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Reliable Hub Location Problems and Network Design

Hyun Kim*

Abstract : The hub and spoke network is a critical network-based infrastructure that is widely applied in current transportation and telecommunications systems, including Internets, air transportation networks and highway systems. This main idea of hub location models is to construct a network system which achieves the economy of scale of flows. The main purpose of this study is to introduce new hub location problems that take into account network reliability. Two standard models based on assignment schemes are proposed, and a minimum threshold model is provided as an extension in terms of hub network design. The reliability and interaction potentials of 15 nodes in the U.S. are used to examine model behaviors. According to the type of models and reliability, hubs, and minimum threshold levels, relationships among the flow economy of scale, network costs, and network resiliency are analyzed.

Keywords : Hub network design, reliability, flow economy of scale, network resiliency

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1. Introduction

The hub and spoke network system is a critical network-based infrastructure that is widely applied in transportation and telecommunications systems, including Internets, air transportation networks, and highway systems (Daskin 1995; Campbell *et al.* 2002). Since the hub location problem was recognized as an important class of location theory during the late 1980s, a number of models and variants have been proposed in the fields of geography, transportation science, telecommunications, operations research, and even computer science (Campbell 1994a; Klincewicz 1998). 'Hubs' are geographically and functionally significant nodes since they serve demands from, as well as provide connection to, 'spoke' nodes within their assigned regional areas. The placement of hubs is a strategic decision considered in geographical space since economic benefits or network costs of a network can be maximized or minimized respectively by identifying optimal locations of hubs and assignments between hubs and spoke nodes.

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* Program Director of GIS Certificate Program, Department of Geography, University of South Florida, hkim1@cas.usf.edu

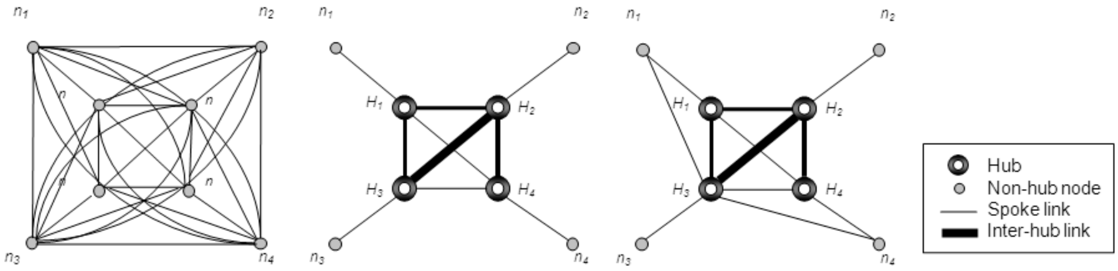
The most critical issue in hub location problem is the system reliability when any failures or disruptions are involved in hubs or inter-hub links. In particular, after 9/11, hubs are regarded as a critical facility to be protected from any malfunction or disruptions (Grubesic *et al.* 2003; NSTAC 2003). For this reason, designing more reliable hub networks becomes a critical issue in current network design models (Klincewicz 2006; Skorin-Kapov *et al.* 2006). Two incidents during last several years highlight the importance of the issue. First, an adverse weather condition in a single hub airport can cause a number of aircrafts delays throughout the network that affect passengers' socio-economic costs (Shavell 2000). Additionally, severe virus attacks on the critical hub of an Internet backbone provider brought disastrous malfunctions over telecommunication systems, which raised the question of optimal hub location in order to ensure network's reliability at a desirable level (NCA 2004; Kim and O'Kelly 2004).

In this context, this paper aims to introduce a new class of hub location models - namely the reliable hub location problem (hereafter RHLP). The main objective of the RHLP is to design a hub network system which can maximize network performance under disruptive conditions (Kim and O'Kelly 2009). The basic formations of the RHLP are provided and model behaviors in terms of scale of network economies and network resiliency are discussed. The paper is organized as follows. The next section reviews the fundamental characteristics of classical hub location models and the RHLP models. The section 3 provides the model formulations of the RHLP, and the model behavior with results will be discussed in section 4. Concluding remarks are provided in the final section, along with geographical implications and future research directions for the reliable hub location problems.

2. Evolution of hub location problems

1) Classical hub location models

Classical location problems, such as *p*-median and covering problems assume that facilities are located ideally in order to satisfy an objective function under certain given restrictions. The demand nodes or regions are assigned to *near* supply facilities which minimize cost or distance (Campbell 1994b; Revelle and Williams 2002). In hub location problems, the demand (node or area) is characterized as flows among origins and destinations. A hub facility is a special type of node where the *movement* of people, goods, traffic, or information among a set of origin-destinations are aggregated, re-sorted, and transferred to their destinations (Campbell *et al.* 2002). Most hub location models deal with optimization in two levels simultaneously - 1) to locate *p*-hubs and 2) to allocate non-hub nodes to one or more hubs. As a variant, the number of hubs can be determined *endogenously* as an internal part of optimization with given constraints and cost structure. Due to this property, hub location problems require higher computational complexity in the optimization process than the other classes of location problems. In detail, hub location problems are classified based on the type of assignment among hub and non-hub nodes.



(a) Completely connected network (b) Single Assignment (c) Multiple Assignment

Figure 1. Network structure: (a) Completely connected network, (b) Hub network: Single Assignment (SA), (c) Hub network: Multiple Assignment (MA).

As illustrated in Figure 1, *single assignment* requires that each non-hub node should be connected to a single hub. All traffic for a given non-hub node must travel through the single hub to which it is connected. In contrast, the *multiple assignment* form allows each node to be connected more than one hub. Each origin-destination (O-D) pair should be routed via at least one hub to carry out its demand from origin to destination. A route between an O-D pair with more than one hub utilizes hub to hub linkages, which are termed *inter-hub links*. It is assumed that the hubs are fully connected throughout inter-hub links unless such a restriction as a capacity constraint to limit the amount of flows is imposed. The single assignment form is commonly applied to design physical network infrastructures such as telecommunications where network budget is restrictive to construct network facilities. In contrast, the multiple assignment model is fit well to non-planar type of network systems (e.g. air transportation networks) where flexible routings are allowed among O-D pairs. It should be noted that economies of scale in a hub network are achieved when interacting flows among O-D pairs are aggregated in hubs and traveled on the inter-hub links since the aggregation of flows are encouraged by discount factor on inter-hub links. The more the flows are agglomerated, the greater the savings of travel or network costs are expected. For this reason, a number of hub location models focus on developing models that reduce total flow-costs and network-costs for building facilities in objective function. Due to its conceptual elegance as mathematical model and applicability the hub location problems have attracted many fields with a number of model variants (Alumur and Kara 2008). As shown in Table 1, the classes of hub location problems are summarized in terms of design components.

2) Reliable hub location problems

Traditionally, the issue of a high level of reliability in network systems has been recognized as a critical factor in designing network infrastructure (Gavish and Neuman 1992; Grover and Tipper 2005). The broadest definition of reliability is the ability of a network to carry out a desired network operation or resiliency to potential network disruptions (Shier 1991). It is expressed as probability ($0 \leq r \leq 1$; 0 for total loss and 1 for perfect delivery) by measuring availability of traffic with a given time. In general, reliability of a link or facility can be estimated based on empirical data by monitoring latency, traffic loss or delay. A calculation of reliability for an *i-j* pair is to multiply reliabilities of disjoint links and facilities which constitute the path between *i* and *j* (Colbourn 1987). It should be noted that little attention has been paid to hub-and-spoke network design associated with network reliability. This is because the concept of a reliable network system is often recognized as a counter-concept to the principle of a hub-and-spoke network design, which mainly intends to construct network for economies of scale with less-redundant network structure (Klincewicz 1998, 2002; Campbell *et al.* 2002). For example, Table 2 presents a relationship between network reliability as well as resiliency and network costs between three different types of networks from Figure 1. The length of linkages is simply translated as network cost since building cost for linkages linearly increases with the

Table 1. Classes of hub location problems

Key design components	Description	Representative works
Objective function		
Fixed costs	• Physical facility building costs	Gavish (1992)
Variable costs	• Flow-based costs for linkages	O'Kelly and Bryan (1998)
Performance related costs	• Latency / Travel time	Kim (2008) / Kara and Tansel (2001)
	• Reliability / Congestions	Kim and O'Kelly (2009) / Elhedhli and Hu (2005)
Exogenous factors	• Game theory	Grove and O'Kelly (1986)
	• Scheduling	Kim (2004), Marianov and Serra (1999)
Constraints		
Hubs	• The number of hubs are fixed	O'Kelly (1986)
	• Hubs are endogenously determined	Bryan (1998)
Assignment types	• Single Assignment (SA)/ Multiple Assignment (MA)	O'Kelly (1987) / O'Kelly *et al.* (1996)
Topological variation	• Steiner tree/ Spanning tree/ Ring/ Star structure/ Hierarchical structure	Kim *et al.* (1995), Chamberland and Sansò (2000), Chung *et al.*, (1992)
	• Direct links among non-hub to hub	Aykin (1994)
Capacity restriction	• Uncapacitated/ Capacitated on Links or hubs	Ernst and Krishnamoorthy (1998), Bryan (1998), Ebery *et al.*(2000), Podnar *et al.* (2002)
Solution Approach		
QAP	• Quadratic Assignment Programming	O'Kelly (1987)
Linear programming relaxation	• Integer Programming	Campbell (1994b)
	• Linear Programming	Sohn and Park (1997)
	• Tight LP relaxation techniques	Skorin-Kapov *et al.* (1996)
Exact solution approach	• Dual Ascent or Lagrangean relaxation	Pirkul and Schilling (1998)
Heuristic approach	• Branch-and cut	Ernst and Krishnamoorthy(1998)
	• Greedy Search Algorithm	Klincewicz (1992)
	• Tabu Search Algorithm	Skorin-Kapov and Skorin-Kapov (1994)

Table 2. Comparison of network property according to structure

	Complete Network	Single Assignment	Multiple Assignment
Network reliability*	0.999	0.901	0.972
Network cost**	51.8	10.8	14.5
Resiliency***	28	6	8

* Network reliability is calculated based on the average value of reliabilities for all i-j pairs on the network. In order to compute the reliability for each i-j pair, this paper uses the inclusion-exclusion method, which calculates the reliability by including all possible paths' probabilities and excluding complimentary probability for given i-j pair (see Shier 1991 in detail).

** Network cost is simply computed by the total length of links in a geometric space.

*** Resiliency is measured by the smallest number of link failures, which result in total network flow loss.

physical lengths of links so that it is a major concern in network design (Campbell *et al.* 2002).[1] Not surprisingly, a fully connected network (Figure 1-(a)) has always a better reliability and resiliency than single- and multiple assignment hub networks (Figure 1-(b) and (c)). However, a fully connected network would not be realistic since too much redundancy is prohibitive in most network systems. Moreover, a fully connected network system is not encouraged in terms of scale of economies if a link is dedicated to serve a small amount of flows. In contrast, single assignment network model is comparatively a cost-saving strategy although the structure is the least resilient to possible network failure.

Two design components are stressed in reliable hub network design. First is to embed performance -related measures or variables into the model structure. As current network systems become more delay sensitive, Quality of Service (QoS) related variables such as reliability and traffic latency are reflected in hub network design (Skorin-Kapov *et al.* 2006; Klincewicz 2006). Second, to reflect the level of economic of scale, a *Minimum Threshold* (MT) on inter-hub links is suggested for the multiple assignment models. Exploring the model behaviors with regard to network resiliency and MT level provide insight to identify the appropriate level of capacity of linkages for network design (Bryan 1998).

3. Modeling RHLP

This paper presents two standard models of the RHLP based on the assignment scheme, named RHLP-SA (RHLP-Single Assignment model) and RHLP-MA (RHLP-Multiple Assignment model). In order to formulate the models, (1) a performance-related measure, the routing reliability R_{ijkm} and (2) the reliability factors α and γ, both of which are different compared to conventional hub location models, are considered in the objective function. The routing reliability refers the probability of successful traffic delivery rate. Let the decision variable X_{ijkm} denote the route where traffic is delivered between nodes i and j via hubs k and m ($i \rightarrow k \rightarrow m \rightarrow j$), then the coefficient of R_{ijkm} which represents the probability of traffic delivery with the route X_{ijkm} is calculated by multiplying the reliabilities of each links ($R_{ijkm}=r_{ik} \times r_{km} \times r_{ij}$). In order to reflect the facility's capability for transferring flows, two reliability factors alpha (α) and gamma (γ) are involved in this computation in order to reflect the level of transferability of inter-hub link ($r_{km}^{-\alpha}$) and hub ($r_{kk}(=\gamma)$), respectively. Note that the higher value of α or γ encourages use of inter-hubs or intra-hubs. The best route for each OD pair depends on what levels of these two factors are imposed in the model. Accordingly, the behavior of hub selection and the configuration of allocations are affected (see Kim and O'Kelly 2009 in detail). The formulation of the RHLP-SA is as follows.

1) The RHLP-SA

Maximize

$$Z=\sum_i \sum_j \sum_k \sum_m W_{ij} R_{ijkm} X_{ijkm} \tag{1}$$

Subject to

$$\sum_k H_{kk}=p \qquad (2 \le p) \tag{2}$$

$$\sum_k H_{ik}=1 \qquad \forall i \tag{3}$$

$$H_{ik}-H_{kk} \le 0 \qquad \forall i,\ k\ (i \ne k) \tag{4}$$

$$\sum_m X_{ijkm}-H_{ik}=0 \qquad \forall j>i;\ k \tag{5}$$

$$\sum_k X_{ijkm}-H_{jm}=0 \qquad \forall j>i;\ m \tag{6}$$

$$H_{ik} \in \{0,\ 1\} \tag{7}$$

$$0 \le X_{ijkm} \le 1 \tag{8}$$

where

p	the number of hubs to be located
W_{ij}	the amount of flow to travel between i and j
X_{ijkm}	the fraction of flow from origin i to destination j via hub k and m in that order
R_{ijkm}	the routing reliability for the route X_{ijkm}
H_{ik}	1 if node i is allocated to hub k; 0 otherwise
H_{kk}	1 if node k is a hub; 0 otherwise
α	Inter-hub reliability factor ($0 \le \alpha \le 1$)
γ	Intra-hub reliability factor ($0 \le \gamma \le 1$), $\gamma = r_{kk}$ or r_{mm}

The objective function (1) maximizes the total network flows of W_{ij} that can be delivered from origin i to destination j based on $R_{ijkm} \cdot X_{ijkm}$. Constraint (2) requires the number of p hubs to be open. However, this constraint is optional. Without this constraint, the model determines the number of hubs as a part of optimization process. Constraint (3) forces each node i to be allocated to only a single hub k. Constraint (4) requires a hub to be open before a node is assigned to the hub denoted as H_{kk}. Constraints (5) and (6) make sure that flow i to j should be routed via hubs k and m if origin i is assigned to hub k and j is linked to hub m. Integrality condition is given to the variables H_{ik} to prevent partial facility location. The formulation of the RHLP-MA is as follows.

2) The RHLP-MA

Maximize

$$Z=\sum_i \sum_j \sum_k \sum_m W_{ij} R_{ijkm} X_{ijkm} \tag{9}$$

Subject to

$$\sum_k H_k=p \qquad (2 \le p) \tag{10}$$

$$\sum_k \sum_m X_{ijkm}=1 \qquad \forall j>i \tag{11}$$

$$\sum_{m} X_{ijkm} - H_k \leq 0 \quad \forall j > i;\ k \tag{12}$$

$$\sum_{k} X_{ijkm} - H_m \leq 0 \quad \forall j > i;\ m \tag{13}$$

$$H_k \in \{0, 1\} \tag{14}$$

$$0 \leq X_{ijkm} \leq 1 \tag{15}$$

where

H_k 1 if node k is a hub; 0 otherwise

In the RHLP-MA, the objective function (9) is the same with (1). Constraint (10) is also optional but considered if p hubs are selected. Constraint (11) ensures that the flows from i to j should travel through hub(s) k and m, and k can be equal to m for one-hub stop route (e.g. X_{ijkk} or X_{ijmm}). Constraints (12) and (13) together ensure the flow between i to j to be routed via open hubs. Constraint (15) imposes binary integer restrictions on hub facility variables. Determining the optimal route for each ij pair in the RHLP-MA is more flexible since each i can choose the best reliable route out of all possible routes R_{ijkm} for open hub k and m.

3) The RHLP-MT

The standard MA model assumes all inter-hub links to be open and fully connected. However, this assumption would not be realistic if underutilized inter-hub links are a concern in hub network design. In other words, a less-utilized inter-hub should be suppressed to be open to achieve the certain level of economies of scale of flows. As a variant of the RHLP-MA, the RHLP-MT is formulated by adding flow constraints (16) to (18) which impose a threshold level on the inter-hub links.

$$\sum_{i} \sum_{j} W_{ij} R_{ijkm} X_{ijkm} \geq \Omega_{km} Y_{km} \quad \forall j > i;\ k,\ m(k \neq m) \tag{16}$$

$$X_{ijkm} - Y_{km} \leq 0 \qquad \forall j > i;\ k,\ m(k \neq m) \tag{17}$$

$$Y_{km} \in \{0, 1\} \qquad \forall k,\ m \tag{18}$$

where

Ω_{km} the minimum threshold (MT) level over inter-hub link between hubs k and m

Y_{km} 1 if inter-hub link km is open; 0 otherwise

Constraint (16) requires that the inter-hub link Y_{km} cannot be open unless the amount of inter-hub flow is greater than the minimum amount threshold Ω_{km}. In other words, the constraints prevent inter-hub link from being open unless the total amount of flows on the Y_{km} exceed or equal to the given MT level. Constraint (17) ensures that no route utilizing Y_{km} is used without opening the Y_{km}. Constraint (18) imposes the integer restriction for inter-hub link variables.

4) Data and model experiments

To explore the model behaviors, the IP network performance statistics of QWEST Internet service provider as of 2005 (http://stat.qwest.net) was obtained. The data represents the empirical statistics of traffic delivery rate of O-D flows among 15 U.S. cities.[2] It is worthwhile to noting that the reliability (r_{ij}) is highly correlated with physical distance of routes between nodes. As O-D traffic matrix (W_{ij}), the interaction potentials by the size of supply and demand between i and j cities are prepared based on the data by Zook (2000) and Atkinson and Gottlieb (2001).

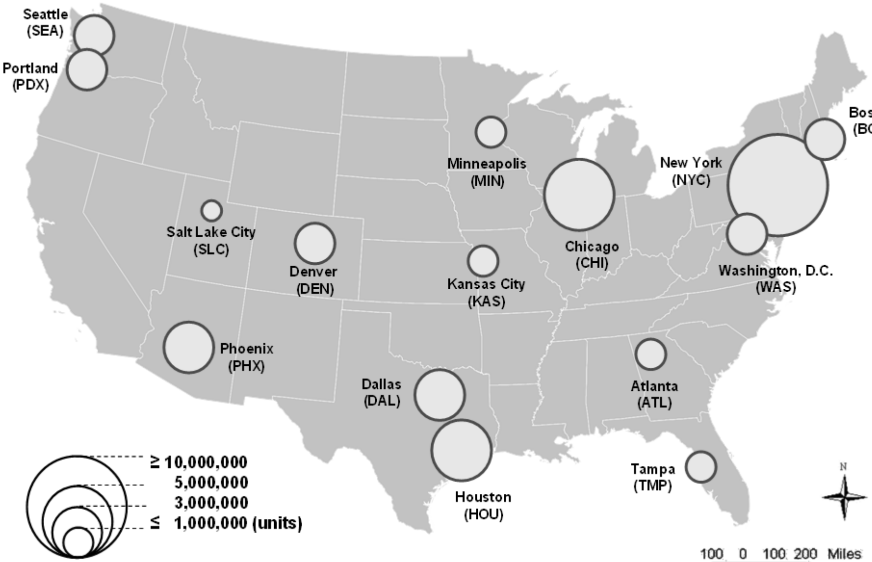


Figure 2. 15 U.S. city nodes and potential flows.

Figure 2 illustrates the location of 15 city nodes and the size of interactions. The experiments were made for four-, five- and six hub models (i.e. p=4, 5, and 6) under the selective parameter settings of both α and γ (=0.10, 0.60, and 0.99). All experiments are carried out on a XP server with Intel Core2 Duo 2.66 GHz processor with 2GB RAM, and all instances are solved with optimality using CPLEX 10.1. Solving time varies from a second in most cases to half an hour for some RHLP-SA instances.

4. Model behaviors

1) Standard Model Results

The model behaviors are examined in terms of the relationship among selection of hub locations, network costs, and flow economies of scale on inter-hub links. Table 3 summarizes the result of the standard SA and MA model response to the level of both factors and different number of fixed hubs (p).

Not surprisingly, the objective function of the model increases with the level of α and γ, which indicates that the performance of network relies on the level of reliability of hubs and inter-hub links. The scale economy of network flows of each instance is summarized by the indicator *INTFLOW*, which calculates a ratio between the total inter-hub flows against the number of inter-hub links. By definition, larger *INTFLOW* means that the flows on the network are more effectively delivered with intense utilization of the inter-hub links, achieving a better economy of scale. Notice that not only the objective value but also the *INTFLOW* of MA model is always greater than (or equal to) that of SA model under given same reliability condition because the network structure of SA model is the lower bound of MA model (O'Kelly and Bryan 1998). In detail, in the case of SA models, an increase of α and γ encourages O-D flows to utilize inter-hub links with fixed number of p-hubs. This behavior is generally experienced in the MA models. However, for some instances of the RHLP-MA models, the *INTFLOW* did not linearly respond to the increase of α and γ. For instance, with p = 6, the *INTFLOW* decreases when both parameter increase from 0.60 to 0.99 although the model facilitates utilization of inter-hub links for a certain level of both parameters. The reason for the model behavior is due to the flexibility of routings where model makes direct allocation between non-hub nodes to hub (i.e. $i \rightarrow k \rightarrow j$) if the benefit is greater than utilizing inter-hub links. As shown in Table 3, optimal hubs are not necessarily placed in the large cities (for example, New York and Chicago in our data). Rather, Atlanta, Dallas, Salt Lake City, and Kansas City, are frequently selected as optimal placement of hubs because the hub selections are influenced by not only the amount of interactions but also the relative locational advantage of a city

Table 3. Results of RHLP-SA and MA models

a) RHLP-SA

p-hub	Level of α, γ	Objective	Hubs	*INTFLOW**	Sol. time (sec)
4	0.10	12,528,690.3	ATL MIN SLC TMP	875,542	1562.52
	0.60	14,658,212.2	ATL MIN SLC TMP	997,494	102.66
	0.99	16,319,267.1	DAL MIN SLC WAS	1,568,971	0.28
5	0.10	12,371,849.6	ATL KAS MIN SLC TMP	548,095	150.36
	0.60	14,466,981.1	ATL KAS MIN SLC TMP	635,698	16.08
	0.99	16,341,661.0	DAL MIN PDX SLC WAS	943,220	0.28
6	0.10	11,794,537.4	ATL KAS MIN SLC TMP WAS	371,429	43.01
	0.60	14,206,771.8	ATL KAS MIN SLC TMP WAS	415,728	2.45
	0.99	16,354,814.5	ATL BOS DAL MIN PDX SLC	462,267	0.45

b) RHLP-MA

p-hub	Level of α, γ	Objective	Hubs	*INTFLOW*	Sol. time (sec)
4	0.10	15,906,047.1	ATL DAL KAS WAS	1,438,856	0.77
	0.60	16,065,629.4	DAL DEN KAS WAS	1,508,669	0.75
	0.99	16,319,267.0	DAL MIN SLC WAS	1,568,971	0.84
5	0.10	15,934,187.6	ATL DAL DEN KAS WAS	822,206	4.02
	0.60	16,102,192.1	ATL DAL KAS SLC WAS	977,845	0.39
	0.99	16,341,661.0	DAL MIN PDX SLC WAS	943,220	0.88
6	0.10	15,956,332.1	ATL BOS DAL DEN KAS WAS	375,565	0.53
	0.60	16,118,431.5	ATL CHI DAL KAS SLC WAS	681,689	0.55
	0.99	16,354,814.5	ATL BOS DAL MIN PDX SLC	462,267	0.52

* *INTFLOW* represents the amount of average flow utilizing Inter-hub links on the network. In detail, the index calculated by [Total amount of inter-hub flows/the number of inter-hub links].

which possess a higher potential to handle interactions. Interestingly, the different selection of hubs is influenced by the level of both factors. As shown in Table 3, for example, the model response is often to locate hubs dispersed with a separation as the level of both factors improves because a sufficient level of reliability in inter-hub links or hubs tends to reduce the distance of assignments between non-hub nodes to hubs, rather than hub to hubs. Note that the different hub selections are also observed between the RHLP-SA and MA models although the same condition (*p* and both factors) is supposed. The main reason of this behavior is due to the flexibility in determining the best routes for all *i-j* pairs. In theory, SA models should place hubs that can maximize routing reliabilities where the best route of each *i-j* pair should be explored within fixed arranged links between non-hub(s) to a hub. In the MA models, the best route for an *i-j* pair can be searched with more routing choices and completed independently regardless of other pairs' routing selection process.

2) Hub network design with minimum threshold

The models in the previous section only focus on the class of *uncapacitated p*-hub problems, which refers the problem condition that *p*-hubs are specified and no capacity constraints are assumed on inter-hub links. Although the characteristics of this class are well studied, however, the assumption is often unrealistic if a large *p* or an inter-hub link handles only a small amount of flows. In other words, the hub location problem often requires both the number of *p*-hub and inter-hub links to be determined under capacity restriction *simultaneously* as internal optimal processes. Within this hub network design, the inter-hubs, which cannot maintain a certain level of flow, are discouraged to be open, and the optimal *p* is also explored from the non-fixed number of hubs. As discussed by Bryan (1998), the idea of minimum threshold is useful for MA models to determine the level of capacity as well as the number of hubs for a cost-effective hub network design. In this paper, the RHLP-MT models for two extreme levels of reliability factors (α, γ=0.10, 0.99) are tested with the three different ranges of minimum threshold (400,000, 800,000, and 1,000,000). Numerical results are reported in Table 4. Of interest is to explore the response of the models relevant to what is the appropriate number of hubs and inter-hub links to the different level of minimum threshold. As shown in Table 4, to given 15 nodes network design, the optimal number of hubs is revealed as four regardless of change of reliability factors and MT levels. The network performance (i.e. objective function) decreases as the MT level increases because the best route of O-D pairs should be explored throughout only opened inter-hub links. In general, the higher MT level, the fewer inter-hub links are allowed to be open. As expected, the network cost (the 5th column) decreases as the level of MT increases, which proves that a larger MT is imposed, the more cost-effective network can be designed. To explore the level of flow economy of scale, the index *UNITFLOW*, the degree of utilization of inter-hub link, is devised. As presented in the last column, the higher MT level results in the higher intense utilization of inter-hub links.

Table 4. Hub network design (RHLP-MA) with minimum threshold

α, γ	MT level	Hubs	Objective	Network cost*	*UNITFLOW***
0.10	-	ATL DAL DEN KAS WAS	15,934,187.6	392,648	20.94
	400,000	ATL DAL DEN KAS WAS	15,932,874.5	215,417	39.77
	800,000	ATL DAL DEN KAS WAS	15,930,358.1	172,090	47.42
	1,000,000	ATL DAL DEN KAS WAS	15,930,284.0	118,050	62.94
0.99	-	DAL MIN PDX SLC WAS	16,341,661.0	302,777	31.15
	400,000	DAL MIN PDX SLC WAS	16,340,173.1	262,031	36.43
	800,000	DAL MIN PDX SLC WAS	16,332,950.8	196,824	48.13
	1,000,000	DAL MIN PDX SLC WAS	16,328,311.2	131,940	68.62

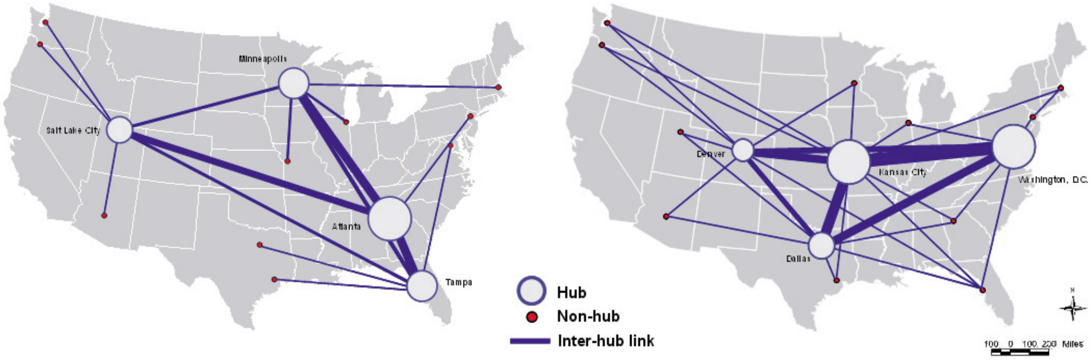
* Network cost is represented by the length of inter-hub links (miles).

** *UNITFLOW* represents the amount of inter-hub flows per unit cost (total length of inter-hub links). This index is computed as [Total amount of flows utilizing inter-hub links / Network cost].

3) Flow economy of scale, network costs, and resiliency

A network configuration of the SA and the MA model is presented in Figure 3 to explore the relationship between network performance and the network cost. As presented in the objective functions under the same condition of reliability factors, the MA model performs better to deliver the flows than the SA model. Not surprisingly, the multiple linkages between non-hub nodes and hubs in the MA model can improve the network's reliability. However, regarding network costs which can be translated into the total length of linkages miles, Figure 3 indicates that the network 3-(a) may be encouraged in hub network design due to its simple assignment structure. In contrast, the network 3-(b) is considered in a network planning where the network effectiveness for scale of economy is more weighted than the network cost.

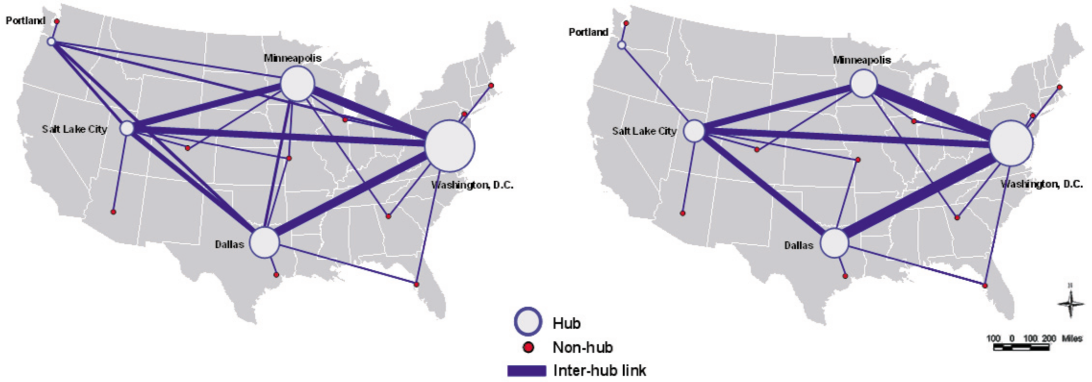
In terms of MT model as presented by Figure 4, for the instance of α, γ =0.99, three inter-hub links (PDX-MIN, PDX-WAS, and PDX-DAL) are suppressed when MT level (=1,000,000) is given because those do not meet the given MT level. In addition, as a response of limited inter-hub links, it is well observed that some multiple assignment links between non-hubs to hubs are suppressed to open or assigned to different hubs. More importantly, imposing a MT level makes O-D flows more agglomerated on opened inter-hub links so that the higher economy of scales of network flows can be achieved. Accordingly, this increased concentration of flows makes some hubs' excessive activity level of flows increase, which may cause a possible congestion on hub facility for transferring flows (Elhedhli and Hu 2005; Kim and O'Kelly 2009). Coupled with the result in Table 4, Figure 3 indicates that the increase of *UNITFLOW* degrades network reliability to possible malfunctions on inter-hub links.



Objective: 14,658,212.2
Network cost: 13,707 (miles)
(a) RHLP-SA

Objective: 16,065,629.4
Network cost: 21,038 (miles)
(b) RHLP-MA

Figure 3. Optimal four-hub location problems: RHLP-SA (a) and RHLP-MA (b) (α, β = 0.60)



Objective: 16,341,661.0
Network cost: 302,777 (miles)
Resiliency: 26
(a) MT level = none

Objective: 16,328,311.2
Network cost: 131,940 (miles)
Resiliency: 21
(b) MT level = 1,000,000

Figure 4. Comparison of hub networks with different MT levels with α, β = 0.99.

5. Concluding remarks

In a network, there would be a number of users and interacting movements, such as traffic or information flows among them, which should be delivered successfully. To satisfy this condition, it is required that the underlying network should ensure reliable transferring routes (or paths) between the nodes. Further, the overall performance of the network is regarded as a function of its ability to satisfy this requirement (Colbourn 1999).

Current network design and location models are required to take the measures representing the

QoS such as delay and loss of traffic into account in the model formulations, coupled with topological requirements such as single and multiple assignment to ensure a reliable network system. In this context, this paper introduces a new set of hub location models, named the RHLP. From this study, several important findings can be highlighted. First, although developing reliable network systems has been a significant issue in many fields, less attention has been paid to applying its concept in hub location problems or network design. The main reason behind this lack of discussion might be due to the property of hub network system where reliability appears to be less important component in the designing procedures. However, it is inevitable to embed the concept of reliability into hub network design since a number of current networks and infrastructures have been evolved into hub and spoke type configuration due to its effectiveness of flow control and economic benefits. Second, the main reason for designing a hub network is to construct a network which reflects flow economy of scale with agglomeration and re-distribution of O-D flows by operating a handful of hubs (Gavish 1992; Taaffe *et al*. 1996). As proposed in this paper, two indicators, *INTFLOW* and *UNITFLOW* can be used to gauge the intensity and cost-effectiveness of economy of scale of flows in hub location models. According to the results presented in the paper, the hub network system is an ideal network design process to achieve economy of scale of flows with minimum threshold on inter-hub links which suppress the opening of underutilized inter-hub links. In general, the number of inter-hub links decreases with increase of the MT level. This constraint is well adapted in telecommunications network or computer network systems (Chung *et al*. 1992; Kim *et al*. 1995). Third, the topological characteristic of hub network system can impact on the network resiliency to the possible malfunction on network components such as hub and inter-hub links. Basically, network resiliency is associated with the number of redundant links. As shown in our results, allowing multiple assignment models in hub location problems can improve network resiliency to possible malfunctions; however, it entails increased network costs. For better decision-making to determine the optimal level among network resiliency, reliability, MT level and costs, a multi-objective type hub model where these variables are considered in objective function can be suggested. By imposing different weights on each variable, the point to reconcile the contradictory components in network design can be explored.

Given that the paper focuses on the model behavior, further research could be extended by considering other modeling components with a large size of data. First of all, different levels of reliability factors would affect the optimization process to determine the number of hubs and linkages, and the locational change of hubs and assignment of linkages. Note that a level of reliability of α and γ may represent a stage of network evolution. For instance, a network with lower α and γ can represent an initial copper-based system with a lower technological level of hub facilities in telecommunications. In contrast, higher α and γ may reflect a current fiber-optic cable system such as Internet whose transferability is enhanced for wide networks (Grötschel *et al*. 1995). Finally, reliable hub location models are differently designed based on the field of network design. For example, the model employing single assignment scheme makes good sense for telecommunication network design because the cost for building hubs and linkages (i.e. physical facilities) is the most important concern in telecommunication networks. In contrast, multiple assignment models are considered in air-transportation network if reliability of the system is more of an issue and flow-based costs are the main concern of the model (O'Kelly and Bryan 2002; Klincewicz 1998). As a constraint to control of flow level, not only minimum threshold model, but also the capacitated model which impose the maximum amount of flows on inter-hub links or hub itself can be extended in future research.

Notes

1) Network cost can be classified into two categories, fixed cost and variable cost. The fixed cost refers the establishment cost including building cost for physical facilities such as cables, routers, and hubs. Variable cost is often called flow-dependent cost whose unit cost is changed with the level of utilization of flows on the links because of variable discounts. In general, fixed cost increases as more facilities or physical linkages are added or new travel routes are provided in the network. Accordingly, the network cost is a function of the length of total paths (or routes) and the number (or capacity) of established facilities. Flow-dependent cost can be reduced by achieving economies of scales of flows with less inter-hub links.

2) In this paper, only 15 nodes are selected from 25 nodes in original reliability matrix because other nodes except 15 nodes are ambiguous in its locations, which make the estimation of flows among nodes difficult.

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교신: 김현, 사우스 플로리다 주립대학교 지리학과, 전화: +1-813-974-9490, 이메일: hkim1@cas.usf.edu

Correspondence: Hyun Kim, Department of Geography, University of South Florida, 4202 E Fowler Ave, NES 107, Tampa, FL 33620 USA, Tel: +1-813-974-9490, E-mail: hkim1@cas.usf. edu

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신뢰성에 기반한 허브 입지 모델과 네트워크 디자인

김현*

요약: 허브-스포크 망은 항공, 육상 교통 및 정보통신분야에서 유동의 규모의 경제를 고려하여 기간망을 설계하는 대표적인 입지 최적화 문제이다. 이 연구는 우선, 기존의 허브 스포크 망에 관한 지리학 및 타 분야의 연구 경향과 함께 최근 논의되고 있는 신뢰성에 기반한 허브 입지 모델을 소개한다. 기본 모델은 입지 배분 방식에 따라 단일 배분 및 다중 배분 입지 모델로 구분되고, 기본 모델의 확장 형태로서 최소 유동량 제약을 적용한 입지 디자인 모델이 제시되었다. 모델의 행태를 살펴보기 위해 미국의 15개 도시의 신뢰성 및 유동 자료를 이용하여 네트워크의 규모의 경제성과 네트워크 비용, 그리고 신뢰성간의 관계에 초점을 두어 분석을 하였다. 이 연구는 경제성과 신뢰성에 바탕을 둔 입지 문제 및 네트워크 최적화 분야에 고려해야 할 요인의 특성을 밝혔다는 점에서 그 중요성이 강조된다.

주요어: 허브 입지 모델, 네트워크 디자인, 신뢰성, 유동의 규모의 경제, 네트워크 탄력성

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* GIS 프로그램 디렉터, hkim1@cas.usf.edu