}^2W,X)W\\
+ \phi A_1\phi A_1 X + \psi A_1\psi A_1 X + \theta A_1\theta A_1 X.
\end{aligned}
$$

Differentiating the first equation of (5.3) covariantly and using (3.1)-(3.6) and (5.3) itself, we have

$$
\begin{aligned}
&(\nabla_Y\nabla_X A_1)U - (\nabla_{\nabla_Y X}A_1)U + (\nabla_X A_1)\phi A_1 Y\\
&= \frac{c}{4}\{g(A_1Y,X)U - g(\theta A_1Y,X)V + g(\psi A_1Y,X)W\\
&\quad - u(X)A_1Y - w(X)\psi A_1Y + v(X)\theta A_1Y\}
\end{aligned}
$$

and consequently

$$
(5.5) \quad
\begin{aligned}
&R(Y,X)A_1U - A_1R(Y,X)U + (\nabla_X A_1)\phi A_1Y - (\nabla_Y A_1)\phi A_1X\\
&= \frac{c}{4}\{u(Y)A_1X - u(X)A_1Y - v(Y)\theta A_1X + v(X)\theta A_1Y\\
&\quad + w(Y)\psi A_1X - w(X)\psi A_1Y + g(\theta A_1X,Y)V\\
&\quad - g(\theta A_1Y,X)V + g(\psi A_1Y,X)W - g(\psi A_1X,Y)W\}.
\end{aligned}
$$

Similarly from the other equations of (5.3) we have

$$
\begin{aligned}
&R(Y,X)A_1V - A_1R(Y,X)V + (\nabla_XA_1)\psi A_1Y - (\nabla_YA_1)\psi A_1X\\
&= \frac{c}{4}\{v(Y)A_1X - v(X)A_1Y + u(Y)\theta A_1X - u(X)\theta A_1Y\\
&- w(Y)\phi A_1X + w(X)\phi A_1Y - g(\phi A_1Y,X)W\\
&+ g(\phi A_1X,Y)W + g(\theta A_1Y,X)U - g(\theta A_1X,Y)U\},
\end{aligned}
\tag{5.6}
$$

$$
\begin{aligned}
&R(Y,X)A_1W - A_1R(Y,X)W + (\nabla_XA_1)\theta A_1Y - (\nabla_YA_1)\theta A_1X\\
&= \frac{c}{4}\{w(Y)A_1X - w(X)A_1Y - u(Y)\psi A_1X + u(X)\psi A_1Y\\
&+ v(Y)\phi A_1X - v(X)\phi A_1Y - g(\psi A_1Y,X)U\\
&+ g(\psi A_1X,Y)U + g(\phi A_1Y,X)V - g(\phi A_1X,Y)V\}.
\end{aligned}
\tag{5.7}
$$

Now we take an inner product of (5.5) with V and W, respectively. Then by using (2.23) and (5.3) we have

$$
\begin{aligned}
0 = &-\frac{c}{4}\{v(X)u(A_1Y) - v(Y)u(A_1X) - w(X)g(\phi Y,A_1U)\\
&+ w(Y)g(\phi X,A_1U) - 2g(\phi Y,X)w(A_1U)\\
&+ u(X)g(\theta Y,A_1U) - u(Y)g(\theta X,A_1U) + 2g(\theta Y,X)u(A_1U)\}\\
&- v(A_1X)u({A_1}^2Y) + v(A_1Y)u({A_1}^2X)\\
&- \frac{c}{4}\{u(X)v(A_1Y) - u(Y)v(A_1X) + w(X)g(\psi Y,A_1V)\\
&- w(Y)g(\psi X,A_1V) + 2g(\psi Y,X)w(A_1V)\\
&- v(X)g(\theta Y,A_1V) + v(Y)g(\theta X,A_1V) - 2g(\theta Y,X)v(A_1V)\}\\
&- u(A_1X)v({A_1}^2Y) + u(A_1Y)v({A_1}^2X),
\end{aligned}
\tag{5.8}
$$

$$
\begin{aligned}
0 = &-\frac{c}{4}\{w(X)u(A_1Y) - w(Y)u(A_1X) + v(X)g(\phi Y,A_1U)\\
&- v(Y)g(\phi X,A_1U) + 2g(\phi Y,X)v(A_1U)\\
&- u(X)g(\psi Y,A_1U) + u(Y)g(\psi X,A_1U) - 2g(\psi Y,X)u(A_1U)\}\\
&- w(A_1X)u({A_1}^2Y) + w(A_1Y)u({A_1}^2X)\\
&- \frac{c}{4}\{u(X)w(A_1Y) - u(Y)w(A_1X) + w(X)g(\psi Y,A_1W)\\
&- w(Y)g(\psi X,A_1W) + 2g(\psi Y,X)w(A_1W)\\
&- v(X)g(\theta Y,A_1W) + v(Y)g(\theta X,A_1W) - 2g(\theta Y,X)v(A_1W)\}\\
&- u(A_1X)w({A_1}^2Y) + u(A_1Y)w({A_1}^2X).
\end{aligned}
\tag{5.9}
$$

Taking the inner product of (5.6) with W and using (2.23) and (5.3), we also have

$$
\begin{aligned}
0 = & -\frac{c}{4}\{w(X)v(A_1Y) - w(Y)v(A_1X) + v(X)g(\phi Y, A_1V) \\
& - v(Y)g(\phi X, A_1V) + 2g(\phi Y, X)v(A_1V) \\
& - u(X)g(\psi Y, A_1V) + u(Y)g(\psi X, A_1V) - 2g(\psi Y, X)u(A_1V)\} \\
& - w(A_1X)v({A_1}^2Y) + w(A_1Y)v({A_1}^2X) \\
& - \frac{c}{4}\{v(X)w(A_1Y) - v(Y)w(A_1X) - w(X)g(\phi Y, A_1W) \\
& + w(Y)g(\phi X, A_1W) - 2g(\phi Y, X)w(A_1W) \\
& + u(X)g(\theta Y, A_1W) - u(Y)g(\theta X, A_1W) + 2g(\theta Y, X)u(A_1W)\} \\
& - v(A_1X)w({A_1}^2Y) + v(A_1Y)w({A_1}^2X).
\end{aligned} \tag{5.10}
$$

On the other hand, using (3.1)-(3.6) and (5.3), we can easily see that the differential of the function λ appeared in (3.10) is given by

$$\frac{1}{2}X\lambda = -g(A_1X, T). \tag{5.11}$$

From now on we assume that U, V and W are the eigenvectors of A_1, that is,

$$A_1U = \alpha U, \quad A_1V = \beta V, \quad A_1W = \gamma W, \tag{5.12}$$

where $\alpha = u(A_1U)$, $\beta = v(A_1V)$ and $\gamma = w(A_1W)$. We first notice that $T = 0$ by the aid of (3.7) and (5.12). Thus (5.11) yields that λ is constant.

Substituting (5.12) into (5.8) and using (2.13)-(2.16), we can easily see that

$$(\alpha - \beta)[(\frac{c}{2} + \alpha\beta)\{u(X)v(Y) - v(X)u(Y)\} - \frac{c}{2}g(\theta Y, X)] = 0. \tag{5.13}$$

Putting $Y = V$ in (5.13) and using (2.16), we have

$$(\alpha - \beta)(c + \alpha\beta) = 0. \tag{5.14}$$

Putting θY instead of Y in (5.13) and using (2.9) and (2.16), we also have

$$(\alpha - \beta)[(\frac{c}{2} + \alpha\beta)\{u(X)u(Y) + v(X)v(Y)\} + \frac{c}{2}\{g(Y, X) - w(X)w(Y)\}] = 0,$$

which and (5.14) yield

$$\frac{c(\alpha - \beta)}{2}\{g(Y, X) - u(X)u(Y) - v(X)v(Y) - w(X)w(Y)\} = 0$$

and consequently

$$(n - 3)c(\alpha - \beta) = 0. \tag{5.15}$$

By the quite similar method we can obtain from (5.9) and (5.10) that

$$(n - 3)c(\beta - \gamma) = 0, \quad (n - 3)c(\alpha - \gamma) = 0. \tag{5.16}$$

From (5.15) and (5.16), it is clear that

$$\alpha = \beta = \gamma = \lambda/3, \tag{5.17}$$

provided that $n > 3$ and $c \neq 0$.

Differentiating the first equation of (5.12) covariantly and using (3.4), (5.3), (5.12) and (5.17), we have

$$\alpha\phi A_1 X = -\frac{c}{4}\{\phi X + w(X)V - v(X)W\} + A_1\phi A_1 X,$$

which implies

$$\alpha(\phi A_1 - A_1\phi)X = 0.$$

Similarly the other equations of (5.12) yield

$$\alpha(\psi A_1 - A_1\psi)X = 0, \quad \alpha(\theta A_1 - A_1\theta)X = 0.$$

Thus we have

Lemma. *Let M be an $n(>3)$-dimensional QR-submanifold of $(p-1)$ QR-dimension in a quaternionic space form $\overline{M}^{(n+p)/4}(c)$ $(c \neq 0)$ and let the normal vector field ξ be parallel with respect to the normal connection. If U, V and W are the eigenvectors of the shape operator A_1 and if* (5.2) *is established on M and $\lambda \neq 0$, then*

$$A_1\phi = \phi A_1, \quad A_1\psi = \psi A_1, \quad A_1\theta = \theta A_1.$$

Combining Theorem K-P and the above lemma, we obtain the required result in Theorem 2. □

Corollary 2. *Let M be a real hypersurface of a quaternionic projective space $QP^{(n+p)/4}(4)$. Suppose that*

$$\overset{\circ}{\mathcal{L}}_U\, h_1 = 0, \quad \overset{\circ}{\mathcal{L}}_V\, h_1 = 0, \quad \overset{\circ}{\mathcal{L}}_W\, h_1 = 0$$

on M, where $\overset{\circ}{\mathcal{L}}_U$, $\overset{\circ}{\mathcal{L}}_V$ and $\overset{\circ}{\mathcal{L}}_W$ denote the operators defined by (5.1), *respectively and $h_1(X, Y) = g(A_1X, Y)$, A_1 being the shape operator corresponding to a unit vector field ξ normal to M. If the function λ appeared in* (3.10) *is not zero at some point of M and if U, V and W are the eigenvectors of A_1, then $\pi^{-1}(M)$ is locally a product of $M_1 \times M_2$ where M_1 and M_2 belong to some $(4n_1+3)$- and $(4n_2+3)$-dimensional spheres.*

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Hyang Sook Kim
Department of Computational Mathematics
School of Computer Aided Science
Institute of Basic Science
Inje University
Kimhae 621-749, Korea
E-mail address: mathkim@inje.ac.kr

Jung-Hwan Kwon
Department of Mathematics Education
Daegu University
Kyongsan 712-714, Korea
E-mail address: jhkwon@daegu.ac.kr

Jin Suk Pak
Department of Mathematics Education
Kyungpook National University
Daegu 702-701, Korea
E-mail address: jspak@knu.ac.kr