Korean Journal of Remote Sensing, Vol.24, No.5, 2008, pp.437~444

Selecting Optimal Basis Function with Energy Parameter in Image Classification Based on Wavelet Coefficients

Hee Young Yoo*†, Kiwon Lee, Hong Sung Jin***, and Byung-Doo Kwon***

*Dept. of Earth Science Education, Seoul National University
**Dept. of Information System Engineering, Hansung University
***Dept. of Applied Mathematics, Chonnam National University

Abstract : Land-use or land-cover classification of satellite images is one of the important tasks in remote sensing application and many researchers have tried to enhance classification accuracy. Previous studies have shown that the classification technique based on wavelet transform is more effective than traditional techniques based on original pixel values, especially in complicated imagery. Various basis functions such as Haar, daubechies, coiflets and symlets are mainly used in 2D image processing based on wavelet transform. Selecting adequate wavelet is very important because different results could be obtained according to the type of basis function in classification. However, it is not easy to choose the basis function which is effective to improve classification accuracy. In this study, we first computed the wavelet coefficients of satellite image using ten different basis functions, and then classified images. After evaluating classification results, we tried to ascertain which basis function is the most effective for image classification. We also tried to see if the optimum basis function is decided by energy parameter before classifying the image using all basis functions. The energy parameters of wavelet detail bands and overall accuracy are clearly correlated. The decision of optimum basis function using energy parameter in the wavelet based image classification is expected to be helpful for saving time and improving classification accuracy effectively.

Key Words : Wavelet transform, Basis function, Energy parameter, Image classification.

1. Introduction

Land-cover or land use classification is one of the important applications in the field of remote sensing. The classification results are often used to basic information for integrating and analyzing with other data. Therefore, many efforts have been tried to obtain more accurate classification results. Recently, image classification using spatial information as well as spectral information is one of the most active areas of remote sensing. Spatial information based classification is non-traditional method and considers neighbour pixels. Image classification using wavelet coefficients is also one of the spatial information based classification methods to improve classification accuracy. Yoo *et al.* (2007) showed that wavelet coefficients based image classification is known for more effective classification method than traditional

Received September 25, 2008; Revised October 10, 2008; Accepted October 16, 2008.
† Corresponding Author: Hee Young Yoo (skyblue@mantlc.snu.ac.kr)

pixel values based classification method.

Wavelet transform is able to use basis functions having various forms and lengths in contrast to Fourier transform using sinusoidal functions to decompose data. If different basis functions are used, totally different wavelet coefficients will be acquired. And the accuracy of image classification is greatly influenced by the type of basis function. Thus the selection of a suitable basis function is important for the image classification in wavelet domain. A study has focused on selecting basis function using local maxima of electrocardiogram (ECG) signal in wavelet domain (Singh and Tiwari, 2006). Most previous studies tested some basis functions which are frequently used to process images and one basis function is suggested as the best basis function. After classifying the image in wavelet domain using all basis functions, we can choose the best basis function that is shown the best result however this method is very time-consuming. If the optimal basis function is selected before the image classification based on wavelet coefficients, we may save time and classify effectively satellite images. In this study, we analyze relationship between the energy parameter in wavelet domain and classification accuracy by the type of basis function and intend to help to select optimal basis function before classification.

2. Methodology

1) The fundamentals of wavelet transform

The basic idea of wavelet transform is to represent any arbitrary function as a superposition of basis function, that is, wavelets (Equation 1). Wavelets are functions that satisfy certain requirements of being integrated to zero, especially the wave above and below the x-axis (y=0) and insure quick and easy calculation of the direct and inverse wavelet transform.

$$f(a,b) = \frac{1}{|a|^{1/2}} \int \psi\left(\frac{x-b}{a}\right) f(x)dx \tag{1}$$

where $\psi(t)$ is a continuous function in both the time domain and the frequency domain called the mother wavelet, a is positive and defines the scale and b is any real number and defines the shift.

It is computationally difficult to analyze a signal using all wavelet coefficients. A discrete wavelet transform (DWT) is any wavelet transform for which the wavelets are discretely sampled. Discrete wavelet transform analyzes from signal to approximation component and detail component using a sort of low pass filter and high pass filter (Fig. 1).

$$f(x) = \sum_k c_{Jk}\phi_{Jk}(x) + \sum_{j=1}^{J} \sum_k d_{jk}\psi_{jk}(x) \tag{2}$$

Where c_{jk} are the scaling coefficients and d_{jk} are the wavelet coefficients. The first term (ϕ_{Jk}) in

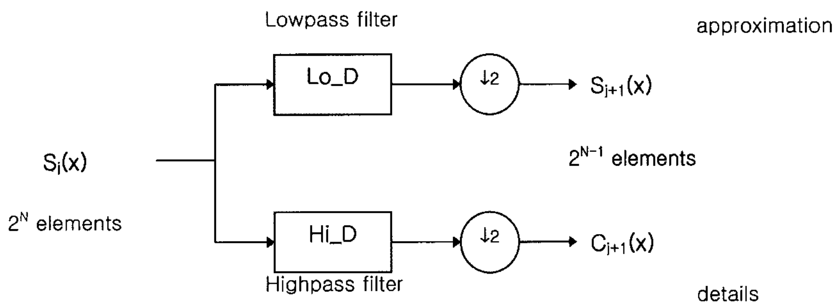


Fig. 1. The diagram of discrete wavelet transform.

Equation 2 gives the low-resolution approximation of the signal while the second term gives the detailed information at resolutions from the original down to the current resolution (Pajares and Cruz, 2004).

The 2D DWT results are obtained by first filtering the signal in row direction then re-filtering the output in column direction by the same filters. Digital Image processing is based on the 2D DWT. When an image is decomposed by 2D DWT in one level, four sub-bands are formed: LL, LH, HL and HH. Each sub-band contains approximation component, horizontal detail component, vertical detail component and diagonal detail component, respectively. The previous works of wavelet based texture image processing were focused on classifying multi-level images followed by texture measures and segmentation of wavelet coefficients (Arivazhagan and Ganesan, 2003; Myint, 2003).

2) The properties of basis functions

There are different types of wavelet families whose qualities vary according to several criteria. The main criteria are the support of forward and inverse transform, the symmetry, the number of vanishing moments, the regularity, the existence of a scaling function and the orthogonality or the biorthogonality (Daubechies, 1992). The details on each criterion are illustrated in Yoo *et al.* (2008). Among various basis functions, we used the ten different basis functions for wavelet decomposition in horizontal and vertical directions: Haar, db4, db8, sym1, sym4, sym8, coif1, coif2, bior3.5 and bior3.7 (Fig. 2). The used basis functions are explained with these criteria in detail (Table 1). Biorthogonal wavelet family exhibits the property of linear phase, which is needed for signal and image reconstruction. By using two wavelets, one for decomposition (on the left side in Fig. 2 (i)

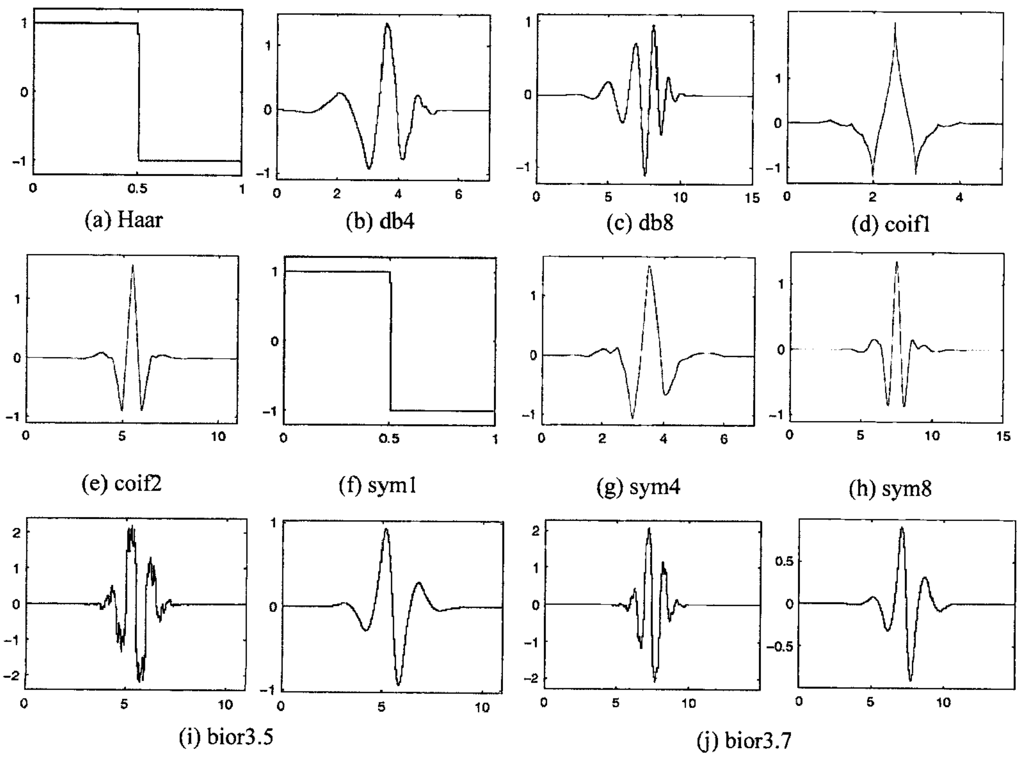


Fig. 2. Ten basis functions used for wavelet decomposition.

Table. 1. Summary of wavelet families and associated properties

Property	Haar	dbN	symN	coifN	biorNr.Nd
Arbitrary regularity		●	●	●	●
Orthogonal	●	●	●	●	
Biorthogonal					●
Symmetry	●				●
Asymmetry		●			
Near symmetry			●	●	
Arbitrary number of vanishing moments		●	●	●	●
Vanishing moments for Φ				●	
Existence of Φ	●	●	●	●	●
Filters length	2	2N	2N	6N	max(2Nr,2Nd)+2

and (j)) and the other for reconstruction (on the right side in Fig. 2 (i) and (j)) instead of the same single one, interesting properties are derived. Ten basis functions have different forms and lengths. Even though same images are used, but the used basis functions are different, consequently wavelet coefficients will be different.

3) Energy parameter for selecting the optimal basis function

To select the optimal basis function, we used the energy parameter in this study. The energy parameter of signal is the sum of the squares of wavelet coefficients and it is computed from sub-bands after wavelet decomposition process. The energy parameter is calculated by the Equation 3.

$$\varepsilon_f = \sum_{i=1}^{l} \sum_{j=1}^{m} f_{i,j}^2 \tag{3}$$

Where $f_{i,j}$ is wavelet coefficient of sub-band in wavelet domain.

Wavelet transformation conserves the energy of original signal or image. Even if one image is decomposed by wavelet transform via different basis functions, the sum of energy is always equal each other. The energy parameter of the trend sub-band, L band, usually accounts for a large percentage of the energy of the transformed signal.

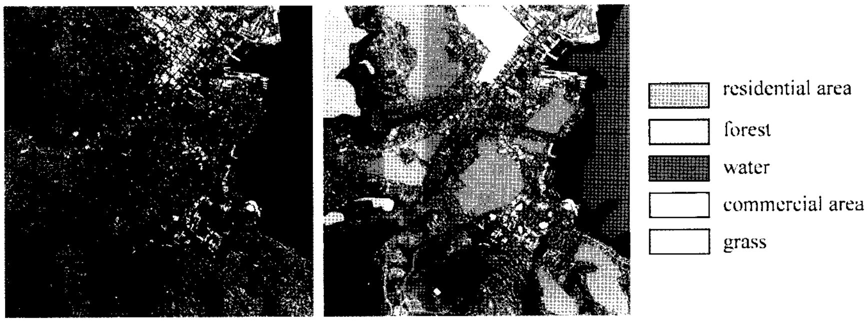
The energy parameter is calculated by sub-bands after the wavelet decomposition and the energy parameter of each sub-band can be a favorable feature of texture because it indicates dominant spatial and spectral channels of the original image (Fukuda and Hirosawa, 1999). Pixel value's changes in image are indicated by energy parameter in wavelet domain and energy parameter is expected to relate to classification accuracy.

In this study, satellite imagery is decomposed using ten basis functions and their wavelet coefficients are used for image classification. And then the relationship between classification accuracy and the energy parameter is analyzed.

3. Results and Discussion

1) The test data set

The Ikonos imagery in Hobart, Australia used for this study is one of ISPRS dataset collections (http://www.isprs.org/data/index.html) and is composed of red, green, blue, near infrared bands (4m). The training and reference data sets for supervised classification are indicated on the test image in Fig. 3. The reference data is divided into residential area, forest, water, commercial area and



(a) (b)

Fig. 3. Training and reference data sets for supervised classification with class indexes: (a) training data set, (b) reference data set.

grass (Fig. 3(b)). The number of reference pixels by class is 142277 in residential area, 42546 in forest, 84391 in water, 29224 in commercial area and 1892 in grass, respectively. This reference set was collected from the image by visual check because the classes in high resolution image can be divided visually. The training data set is obtained 10% random sampling of each reference data set (Fig. 3 (a)). The explanation between the class and the color is given in the right part of the reference data set.

2) The correlation between energy parameter and classification accuracy

The used image has four spectral bands: blue, green, red and near infra-red and each band is decomposed in one level with 10 basis functions. The energy of each sub-band is calculated by the basis function. We can get 4 energy parameters by 4 spectral bands. Four energy parameters are obtained from LL, LH, HL and HH sub-band. The energy parameters in each spectral band are changed with the basis function used but trend or relative measure does not change as a Fig. 4 indicates. Thus the sum of each sub-band's energy parameter of all spectral bands can be used to compare with the classification accuracy (Fig. 5). Fig. 6 shows the accuracy of the test image. Overall accuracy is displayed by Fig. 6 (a) and Fig. 6 (b) shows the accuracy of each class. Table 2 shows accuracies in number. In cases of residential area and water classes, the accuracies shows very small changes by basis functions but forest and grass class shows remarkable changes. In case of forest class, sym8 basis has the lowest accuracy but Haar and sym1 have the highest accuracy. The accuracy of coif1 is the highest but that of db4 is the lowest in commercial area class. In case of using coif1, the grass class has the highest accuracy. Meanwhile, the lowest accuracy is shown using db8. In overall accuracy, the case of coif1 basis function shows the highest accuracy but bior3.7 has the lowest accuracy. On the contrary, the energy parameter of bior3.7 is the highest and the energy parameters of Haar and sym1 are the lowest in the energy of LL band as we can see from Fig. 5 and Table 3. The energy parameter of coif1 is relatively low. The energies of LH, HL, HH sub-bands show similar tendency. Contrary to LL sub-band, the energies of Haar, sym1, coif1 have high figures while that of bior3.7 is low. The energies of LH, HL, HH sub-bands are not the same to classification accuracy but almost similar. A correlation between the energy parameter of each sub-band and overall accuracy is calculated for confirming how similar the trend of energy parameter and overall accuracy is. Table 4 summarizes the

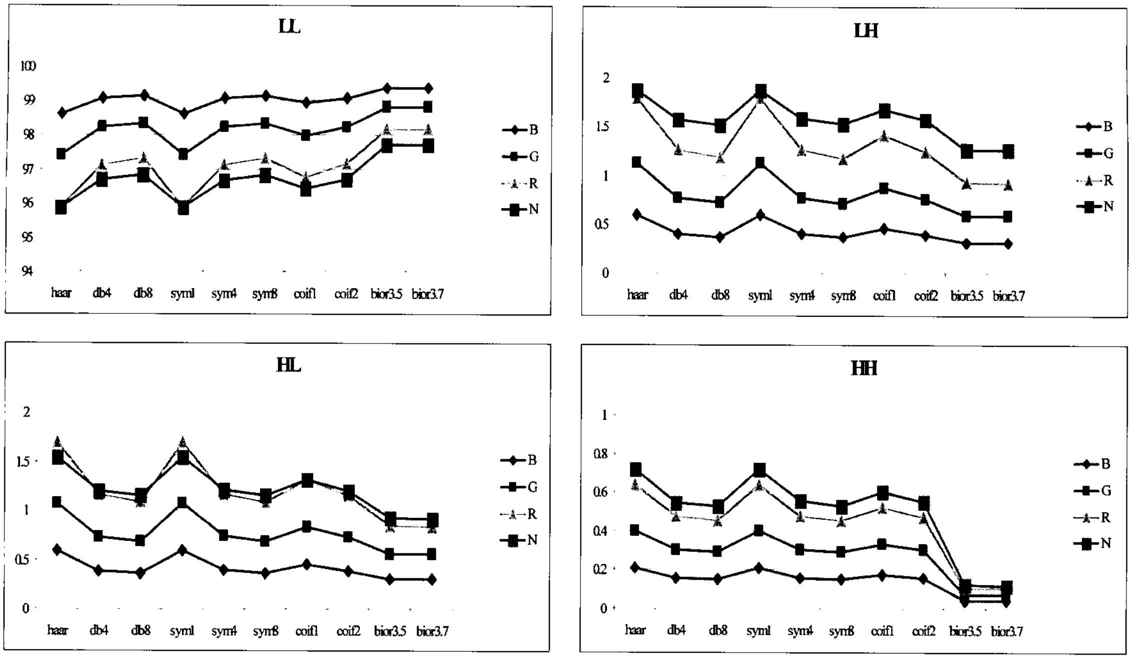


Fig. 4. The energy parameters of wavelet decomposition's sub-bands (LL, LH, HL and HH) about each spectral band (blue, green, red and near infra-red) expressed as percentages.

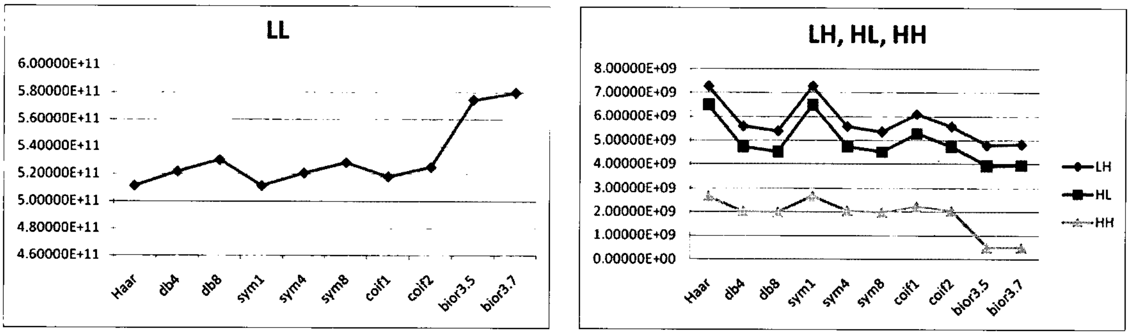


Fig. 5. The energy parameters of wavelet decomposition's sub-bands: the energy parameter of LL band according to basis function (left) and the energy parameter of LH, HL, HH bands according to basis function (right).

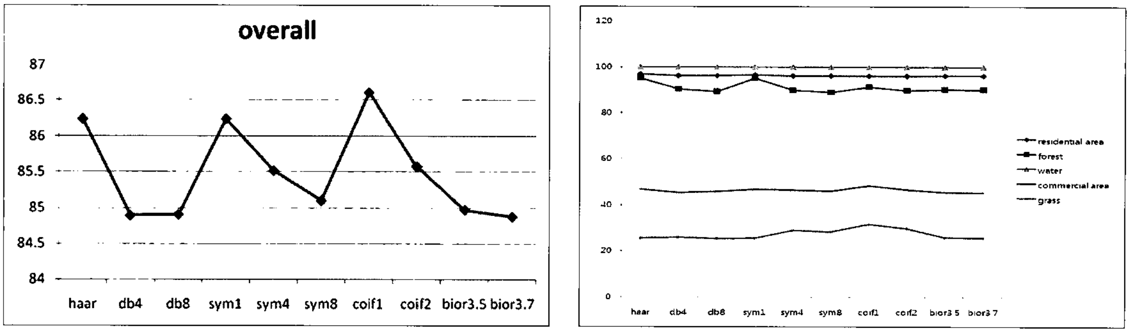


Fig. 6. The classification accuracy of test image: overall accuracy (left) and accuracies by classes (right).

Table. 2. The user's accuracies according to basis function

	residential	forest	water	commercial	grass	overall
Haar	96.90	95.22	100.00	46.67	25.43	86.24
db4	96.23	90.40	99.98	45.29	25.72	84.89
db8	96.25	89.37	99.98	45.75	25.31	84.91
sym1	96.90	95.22	100.00	46.67	25.43	86.24
sym4	96.17	89.87	99.98	46.46	28.89	85.52
sym8	96.18	88.97	99.98	45.86	28.23	85.10
coif1	96.13	91.44	99.98	48.30	31.67	86.61
coif2	96.14	89.83	99.98	46.53	29.67	85.58
bior3.5	96.25	90.30	99.99	45.49	25.76	84.97
bior3.7	96.23	90.16	99.98	45.34	25.67	84.88

Table. 3. The energy parameters of wavelet sub-bands by basis function

	LL	LH	HL	HH
Haar	5.11615E+11	7.26608E+09	6.50720E+09	2.68796E+09
db4	5.21887E+11	5.59201E+09	4.74385E+09	2.04345E+09
db8	5.30312E+11	5.38523E+09	4.54034E+09	1.98828E+09
sym1	5.11615E+11	7.26608E+09	6.50720E+09	2.68796E+09
sym4	5.20755E+11	5.58504E+09	4.75925E+09	2.06307E+09
sym8	5.28373E+11	5.36918E+09	4.52230E+09	1.99254E+09
coif1	5.18172E+11	6.09737E+09	5.28874E+09	2.24039E+09
coif2	5.25024E+11	5.58311E+09	4.74375E+09	2.05982E+09
bior3.5	5.74210E+11	4.80296E+09	3.94630E+09	5.03877E+08
bior3.7	5.79390E+11	4.84169E+09	3.96520E+09	5.06315E+08

Table. 4. The correlations between overall accuracy and the energy parameters of wavelet sub-bands

	LL	LH	HL	HH
Correlation	-0.62115	0.808677	0.810861	0.656627

correlation coefficients between energy parameters of sub-bands and overall accuracy. There is a negative correlation between overall accuracy and the energy parameter of LL band (-0.62115). The correlations of overall accuracy and the energy parameters of LH band and HL band are higher than 0.8 and there are strong correlations. The correlation with the energy parameter of HH band is relatively low but positive. Consequently, the energy parameter especially in LH and HL bands helps to select optimal basis function before image classification based on wavelet coefficient.

4. Conclusions

In this study, the relationship between classification accuracy and basis functions was analyzed when images are classified using wavelet coefficients. We studied whether it is possible to decide optimal basis function for wavelet based image classification. Wavelet decompositions were carried out using 10 different basis functions and the energy parameters by the type of basis function were compared with the classification accuracies based on their wavelet coefficients. It was found from the result that energy parameter has a connection with the accuracy of image classification. The energy of LL sub-band and overall accuracy are negative correlation while the energy of LH, HL and HH sub-bands and overall accuracy are positive one. Especially, the energy parameters of LH and HL band have strong correlation with overall accuracy. The energies of LH, HL, HH sub-bands are not the same order as classification accuracy but remarkably consistent with accuracy. In this study, only one image is classified using various basis function and its result is analyzed with energy parameter. Further applications on many images with different locations and resolution would clarify the association of classification accuracy and basis function and it is expected to be useful for selecting optimal basis function in wavelet based image classification.

Acknowledgement

This research was partly supported by a grant (08KLSGC03) from Cutting-edge Urban Development - Korean Land Spatialization Research Project funded by Ministry of Land, Transport and Maritime Affairs of Korean government.

References

Arivazhagan, S. and L. Ganesan, 2003. Texture segmentation using wavelet transform, *Pattern Recognition Letters*, 24, 3197-3203.

Daubechies, I., 1992. Ten lectures on wavelets, *CBMS, SIAM*, 61: 194-202.

Fukuda, S. and H. Hirosawa, 1999. A wavelet-based texture feature set applied to classification of multifrequency polarimetric SAR images, *IEEE Transactions on Geoscience and Remote Sensing*, 37(5): 2282-2286.

Pajares, G. and J. M. de la Cruz, 2004. A wavelet-based image fusion tutorial, *Pattern Recognition*, 37 (9): 1855-1872.

Myint, S. W., 2003. *The Use of Wavelets for Feature Extraction of Cities in Satellite Images*, Remotely Sensed Cities (Victor Mesev, editor), Taylors, Frances

Singh, B. N. and A. K. Tiwari, 2006. Optimal selection of wavelet basis function applied to ECG signal Denoising, *Digital Signal Processing*, 16: 275-287.

Yoo, H. Y., K. Lee and B. D. Kwon, 2007. Application of the 3D Discrete Wavelet Transformation Scheme to Remotely Sensed Image Classification, *Korean Journal of Remote Sensing*, 23(5): 355-363.

Yoo, H. Y., K. Lee and B. D. Kwon, 2009. A Comparative Study of 3D DWT Based Space-borne Image Classification for Different Types of Basis Function, *Korean Journal of Remote Sensing*, 24(1): 57-64.