Adaptive States Feedback Control of Unknown Dynamics Systems Using Support Vector Machines

Fa-Guang Wang, Min-Chan Kim, Seung-Kyu Park and Gun-Pyong Kwak, *Member, KIMICS*

Abstract—This paper proposes a very novel method which makes it possible that state feedback controller can be designed for unknown dynamic system with measurable states. This novel method uses the support vector machines (SVM) with its function approximation property. It works together with RLS (Recursive least-squares) algorithm. The RLS algorithm is used for the identification of input-output relationship. A virtual state space representation is derived from the relationship and the SVM makes the relationship between actual states and virtual states. A state feedback controller can be designed based on the virtual system and the SVM makes the controller with actual states. The results of this paper can give many opportunities that the state feedback control can be applied for unknown dynamic systems.

Index Terms—Recursive Least Squares Algorithm, State Feedback Controller, Support Vector Machines.

I. INTRODUCTION

THE SVM is one of the methods by which the statistical learning theory can be introduced into solving the pattern recognition problem with small samples and learning problems such as function estimation [1]-[5].

There are classification problems and regression problems in SVM. Classification and regression still can be classified as linear and nonlinear problem. So there are linear classification, nonlinear classification, linear regression and nonlinear regression respectively. In the linear classification part, there also have hard margin for easy classification problems and soft margin for complex classification problems [6]. In nonlinear classification and nonlinear regression SVM, there are different nonlinear kernel function can be applied into SVM. Different kernel function can supply different properties for SVM. How kernel function is chosen depends on the designer and his experience. The difference between classification SVM and regression SVM is the choosing of optimal algorithms [7]. In classification, it is working for getting the minimization margin between different points which need to be classified. The regression SVM are controlled by loss functions defined at first. The benefit of SVM is clearly for controller designing and system identification [8]. So it is widely used.

Only the support vector regression problem is used in this paper.

The main idea of this paper is to make a virtual system by using some adaptive identification algorithm and the virtual states can be related to actual states by using the trained SVM. This paper is about linear systems and the RLS is used to derive an input-output relationship and virtual state space representation is derived from it. A state feedback controller can be designed based on the virtual system and virtual states can be substituted by actual states by using the SVM relationship. As a result, an actual state feedback controller can be designed for unknown dynamic systems.

In this paper, part one introduces SVM briefly and the main idea of this paper. Part two focuses to system model formulation. Part three applies SVM into system model. Part four is the computer simulation result. Last part is the conclusion.

II. PROBLEM FORMULATION

Each Consider the following system

$$X(k+1) = AX(k) + BU(k) \tag{1}$$

The states are measurable. The above system dynamic is unknown except that it is linear. It is required that a state feedback controller is designed without the knowledge of system dynamics. First thing to do is to identify the input-output relationship of the unknown system as a transfer function. The following well-known RLS algorithm can be used for the identification with input-output data [5].

$$y(i) = \varphi_1(i)\theta_1^0 + \varphi_2(i)\theta_2^0 + \ldots + \varphi_n(i)\theta_n^0 = \varphi^T(i)\theta^0 \tag{2}$$

where $\varphi_j(i)$ is regression vector and θ_j^0 is parameter vector.

The algorithm is as follows.

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$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)(y(t) - \varphi^T(t)\hat{\theta}(t-1))$$
$$K(t) = P(t)\varphi(t) = P(t-1)\varphi(t)(\lambda t + \varphi^T(t)P(t-1)\varphi(t))^{-1} \quad (3)$$
$$P(t) = (I - K(t)\varphi^T(t))P(t-1)/\lambda$$

where P is defined as

$$p(t) = (\phi^T(t)\phi(t))^{-1} \quad (4)$$

and λ is the exponential increases rate of P.

Based on the transfer function derived by RLS, a virtual system can be determined as follows [9].

$$Z(k+1) = A_Z Z(k) + B_Z U(k) \quad (5)$$

where $A_z = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & -a_3 & \cdots & -a_n \end{bmatrix}$ and $B_z = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$.

Next step is to derive the relationship between the actual states and virtual states by using SVM.

The following flow chart shows that overall procedure in this paper.

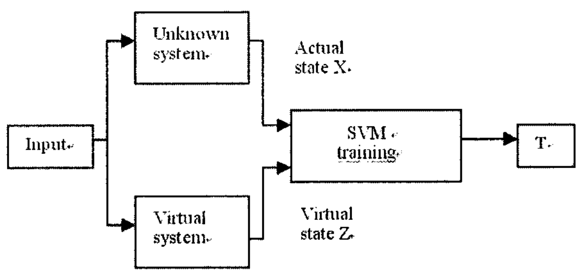

Fig. 1 overall procedure in this paper.

In the above block diagram, T is the transformation matrix as follows.

$$Z = TX \quad (6)$$

In a known system, the transformation matrix can be obtained by using system matrix A and B. However, the system parameters are assumed unknown in this paper. So SVM must be used to obtain the transformation matrix T with measured actual states.

III. SUPPORT VECTOR MACHINES AND ESTIMATION OF TRANSFORM MATRIX

The SVM is one of the statistical learning theory which can be introduced into solving the pattern recognition problem with small samples and the learning problems such as function estimation [2].

The basic idea in SVM is to map the input data x into a higher dimensional feature space.

In SVM method, the basic function is approximated by the following function formation.

$$y(x) = w^T f(x) + b \quad (7)$$

where $x \in R^n$ are inputs, w and b are coefficients. The coefficients are estimated by maximizing the margin width which defines the nearest distance between the points (support vectors) belongs to different groups respectively. The margin width is $M = \frac{2}{|w|}$.

The maximization problem is the same problem of minimizing

$$\frac{1}{2} w^T w \quad (8)$$

This problem leads to a following dual problem where a Lagrange multiplier α is associated with every constraint. The problem is to find $\alpha_1 \ldots \alpha_N$ such that

$$\begin{cases} \text{maximize } Q(\alpha) = \sum \alpha_i - \frac{1}{2} \sum\sum \alpha_i \alpha_j y_i y_j x_i^T x_j \\ \sum \alpha_i y_i = 0 \end{cases} \quad (9)$$

where $\alpha_i \geq 0$.

The solution has the form as follows.

$$w = \sum \alpha_i y_i x_i^T, \; b = y_k - w x_k \quad (10)$$

for any x_k the corresponding $x_k \alpha_k \neq 0$.

To a non-zero α_i, a corresponding $\mathbf{x}_i$ is a support vector. Then the classifying function will have the form:

$$f(x) = \sum \alpha_i y_i x_i^T x + b \quad (11)$$

let $\phi(x) = x_i^T x$, then the $f(x)$ is expressed as

$$f(x) = \sum \alpha_i y_i \varphi(x) + b \quad (12)$$

If $\phi(x) = x_i^T x$, it is a linear SVM. Here $\phi(x)$ also can use a kernel function K(x) as a substitute, then

$$f(x) = \sum \alpha_i y_i K(x) + b \quad (13)$$

It is called as a nonlinear SVM. $K(x): R^n \to R^k (k > n)$

maps the input space into a higher dimension feature space. And the advantage of this kind of kernel function is that many classifying problem will be easily solved in high dimension. Only support vectors are used to specify a hyper-plane which classifies some groups.

The SVM can also be applied to regression problems by the introduction of an alternative loss function. The loss function must be modified to include a distance measure. The following figure illustrates four possible loss functions.

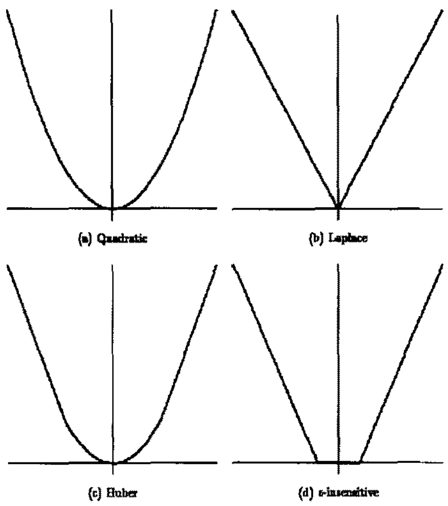

Fig. 2 four loss function

Karl Friedrich Gauss formulated the principle of least squares at the end of the 18th century. Gauss stated that, according to this principle, the unknown parameters of a mathematical model should be chosen in such a way that the sum of the squares of the differences between the actually observed and the computed values, multiplied by numbers that measure the degree of precision, is a minimum. Using the SVM, the procedure to obtain the relationship between actual states and virtual states is shown as following flow graph.

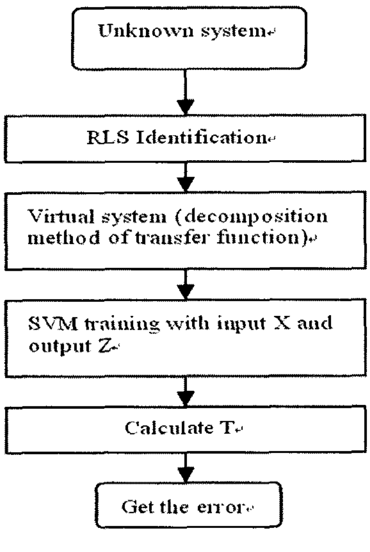

Fig. 3 flow graph

By using the estimated transformation matrix P, the system parameter A, B can be derived from the virtual system. A state feedback controller can be designed based on the virtual states and the virtual states can be replaces by actual states with the relationship established by SVM. And another way of state feedback controller design is based on the estimated actual state equation.

IV. COMPUTER SIMULATIONS

To describe the procedure of state feedback controller design is trivial. So the computer simulation is focused on the derivation of transformation matrix T.

The following system is considered

$$\begin{aligned} x_1(k+1) &= 0.9x_1(k) + u(k) \\ x_2(k+1) &= 0.8x_2(k) + u(k) \\ y(k) &= x_1(k) + x_2(k) \end{aligned} \tag{14}$$

where the initial value is X= [0, 0].

First step is to obtain input-output relationship by using the RLS algorithm. The input u for identification is pulse train.

The following input-output relationship is derived by using the RLS algorithm.

$$y(k+2) = 1.7y(k+1) - 0.72y(k) + 2u(k+1) - 1.7u(k) \tag{15}$$

From the above relationship, the following states equations are obtained by using the decomposition method of a transfer function.

$$\begin{aligned} &Z_1(k+1) = Z_2(k) \\ &Z_2(k+1) = 0.799Z_2(k) + 0.0684Z_1(k) + u(k) \\ &Y(k) = 1.927Z_2(k) + 0.0556Z_1(k) \end{aligned} \tag{16}$$

For the training of SVM, the virtual states are obtained from the simulation. Considering virtual states as target data and actual states as input data, the SVM can be trained.

By using the linear SVM, the relationship between Z and x can be obtained as follows.

$$\begin{aligned} Z_1(k) &= 0.998x_1(k) + 1.001x_2(k) \\ Z_2(k) &= 0.890x_1(k) + 0.801x_2(k) \end{aligned} \tag{17}$$

$$Z = T_{est}X = \begin{bmatrix} 0.998 & 1.1001 \\ 0.890 & 0.801 \end{bmatrix}$$

To check the SVR approximation, the following state transformation matrix is derived.

$$\begin{aligned} Z &= TX \\ A_Z &= TAT^{-1} \end{aligned} \tag{18}$$

where $T=\begin{bmatrix} 1 & 1 \\ 0.9 & 0.8 \end{bmatrix}$

The above T is almost same with the estimated T_{est}.

By using the above equation, the actual state Z can be calculated and compared with the SVR approximated states.

The following figures shows that the actual state Z can be derived from the actual states X by using the SVC and this means that the state feedback controller can be designed for unknown dynamic systems.

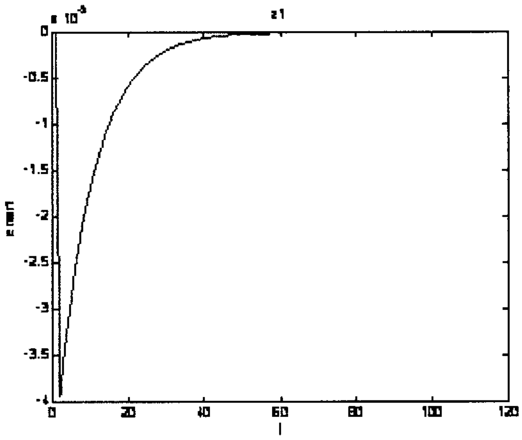

Fig. 4 Errors between actual state z1 and its estimated state using the SVM

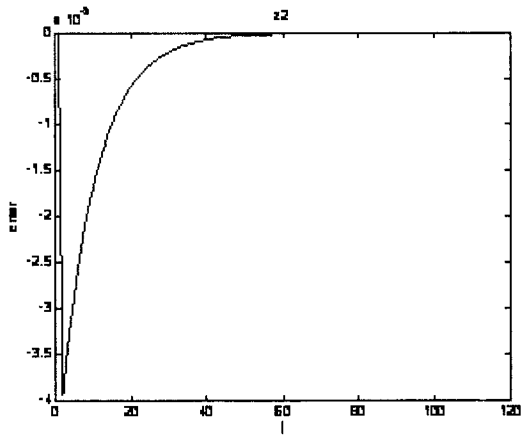

Fig. 5 Errors between actual state z2 and its estimated state using the SVM

V. CONCLUSIONS

A novel method, which makes it possible that a state feedback controller can be designed for unknown dynamic system with measurable states, is proposed. The RLS algorithm is used for the identification of input-output relationship and the SVM is used to obtain the relationship between actual states and virtual states. The results of this paper make it is possible to the state feedback control theory to be used for unknown dynamic systems. This result can be expected to be applied to unknown nonlinear systems.

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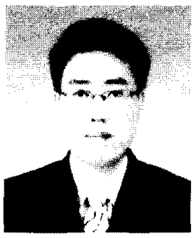

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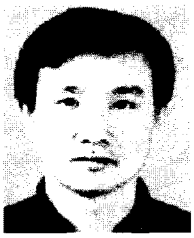

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