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자유수면 밑을 전진하는 세장체에 작용하는 수면흡입력의 추정

윤범상†*, 담반퉁**

울산대학교 조선해양공학부*

Free Surface Suction Force Acting on a Submerged Slender Body Moving Beneath a Free Surface

Bum-Sang Yoon†* and Dam Vam Trung*

School of Naval Architecture & Ocean Engineering, Univ. of Ulsan*

Abstract

In this paper, the steady lift force acting on a slender body moving beneath regular wave systems of arbitrary wavelengths and directions of propagation is considered. The momentum conservation theorem and the strip method are used to obtain the hydrodynamic forces acting on the body and affecting its motions on the assumption that the body is slender. In order to obtain the vertical steady force acting on it, or the free surface suction force, the second-order hydrodynamic forces caused by mutual interactions between the components of the first-order hydrodynamic forces are averaged over time. The validity of the method is tested by comparison of the calculated results with experimental data and found to be satisfactory. Through some parametric calculations performed for a typical model, some useful results are obtained as to the depth of submergence of the body, wavelengths, directions, etc.

※Keywords: Free surface suction force(수면흡입력), Second order hydrodynamic force(2차 동유체력), Momentum theorem(운동량법칙), Wave-body interaction(파도-물체상호작용), Experimental validation(실험검증)

1. Introduction

Analytical methods have been developed by various authors to estimate the forces and moments acting on a submerged body moving beneath a free surface. This problem was first addressed by Ogilvie (1963), who obtained the second-order vertical force for a two-dimensional body. Newman (1970) proposed a basic theory to estimate the second-order

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✝교신저자: bsyoon@ulsan.ac.kr, 052-259-2160

steady force including two main components thereof. The first component is caused by interaction between the wave diffraction and incident wave potentials, and the second is due to interaction of the body motions and the incident waves. A methodology to estimate the first-order oscillatory motion of the body, the second-order time-average vertical force and the pitching moment proposed in terms of Kochin's function by Lee & Newman (1971). However, those studies were restricted to two-dimensional cases, and no calculated results were presented.

In the present study, a calculation algorithm based on a combination of the momentum conservation theorem and the strip method is developed to obtain the wave-induced motions of the body (heave, pitch, sway, yaw and roll). Using motion responses, the free surface suction force is calculated and discussed. Experimental measurements are carried out to validate the calculation algorithms developed in the present study. Agreement between the experimental and calculated results is good enough to say that the method is valid and useful. The theory, calculation algorithm, results and discussions are presented in the following chapters.

2. Mathematical formulation

2.1 Coordinate systems

As shown in Fig.1, the O-XYZ coordinate system is fixed in space, the XY plane coinciding with the undisturbed free surface. G-xyz is a coordinate system moving at the average velocity of the body, and its origin G is located at the mean position of the center of mass of the body distance h beneath the free surface. The Z- and z-axes are directed vertically upward. An incident wave of wavelength λ and amplitude A propagates in the X-direction. The body moves with velocity U, angle of attack α and wave encounter angle β.

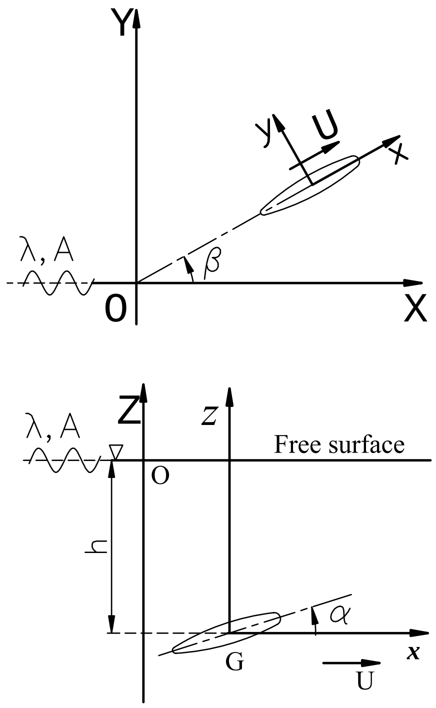


Fig. 1 Coordinate systems

As mentioned above, the strip theory is employed assuming that the body is slender. The fluid flow is assumed to be incompressible, inviscid and irrotational, and the motion of the free surface is very small.

2.2 Force acting on a strip in z-direction

For harmonic wave motions, the velocity potential of the flow around the strip of the body located at $x = x*$ is

$$\Phi(x*, y, z, t) = U\psi(x*, y, z) + \mathrm{Re}\{\varphi(x*, y, z)e^{-i\omega_e t}\} \quad (1)$$

Here, ψ is the velocity potential at the coordinate x=$x*$ for the body advancing steadily with unit velocity beneath the still water surface. The second term is the unsteady part due to the waves and the wave-induced body motions. That is,

$$\varphi = \varphi_0 + \varphi_7 + \sum_{j=2}^{4} V_j \varphi_j \tag{2}$$

Here, φ_0 is the potential of the incident wave written as,

$$\varphi_0 = \frac{gA}{\omega} e^{-kh} e^{k\left(Z + ix^* \cos\beta - iy \sin\beta\right)} \tag{3}$$

where $k = \frac{\omega^2}{g}$ is the wave number and $\omega_e = \omega - kU\cos\beta$ is the wave encounter frequency, φ_7 is the wave diffraction potential, and V_2, V_3 and V_4 are the strip sway, heave and roll velocities, respectively. $\varphi_2, \varphi_3, \varphi_4$ are the sway, heave and roll potentials due to the corresponding motions with unit velocities, respectively.

As seen in Fig.2, the fluid domain v_o is bounded by the boundaries S_c and S_o, which are the boundary at infinity and the body boundary, respectively. The outward unit normal vector to the boundary surface is denoted by $\hat{n}(n_y, n_z)$, and the fluid velocity is denoted by $\hat{V}(v, w)$. The fluid momentum equation in the z-direction is,

$$\frac{d}{dt}\iint_{v_0} \rho w \cdot dv_0 = \frac{\partial}{\partial t}\iint_{v_0} \rho w \cdot dv_0 + \int_S \rho w \cdot \hat{V} \cdot \hat{n} \cdot dS \tag{4}$$
$$= \int_S \sigma_z \cdot dS + \iint_{v_0} \rho f_z \cdot dv_0$$

Considering that

$$\sigma_z = -P_z \quad , \iint_{v_0} \rho f_z dv_0 \quad = -\int_S \rho g Z \cdot n_z dS$$

eq. (4) can be written as

$$\frac{d}{dt} M_z = -\int_{S_o + S_c} \left[(P + \rho g Z) n_z + \rho w v_n \right] dS$$
$$= -\int_{S_o + S_c} \left[(P + \rho g Z) n_z + \rho \Phi_z (\Phi_n - U_n) \right] dS \tag{5}$$

Furthermore, the following boundary conditions are given on the boundaries S_o and S_C.

$$U_n = \begin{cases} \Phi_n & on\ S_o \\ 0 & on\ S_c \end{cases}$$

The total hydrodynamic force acting on the surface S_o is obtained as follows:

$$f_z(on\ S_o) = \int_{S_o} (P + \rho g Z) n_z dS$$
$$= -\int_{S_c} \left[(P + \rho g Z) n_z + \rho \Phi_z \Phi_n \right] dS - \frac{dM_y}{dt} \tag{6}$$

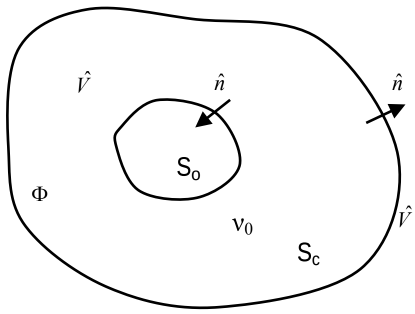


Fig. 2 Scheme of the flow: the fluid domain v_o is bounded by the boundaries S_c and S_o

2.3 Steady time-average force acting on the strip in the z-direction

The free surface suction force in the z-direction can be obtained by averaging eq. (6) over time. The time-average value of the last term in eq. (6) equals zero, and hence we can derive the following equation

$$\overline{f_z}(on\ S_o) = -\int_{S_c} \left[\left(\overline{P + \rho g Z}\right) n_z + \rho \overline{\Phi_z \Phi_n} \right] dS \tag{7}$$

From Bernoulli's equation it follows that

$$P + \rho g Z = -\rho \frac{\partial \Phi}{\partial t} - \frac{1}{2}\rho (\nabla \Phi)^2 \tag{8}$$

The time-average value of the first-order hydrodynamic force, i.e. the first term in the right-hand side of eq. (8), equals zero. Therefore, eq. (7) becomes

$$\overline{f_z} = \rho \int_{S_c} \left[\frac{1}{2} \overline{(\nabla \Phi)^2} n_z - \rho \overline{\Phi_z \Phi_n} \right] dS \tag{9}$$

Substituting the expression for the total potential given by eq. (1) into eq. (9), we have

$$\overline{f_z} = \overline{f_{sz}} + \overline{f_z^{(2)}} \tag{10}$$

where,

$$\overline{f_{sz}} = \frac{1}{2} \rho U^2 \int_{S_c} \left[\frac{1}{2} \nabla \psi \cdot \nabla \psi^* n_z - \psi_z \psi_n^* \right] dS \tag{11}$$

$$\overline{f_z^{(2)}} = \rho \int_{S_c} \left[\frac{1}{2} \nabla \varphi \cdot \nabla \varphi^* \cdot n_z - \varphi_z \cdot \varphi_n^* \right] dS \tag{12}$$

The symbol * denotes the complex conjugate. The function $\overline{f_{sz}}$ is the steady suction due to the body advancing with velocity *U* under calm water, and $\overline{f_z^{(2)}}$ is the steady suction due to the waves and the wave-induced body motions. In this study, only the last term $\overline{f_z^{(2)}}$ is considered. Furthermore, only incident wave – diffraction and incident wave – body motions interactions are considered taking into account the linear formulation of the problem. That is,

$$\overline{f_z^{(2)}} \cong \overline{f_{07}} + \overline{f_{0M}} \tag{13}$$

Using the equation of continuity (Laplace's equation) and Stoke's theorem, the following equations are derived without any difficulty:

$$\overline{f_{07}} = \frac{1}{2} \rho \omega A \ \mathrm{Re} \int_{S_c} \left(\varphi_{7n} - \varphi_7 \frac{\partial}{\partial n} \right) e^{k(Z - ix^* \cos\beta + iy \sin\beta)} dS \tag{14}$$

$$\overline{f_{oM}} = \frac{1}{2} \rho \omega A \ \mathrm{Re} \sum_{j=2}^{4} V_j \int_{So} \left(\varphi_{jn} - \varphi_j \frac{\partial}{\partial n} \right) e^{k(Z - ix^* \cos\beta + iy \sin\beta)} dS \tag{15}$$

2.4 Free surface suction force and moment acting on the submerged body

As mentioned above, in order to calculate the motion of the body and the free surface suction force, the strip method is employed.

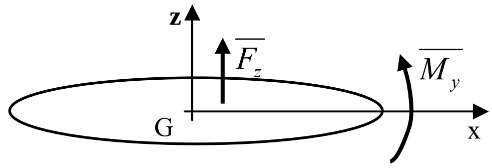


Fig. 3 Definitions of the free surface suction force and moment

The steady vertical force and the moment about *y*-axis acting on the whole body are expressed as follows.

$$\overline{F_z} \cong \int_L \overline{f_z^{(2)}} dx \tag{16}$$

$$\overline{M_y} \cong \int_L f_z^{(2)} \cdot x dx \tag{17}$$

Using the sectional free surface suction force at *x=x** from eqs. (14)–(15) and the strip method (16)–(17), the total force and moment are obtained as,

$$\overline{F_z} = \frac{1}{2} \rho \omega A \, \mathrm{Re} \int_L \left[I_7^* + \sum_{j=2}^{4} V_j I_j^* \right] dx \tag{18}$$

$$\overline{M_y} = \frac{1}{2} \rho \omega A \, \mathrm{Re} \int_L \left[I_7^* + \sum_{j=2}^{4} V_j I_j^* \right] \cdot x dx \tag{19}$$

where, $I_7^* = \int_{S_c} \left(\varphi_{7n} - \varphi_7 \frac{\partial}{\partial n} \right) e^{k(Z - ix^* \cos\beta + iy \sin\beta)} dS$

and $I_j^* = \int_{S_c} \left(\varphi_{jn} - \varphi_j \frac{\partial}{\partial n} \right) e^{k(Z - ix^* \cos\beta + iy \sin\beta)} dS$

When calculating I_7^*, the Haskind relation is used instead of solving the diffraction problem directly.

Here, the equations are derived by use of the strip displacement in z-direction (see Fig. 4).

$$I_7^* = A\omega e^{2k(-h + x^* \alpha)} \left\{ -\int_{S_o} (n_3 + \alpha n_1) e^{2kz} dS \right\} + A\omega e^{2k(-h + x^* \alpha)} \left[m_{33} + \left(\alpha^2 + \cos^2 \beta \right) m_{11} + m_{22} \sin^2 \beta \right] \tag{20}$$

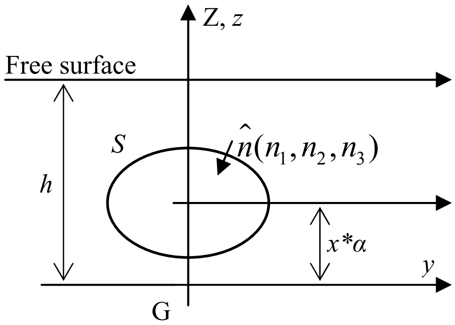


Fig. 4 Section geometry at x = x*

The first term in the above equation can be calculated directly while the second term is evaluated by the use of the added masses of the corresponding strip. m_{ij} is the mass added in direction j by motion mode i. Considering that the strip theory can be employed on the assumption of negligible three-dimensional cross flow effects, the surge-added mass m_{11} of the strip is set to zero here. In the same way, I_j^*'s can be expressed as follows.

$$I_2^* = e^{k(-h+x^*\alpha-ix^*\cos\beta)} \begin{Bmatrix} \int_{S_o} ie^{kz}\sin(ky\sin\beta)n_2 dS \\ -i\sin\beta\dfrac{k}{\rho}\left(m_{22}-\dfrac{N_{22}}{i\omega_e}\right) \end{Bmatrix} \tag{21}$$

$$I_3^* = e^{k(-h+x^*\alpha-ix^*\cos\beta)} \begin{Bmatrix} \int_{S_o} e^{kz}\cos(ky\sin\beta)n_3 dS \\ -\dfrac{k}{\rho}\left(m_{33}-\dfrac{N_{33}}{i\omega_e}\right) \end{Bmatrix} \tag{22}$$

$$I_4^* = e^{k(-h+x^*\alpha-ix^*\cos\beta)} \begin{Bmatrix} \int_{S_o} ie^{kz}\sin(ky\sin\beta)n_4 dS \\ -i\sin\beta\dfrac{k}{\rho}\left(m_{42}-\dfrac{N_{42}}{i\omega_e}\right) \end{Bmatrix} \tag{23}$$

where $n_4=yn_3-zn_2$ is the roll moment arm, from G to the body surface.

2.4 Motion responses

In the linear strip theory, the body motions are divided into two parts. The first is heave-pitch coupling, and the second is sway-yaw-roll coupling. They are both obtained under the assumption of small motions and negligible surge. The motion equations are as follows

$$A_{ij}\ddot{\xi}_j + B_{ij}\dot{\xi}_j + C_{ij}\xi_j = E_j \, , \tag{24}$$

where $\xi_j = \mathrm{Re}[X_j e^{-i\omega_e t}]$ represents the motion for sway ($j=2$), heave ($j=3$), roll ($j=4$), pitch ($j=5$) and yaw ($j=6$). The quantities E_j are the wave exiting forces. The description of the coefficients in equation (24) is omitted here.

Solving the motion equation (24), we obtain the motion responses X_j ($j=2\sim6$). From the responses, the velocities of any strip at $x=x^*$, which are used in eqs. (18) and (19), are derived.

$$V_2 = -\omega_e X_2 + \alpha(i\omega_e x^* + U)X_4 + (-i\omega_e x^* - U)X_6 \tag{25}$$

$$V_3 = -i\omega_e X_3 + \alpha(i\omega_e x^* + U)X_6 \tag{26}$$

$$V_4 = -i\omega_e X_4 \tag{27}$$

3. Experimental validation

The validity of the present prediction method is confirmed by measuring the coordinates of a submerged body and the free surface suction forces in regular waves. As shown in Fig.5, the experiment was carried out in the 2D wave flume of University of Ulsan, which measures *35m(L)x0.5m(W)x0.5m(D)*. The principal parameters of the model and experimental conditions are summarized in Table 1.

As shown in Fig. 5, two springs with spring constant k_1 are attached to the ends of the body, whose weight is balanced by the buoyant force, and the lower ends of the springs are connected to the flume bottom. Measurements are also made for springs with spring constant k_2 in the same experimental conditions. By measuring the

Table 1 Circular cylinder model and experimental conditions

cylinder, *Lxd(cm)*	*100x10*
underwater position	*h/d = 1.0*
wavelength range, $\omega\sqrt{L/g}$	*1.2 ~ 4.0*

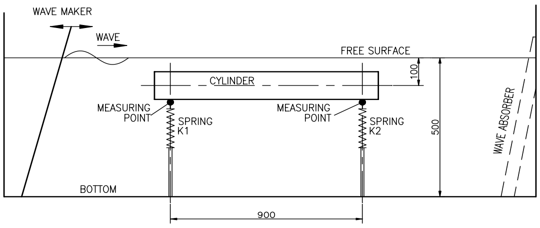


Fig. 5 Experimental setup in 2D wave tank

displacements of the springs, we can obtain not only the heave and pitch motion responses, but also the free surface suction forces in the following way. The experiments are carried out only for the case of head waves and zero forward velocity.

Fig. 6 is an assumed time varying spring displacements, and to be helpful in description of the following equations used in experimental data processing.

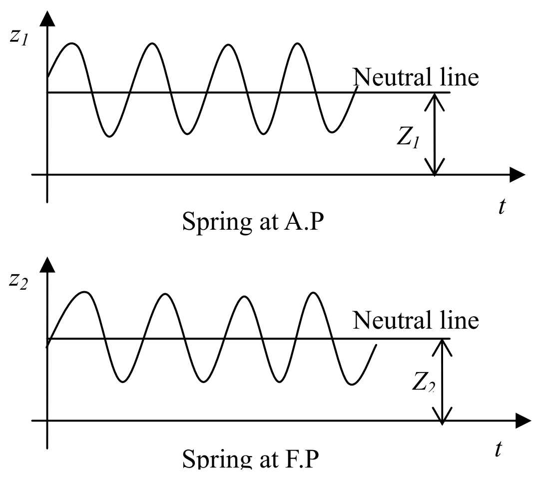


Fig. 6 Assumed spring displacements

(1) The heave is obtained by averaging the displacements of the two springs:

$$z_{av.}(t) = \frac{z_1(t) + z_2(t)}{2}$$

(2) The pitch is determined from the difference of the displacements of the two springs:

$$\theta(t) = \tan^{-1}\frac{z_1(t) - z_2(t)}{l}$$

Where, l is the distance between locations of the two springs.

(3) The steady suction force is obtained as follows.

$$\overline{f_z} = \frac{k_1(Z_1 + Z_2)_{1st\,exp.} + k_2(Z_1 + Z_2)_{2nd\,exp.}}{2}$$

Where, Z_1 and Z_2 are the time-average values of the two spring displacements.

As an example, Fig. 7 gives typical measured displacements of the two springs recorded by a camera, which are shown as a dashed and a dotted line, together with their time-varying average (heave) shown as a dashed-dotted line.

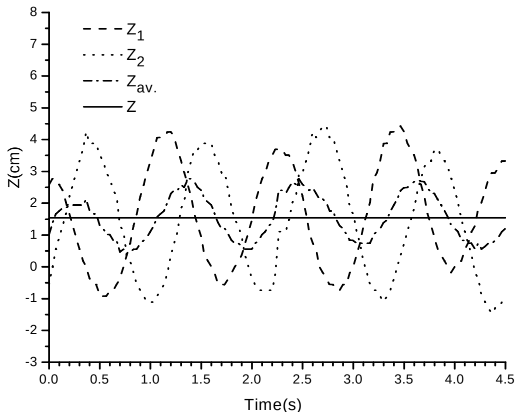


Fig. 7 Measured vertical displacements of the springs and the heave response obtained by averaging them ($\omega\sqrt{L/g}$ =1.8; A=0.0225; h=0.1m)

As seen from Fig.8, in the range of relatively long wavelengths the calculated results are in rather poor agreement with the experimental ones for the heave while a good agreement can be seen for the free surface suction force in almost all wavelength ranges. The predicted heave is overestimated while the predicted pitch is underestimated, especially in long wave length range. But it shows very good agreement in the case of the suction force. The prediction of the free surface suction force, which is the main aim of the present study, can be said to be quite satisfactory.

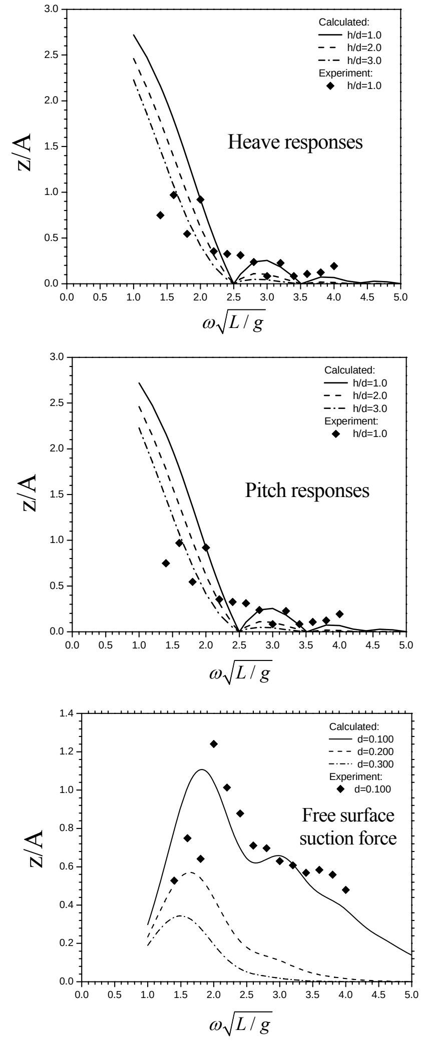


Fig. 8 Comparison of calculation and experiment

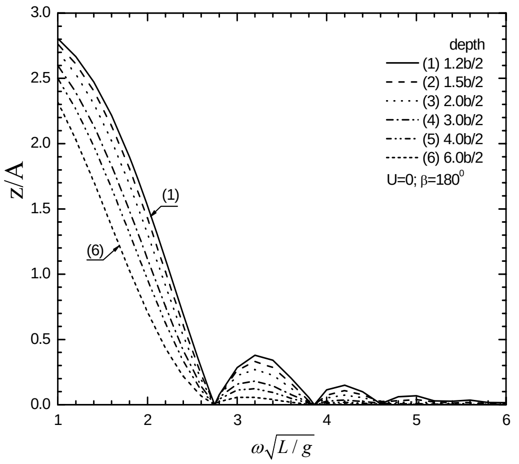
4. Parametric Simulation

Various parametric calculations were carried out for TRIDENT, a well-known submerged axially-symmetric body model, whose geometry is described in Table 2. The heave, pitch, sway, yaw and roll are shown in Figs. 9 to 13. The abscissas are $\omega\sqrt{L/g}$. As expected, the depth of submergence of the body heavily affects all the motion responses. Because of zero roll damping, the roll response becomes infinite at the roll resonant frequency, and its effect is neglected in the calculations of the suction forces and moments. The free surface suction forces and the moments for typical encounter angles are shown in Figs.14 and 15, respectively.

Table 2 Geometry of the Trident Model

Station	Radius(m)
1	0.00100
2	0.10180
3	0.17744
4	0.22621
5	0.25392
6~18	0.27766
19	0.26660
20	0.22723
21	0.00010

L=7.3634 m. LCG=0.3125 m forward of midship



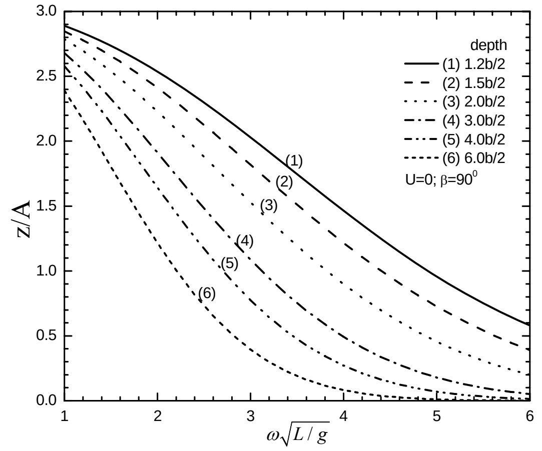


Fig. 9 Heave responses

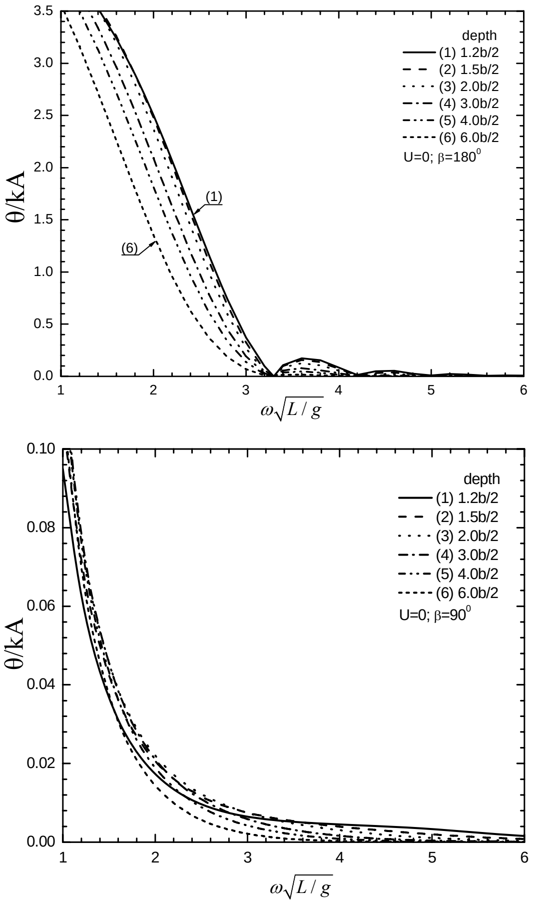


Fig. 10 Pitch responses

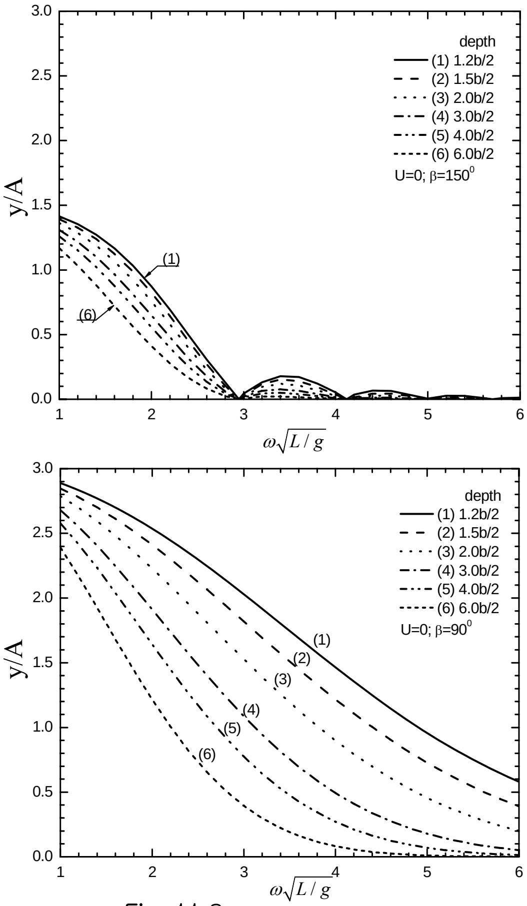
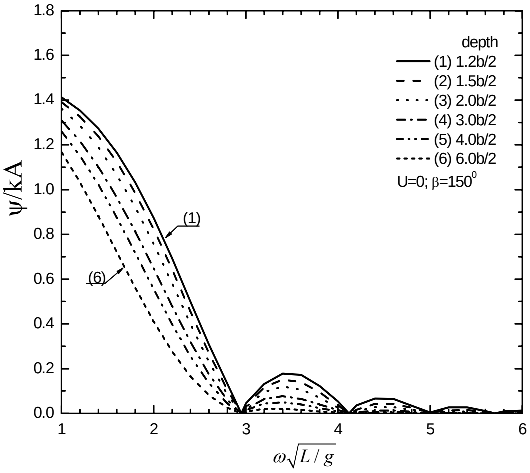


Fig. 11 Sway responses



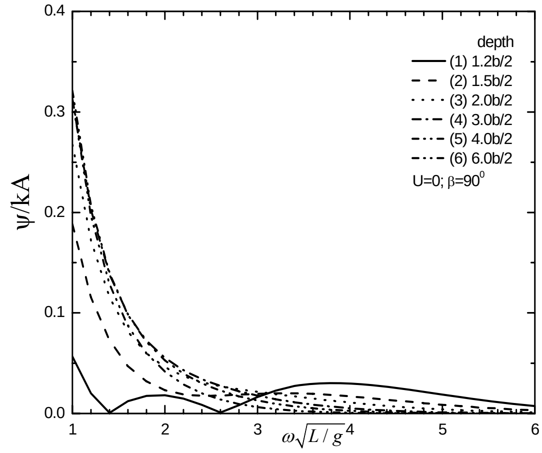


Fig. 12 Yaw responses

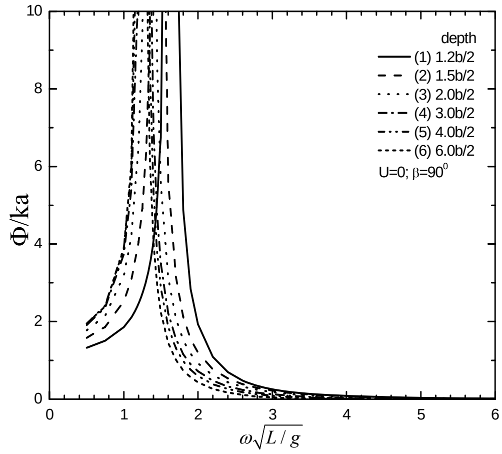
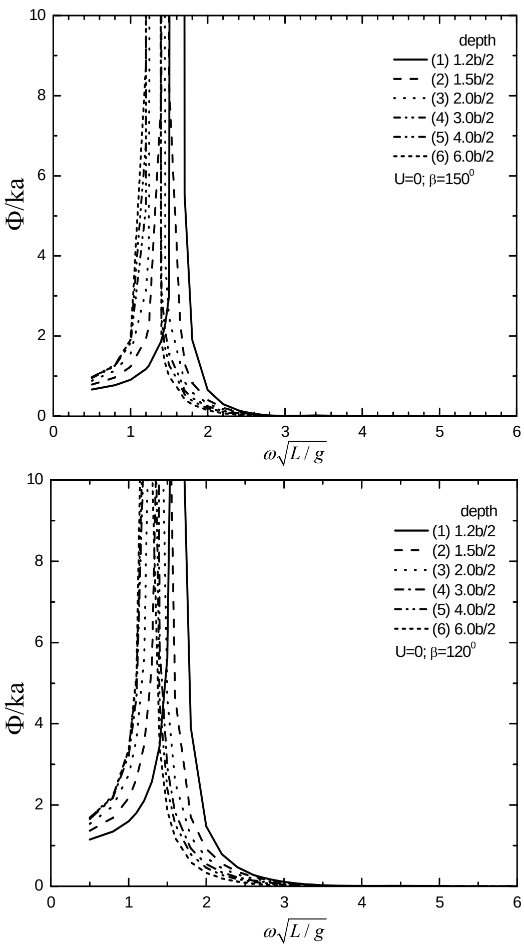


Fig. 13 Roll responses



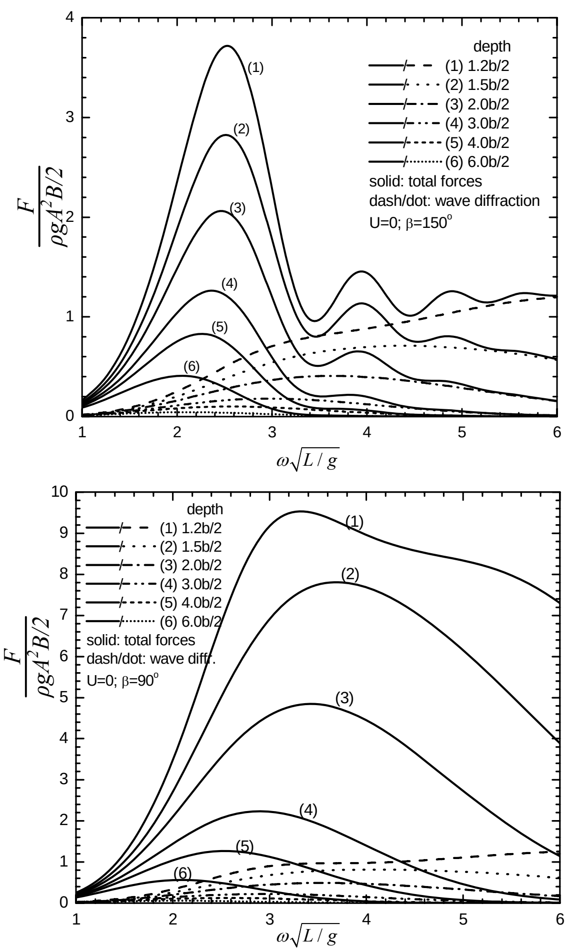


Fig. 14 Free surface suction forces

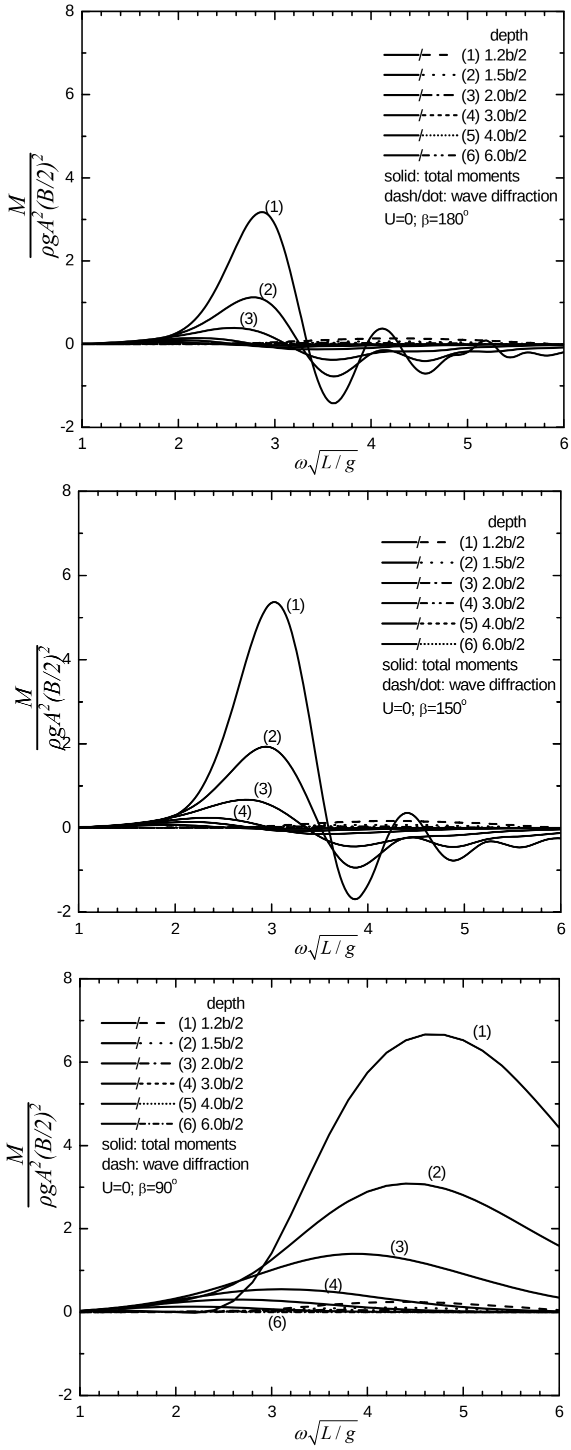


Fig. 15 Free surface suction moments

As would be expected, all the responses increase as the depth of submergence decreases. The free surface suction force consists of two terms: one is due to interaction between the incident wave and diffraction potentials, and the second is due to interaction between the incident wave and body motion potentials. As shown in the figures, the contribution of interaction between the incident wave and the wave diffraction becomes dominant as the wavelength becomes shorter. This corresponds to a small wave-induced motion of the body.

5. Conclusion

From the presented results of mathematical modeling, computations and experimental investigations the following conclusions can be drawn.

(1) The motions of a submerged slender body and the free surface suction force and moment acting on it can be successfully estimated by the proposed method based on the time averaging concept and the momentum conservation theorem.

(2) The validity of the proposed method is confirmed through comparison between the calculated and measured responses of a submerged slender body in regular wave systems

(3) Interaction between the incident wave and wave diffraction potentials turned out to contribute more significantly to the free surface suction force in the short-wavelength range where the wave-induced body motion is small

(4) The proposed method has to be validated through comparison with the near-field approach and the direct pressure calculation method in future studies.

(5) A fully three-dimensional analysis including irregular waves is desirable in future research. It will provide a more adequate description of practical cases involving arbitrary shaped submerged bodies beneath a free surface

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