

### suspension

†

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## Effect of Fluid Viscosity on the Suspension of a Single Particle in Channel Flow

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**Key Words :** Suspension( ), Single Particle( ), Equilibrium Position( ), Viscosity Effect( )

### Abstract

Suspension of a single solid particle in a channel flow with a constant pressure gradient is studied numerically. The interaction of a circular particle with a surrounding Newtonian fluid is formulated using a combined formulation. Numerical results are presented using two dimensionless variables: the sedimentation Reynolds number and the generalized Froude number. From the present results, it has been shown that a solid particle is suspended at a smaller generalized Froude number as the viscosity of the surrounding fluid increases. The time taken for equilibrium position is found to be smaller as fluid viscosity increases when both : the sedimentation Reynolds number and the generalized Froude number are the same while, at the same situation, the dimensionless time taken for equilibrium position is to be nearly the same regardless of fluid viscosity when a dimensionless time variable is introduced

1.

fluidics)

(Micro-가

가

가

(1,2)

가

(3)

Joseph

(5-)

hydrodynamic focusing<sup>(4)</sup>

가

가

(Traction)

( )

Helsa<sup>(9)</sup>

가

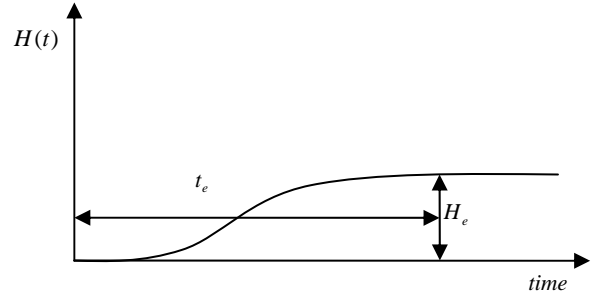
(combined formulation)

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Feng<sup>(6)</sup> Slender body  
 slender body  
 , Joseph Ocano<sup>(7)</sup> Patankar<sup>(8)</sup>  
 sedimentation  
 가  
 가  
 가  
 suspension  
 가  
 (Fig. 1).



**Fig. 1** Equilibrium height and time for a channel flow with a single particle.

(1)  $I$  identity,  $\mu$   
 (2)  $Tr$  (traction)  
 $u, v, \Omega_p$   
 (11)

suspension

2

3

가

ALE

( $\bar{u}_m$ ) Laplace

ALE

Navier-Stokes

P2P1 Galerkin

2.

2.1

Choi<sup>(10)</sup>가

(1) Navier-Stokes  
 (2)

$$\rho \frac{D\bar{u}}{Dt} = \nabla \cdot \bar{\sigma} \quad (1)$$

where  $\bar{\sigma} = -pI + \bar{\tau}$  and  $\bar{\tau} = \eta(\nabla\bar{u} + \nabla\bar{u}^T)$

$$\nabla \cdot \bar{u} = 0$$

$$M \frac{d\bar{U}}{dt} = Tr \text{ where } M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}, \bar{U} = \begin{bmatrix} u \\ v \\ \Omega_p \end{bmatrix} \quad (2)$$

Find  $\hat{u}_h, u_p \in R, v_p \in R, \Omega_p \in R$  such that

$$\int [\bar{w} \cdot \rho \frac{D\hat{u}}{Dt} + \nabla \bar{w} : \bar{\sigma}] d\Omega - \int \bar{w} \cdot \bar{\sigma} \cdot \bar{n} d\Gamma + \delta \hat{U}_p (M \frac{d\hat{U}_p}{dt} - Tr) = 0 \quad (3)$$

for all admissible functions  $w \in V_h$ , where

$$V_h = \{w_h \mid w_h \in H_h^1 \text{ on } \Omega - (\Gamma_g \cup \Gamma_p), w_h = 0 \text{ on } \Gamma_g, w_h = \delta \hat{U}_p + \Omega_p \times (\bar{x} - \bar{X}_c) \text{ on } \Gamma_p\}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\hat{u} - \bar{u}_m) \cdot \nabla$$

(3) P2P1 triangular

가 ALE

$$\begin{bmatrix} \tilde{M} & -\tilde{B} \\ \tilde{B}^T & 0 \end{bmatrix} \begin{bmatrix} \tilde{u} \\ p \end{bmatrix} = \begin{bmatrix} \tilde{f} \\ 0 \end{bmatrix} \quad (6)$$

$V_h$  ,  $\Gamma_p$  가  $\tilde{u}_m$  가 Gradient  $\tilde{u} = [\tilde{u}_i, \tilde{U}_p]^T$  (Rigid body) Nam <sup>(14)</sup> MILU Conjugate <sup>(6)</sup>

(3) [10] Bi-CGStab<sup>(12)</sup>

2.2

가 , shear Reynolds  $R$  gravity(sedimentation) Reynolds  $R_G$  <sup>(8,15)</sup>

$$\begin{aligned} \frac{\tilde{u}^{n+1} - \hat{u}}{\Delta t} &= -\nabla p^{n+1} \\ \nabla \cdot \tilde{u}^{n+1} &= 0 \\ M \frac{\tilde{U}_p^{n+1} - \hat{U}_p}{\Delta t} &= Tr_p \end{aligned} \quad (4)$$

$$\begin{aligned} R &= \frac{\rho_f V d}{\eta} = \frac{\rho_f \dot{\gamma}_w d^2}{\eta} \\ R_G &= \frac{\rho_f V_g d}{\eta} = \frac{\rho_f (\rho_p - \rho_f) g d^3}{\eta^2} \end{aligned} \quad (7)$$

Reynolds  $V = \dot{\gamma}_w d$

P2P1 triangular

W  $\dot{\gamma}_w$   $\bar{p}$  가

$$\dot{\gamma}_w = \frac{W\bar{p}}{2\eta}$$

$$\begin{bmatrix} M & B & E \\ B^T & 0 & 0 \\ Tr_v & Tr_p & D \end{bmatrix} \begin{bmatrix} \tilde{u}^{n+1} \\ p \\ \tilde{U}_p \end{bmatrix} = \begin{bmatrix} f_u \\ 0 \\ f_p \end{bmatrix} \quad (5)$$

$R_G$  Reynolds gravity

(5)  $Tr_p$  (Form drag) ,  $Tr_v$  (Friction drag)  $E$

$$V_g = \frac{(\rho_p - \rho_f) g d^2}{\eta} \quad (8)$$

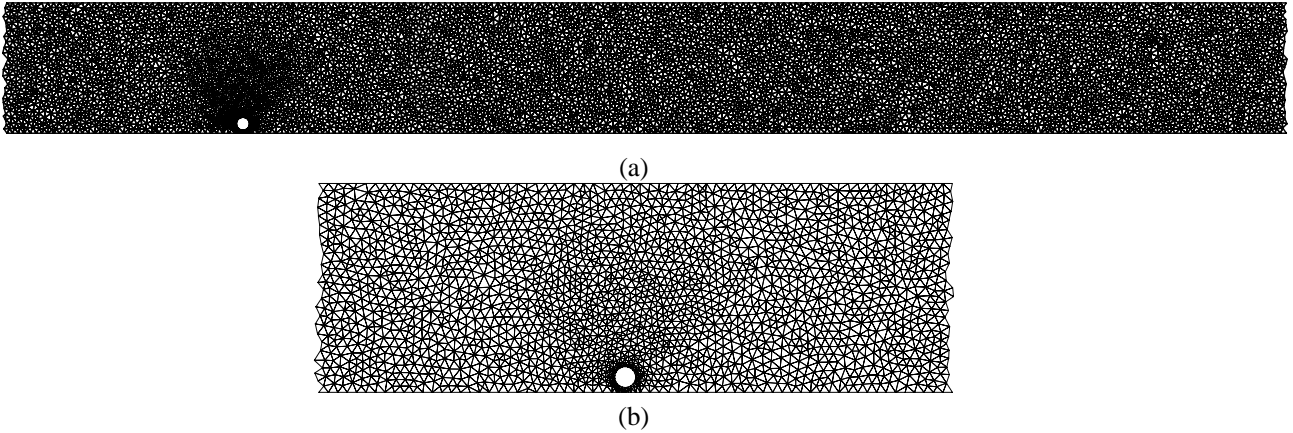
gravity parameter 가

(5) Maury Glowinsky<sup>(13)</sup>가

$$G = \frac{V_g}{V} = \frac{(\rho_p - \rho_f) g d}{\eta \dot{\gamma}_w} = \frac{2(\rho_p - \rho_f) g d}{W\bar{p}} \quad (9)$$

[10]

(7) shear Reynolds parameter (9) gravity generalized Froude



**Fig. 2** (a) Periodic unstructured mesh for a channel flow with a single particle (b) Unstructured mesh around a single particle

. Fig. 2

$$\frac{R}{G} = \frac{d\dot{\gamma}_w^2}{(\rho_p / \rho_f - 1)g} = \frac{\rho_f dW^2 \bar{p}^2}{4g\Delta\rho\eta^2} \quad (10)$$

2

. Fig.

(mid-node)

Generalized Froude

(buoyant weight)

, generalized Froude 가 , gravity parameter 가 suspension 가

(7) gravity(sedimentation) Reynolds

45

R G

$\dot{\gamma}_w$  ( ) 가

3.1

(7) (10)

가 sedimentation

Reynolds

generalized Froude

3.

가 가

$$R_G \sim \frac{\Delta\rho}{\eta^2} \quad \frac{R}{G} \sim \frac{\bar{p}^2}{\eta^2 \Delta\rho} \quad (11)$$

, W = 1.0 cm, d = 0.1 cm,  $\rho_f = 1.0 \text{ g cm}^{-3}$

,  $\Delta\rho$

10W

2.3

3.1

가

suspension

가

가  $0.01 \text{ g cm}^{-3}$

, R/G 가

가

$1000 \text{ dyne cm}^{-3}$   
 $\text{cm}^{-1} \text{ sec}^{-1}$

1.0 g

[7.8]

**Table 1** Dependence of the dimensionless equilibrium height and time on time-step size and grid resolution; the dimensionless equilibrium times are denoted in parenthesis

|            |              |              |              |              |
|------------|--------------|--------------|--------------|--------------|
| $\Delta t$ | 0.005        | 0.01         | 0.03         | 0.05         |
| Mesh       | 2.84 (17.40) | 2.84 (17.40) | 2.84 (17.40) | 2.83 (17.40) |
| Mesh       | 2.84 (17.40) | 2.84 (17.40) | 2.84 (17.40) | 2.84 (17.40) |
| Mesh       | 2.84 (17.40) | 2.84 (17.40) | 2.84 (17.40) | 2.84 (17.40) |

3 가

60%

**Table 2** Effect of sedimentation Reynolds number and generalized Froude number on dimensionless equilibrium height

|  |                       |       |       |       |
|--|-----------------------|-------|-------|-------|
| $\eta$<br>( $\text{gcm}^{-1}$<br>$\text{sec}^{-1}$ ) | $R/G$                 | 112.5 | 159.4 | 255.0 |
|  | $R_G$                 |       |       |       |
| 0.01   | $9.81 \times 10^{-1}$ | ×     | ×     | ×     |
| 0.1  | $9.81 \times 10^{-3}$ | ×     | 0.66  | 0.93  |
| 1.0  | $9.81 \times 10^{-5}$ | 0.62  | 0.76  | 1.06  |

Table 1

$d^2 / \nu$

III

Mesh I, II

30, 45, 60

6200,

13000, 24200

2

Table 1

0.03

Mesh II

Mesh III

0.01

Mesh II

$$R_G = \frac{9.81 \times 10^{-5}}{\eta^2} \tag{12}$$

$$\frac{R}{G} = 0.255 \left( \frac{\bar{p}}{\eta} \right)^2 \sim \dot{\gamma}_w^2$$

Table 2

$R_G$

$R/G$

suspension

, Table

(12)

suspension

3.2

가

Sedimentation Reynolds  
가 suspension

generalized Froude  
가 가

3.3

가

Table 2  
sedimentation Reynolds

generalized Froude  
suspension

suspension

Table 2

$R_G$

$R/G$

3.1

$R_G$

$R/G$

가

×

가

가

가 suspension

가

Table 2

0.01

가

가

$R_G$

, sedimentation Reynolds

generalized Froude

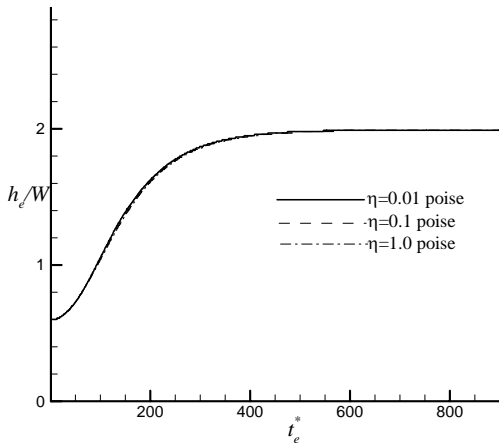
$R/G$  가

Table 3

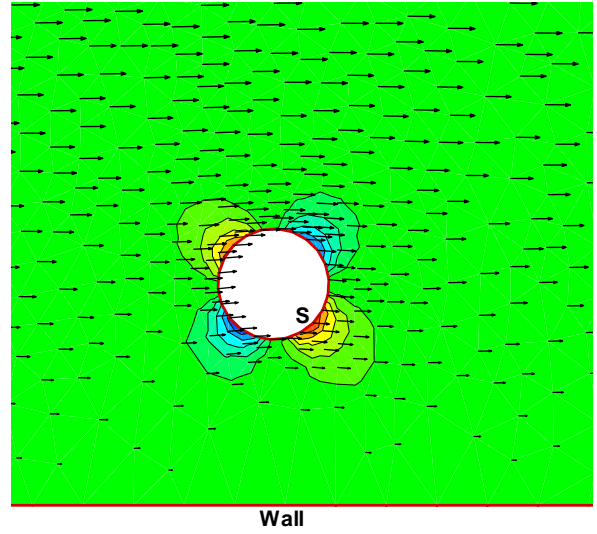
$R_G$

**Table 3** Dependence of equilibrium height on fluid viscosity

| $\eta$<br>( $\text{gcm}^{-1}\text{sec}^{-1}$ ) | $R_G$                 | $R/G$         | $h_e^* =$<br>$h_e/d$ | $t_e$ (sec) | $t_e^* =$<br>$t_e/t_c$ |
|--|-----------------------|---------------|----------------------|-------------|------------------------|
| 0.01   | $9.81 \times 10^{-4}$ | $10^4 / 9.81$ | 1.99                 | 548.0       | 548.0                  |
| 0.1  | $9.81 \times 10^{-4}$ | $10^4 / 9.81$ | 1.99                 | 58.0        | 580.0                  |
| 1.0  | $9.81 \times 10^{-4}$ | $10^4 / 9.81$ | 1.99                 | 5.67        | 567.0                  |



**Fig. 3** Evolution of particle height with a nondimensional time for various fluid viscosities



**Fig. 4** Velocity field and pressure contours around a single particle at equilibrium

$R/G$  가

sedimentation Reynolds suspension

generalized Froude

$d^2/\nu$  가

0.199 가

$d^2/\nu$

(12)

(15)  $R_G$   $R/G$  가

10 가

100 가

$(\gamma_w^{-1})$

Fig. 4 가 0.01 poise

가

Fig. 4 가

S

가

4.

suspension

Reynolds

(1)

(

$R_G$  가

)

) suspension

Sedimentation

generalized Froude

가

$R/G$

,

suspension

(2)  $R_G$  R/G

가

,  $t_e$

(3)  $R_G$  R/G

$d^2 / \nu$

2006 ( )

(KRF-2006-331-D00061).

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A.1 가

$$U_m \dot{\gamma}_w$$

$$U(y) = 4U_m \frac{y}{W} \left(1 - \frac{y}{W}\right) \quad (A1)$$

$$, W \quad , 2\tau_w = \bar{p}W$$

$$U_m$$

$$\bar{p}$$

$$U_m = \frac{W^2 \bar{p}}{8\eta} \quad (A2)$$

$$\dot{\gamma}_w$$

$$\dot{\gamma}_w = \frac{du}{dy} \Big|_w = \frac{4U_m}{W} = \frac{W\bar{p}}{2\eta} \quad (A3)$$