

적외선 탐색 및 추적장비의 동적 특성 및 안정화

Dynamic Characteristics and Stability of an Infrared Search and Track

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ABSTRACT

Current paper investigates the dynamic behavior and stability of an infrared search and track subjected to external disturbance having gimbal structure with three rotating axes keeping constant angular velocity in the azimuth direction. Euler-Lagrange equation is applied to derive the coupled nonlinear dynamic equation of motion of infrared search and track and the characteristics of dynamic coupling are investigated. Two equilibrium points with small variations from the nonlinear coupling system are derived and the specific condition from which a coupled equation can be three independent equations is derived. Finally, to examine the stability of system, Lyapunov direct method was used and system stability and stability boundaries are investigated.

주요기술용어(주제어) : Infrared Search and Track, Dynamic Motion, Stability, Lyapunov

1. Introduction

The maintaining of stability of infrared search and track(IRST) system is important issue to obtain the good image quality of the system. To design and optimize the control of IRST system, the derivation and analysis of dynamic equation of motion becomes the first important procedure.

It has been known that there are general two types of IRST with different operation concepts, scanning and staring array IRST system^[1,2]. In scanning system, the infrared image is obtained

using gimbal structure with three rotating axes instead of using two axes gimbal in staring array IRST. However, current most IRST system is designed using scanning system because of the developing and operational cost even though the scanning system has more dynamic complexity. The scanning mechanism of IRST is constructed with gimbal structure which has three rotational angles, azimuth, pitch and roll motions. The motion of each angle interactively affects each other with its external torque because of system coupling. In the past, most IRST system just has used single input and single out(SISO) control for the convenience without considering the dynamic coupling^[3]. Therefore, we will study the nonlinear dynamic motion including dynamic coupling effect

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and stability.

Conventionally, there are several papers describing the dynamic motion of specific problems such as stability of liquid fluctuation inside, satellite motion using pendulum approach and so on^[4,5]. Then, most papers have used Euler -Lagrange(E-L) equation to obtain the dynamic equation of motion even though some studies used Kane's method, Bond graph model^[5,6]. To describe the stability problem, Chetan used root locus analysis by linearizing the nonlinear system and Z.-M. Ge et al. investigated the behavior of elliptical pendulum using Lyapunov direct method^[4,7]. Park et al.^[8] used momentum equation to derive the dynamic equation of double gimbaled momentum wheel and designed the PID control logic. To design PID control, new variables in roll and yaw angles are introduced without considering a specific condition of inertia through the paper.

In this paper, we first derive the dynamic coupled equation of motion using E-L equation with moment of inertias, motion variables and outer external torques similarly with some papers mentioned above. The equilibrium points are also investigated with small variation from equilibrium state in the motion of IRST.

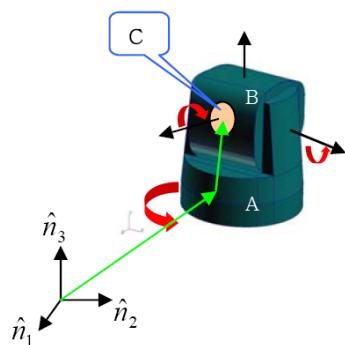
Second, we derived the optimal inertia condition which can derive into independent problem and can be applied to three Single Input Single Output (SISO) control problem for the control of IRST.

Last, we analyze the stability condition with the velocity feedback control torque using Lyapunov stability method^[9] with linearized form of the coupled equation.

2. Dynamic equation of motion

The IRST system is assumed that it moves with three different rotational axes, azimuth, pitch

and roll angles as shown in Figure 1. In this case, the axis of azimuth, \hat{a}_3 , of body A rotates with constant velocity around 1Hz covering whole angle range 360° in a given inertial Newtonian frame, N. The axis of pitch supporting the main infrared(IR) scanning camera module which takes the image of infrared bands, 3~5 μm , 8~12 μm or both bands rotates along the pitch axis, \hat{b}_2 in body frame A. Last IR camera module which rotates through the roll angle axis, \hat{c}_1 constructing the major component of the system and collects the infrared energy that comes out of the target and background rotates in the pitch frame, B. Eventually, these angle make Euler angle. Here, the inertia of each axis is represented using inertia tensor, I^{A/A^*} , I^{B/B^*} and I^{C/C^*} . Here, the superscript means body of each axis in the observed frame. “ A^* ” is the name of a body constructed by its own coordinate frame, “A” located at the center of mass. “ B^* ” and “ C^* ” have similar meaning with previous ones. For the convenience, we just consider inertia of principal axis neglecting terms of product inertia of bodies of each axis even though it does not happen in real physical situation. This is because the product inertia is so small to disregard the additional effects on the dynamic system.



[Figure 1] Configuration of IRST having 3-axis rotational motion

Figure 1 represents the schematic configuration of IRST having three rotational axes. This system should be controlled and robust under the external disturbance and proper control logic should be developed and have stability robustness.

General rotational motions are described with physical parameters, moment of inertias and variations of angular velocity in time called angular accelerations. There are several well-known methods in deriving the dynamic equation of IRST. Euler-Lagrange(E-L) method, Momentum method or Kane's method can be included. However, one of the most well known method, the Lagrange-Euler method^[10] is used in this paper to derive the dynamic motion of IRST which is composed of three axes rotational motion as mentioned above.

First, the Lagrange-Euler equation is represented as the equation (1).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q_i \quad (1)$$

$$L = T - V \quad (2)$$

$$Q_i = F_i \cdot v_{pi} \quad (3)$$

Here, L represents the Lagrangian which is the difference between kinetic energy, T and potential energy, V from the equation (2). The generalized force, Q_i with generalized force F_i and generalized velocities v_{pi} from equation (3) is shown in (1) and (3). q is generalized coordinate and q_1 is the angle of azimuth direction, q_2 is the angle of pitch and q_3 is the angle of roll.

Now, if E-L is considered in the motion of azimuth direction, the kinetic energy in this axis is represented easily with following relations.

$$T_{zh} = \frac{1}{2}(\mathbf{N}\mathbf{w}^A)^T I_3^{A/A^*} \mathbf{N}\mathbf{w}^A = \frac{1}{2} I_3^{A/A^*} (\dot{q}_1)^2 \quad (4)$$

With

$$\mathbf{N}\mathbf{w}^A = \dot{q}_1 \hat{a}_3 \quad (5)$$

Here $\mathbf{N}\mathbf{w}^A$ is the angular velocity vector of body, A in the Newtonian frame, N along the axis, \hat{a}_3 of body A . The kinetic energy is related with angular velocity in a fixed state of linear motion. Second the kinetic energy in the pitch direction is similarly described with following expressions.

$$T_{pt} = \frac{1}{2}(\mathbf{N}\mathbf{w}^B)^T I^{B/B^*} \mathbf{N}\mathbf{w}^B \\ = \frac{1}{2}(I_1^{B/B^*} \dot{q}_1^2 \sin^2 q_2 + I_2^{B/B^*} \dot{q}_2^2 + I_3^{B/B^*} \dot{q}_1^2 \cos^2 q_2) \quad (6)$$

Here, $\mathbf{N}\mathbf{w}^B$ the angular velocity of body "B" in the frame "N" is easily determined with both rotations, azimuth rotation and pitch rotation using the body frame, "B" and its coordinates, \hat{b}_i ($i = 1, 2, 3$)

$$\mathbf{N}\mathbf{w}^B = \mathbf{N}\mathbf{w}^A + {}^A\mathbf{w}^B = -\dot{q}_1 \sin q_2 \hat{b}_1 + \dot{q}_2 \hat{b}_2 + \dot{q}_1 \cos q_2 \hat{b}_3 \quad (7)$$

Last, the kinetic energy in the roll axis using body coordinate, B is

$$T_{rl} = \frac{1}{2}(\mathbf{N}\mathbf{w}^C)^T I_2^{C/C^*} \mathbf{N}\mathbf{w}^C \\ = \frac{1}{2}[I_1^{C/C^*}(-\dot{q}_1 \sin q_2 + \dot{q}_3)^2 + I_2^{C/C^*} \dot{q}_2^2 + I_3^{C/C^*} \dot{q}_1^2 \cos^2 q_2] \quad (8)$$

with

$$\mathbf{N}\mathbf{w}^C = \mathbf{N}\mathbf{w}^A + {}^A\mathbf{w}^B + {}^B\mathbf{w}^C \\ = (-\dot{q}_1 \sin q_2 + \dot{q}_3) \hat{b}_1 + \dot{q}_2 \hat{b}_2 + \dot{q}_1 \cos q_2 \hat{b}_3 \quad (9)$$

Finally, the total kinetic energy representing the dynamic motion is

$$T_{tot} = \frac{1}{2} [(I_3^{A/A^*} + I_1^{B/B^*} \sin^2 q_2 + (I_3^{B/B^*} + I_3^{C/C^*}) \cos^2 q_2) \dot{q}_1^2 + (I_2^{B/B^*} + I_2^{C/C^*}) \dot{q}_2^2 + I_1^{C/C^*} (-\dot{q}_1 \sin q_2 + \dot{q}_3)^2] \quad (10)$$

Next, the potential energy can be expressed as following expression by considering the stiffness of bearing friction between rotating bodies.

$$V_{tot} = \frac{1}{2} k_2 q_2^2 + \frac{1}{2} k_3 q_3^2 \quad (11)$$

Here, k_1 and k_2 are the bearing friction factor not considering the stiffness of azimuth direction since it is assumed that the axis in the azimuth direction rotates with almost constant angular velocity. Furthermore, the gravity force did not considered since the location of the mass center does not change when the IRST rotates. Eventually, the Lagrangian of system can be written as,

$$L = \frac{1}{2} [(I_3^{A/A^*} + I_1^{B/B^*} \sin^2 q_2 + (I_3^{B/B^*} + I_3^{C/C^*}) \cos^2 q_2) \dot{q}_1^2 + (I_2^{B/B^*} + I_2^{C/C^*}) \dot{q}_2^2 + I_1^{C/C^*} (-\dot{q}_1 \sin q_2 + \dot{q}_3)^2] - \frac{1}{2} k_2 q_2^2 - \frac{1}{2} k_3 q_3^2 \quad (12)$$

Now a nonlinear dynamic equation shown in (13) is derived by several manipulations with L-E equation, (1).

$$M(\mathbf{q})\ddot{\mathbf{q}} + N(\mathbf{q}, \dot{\mathbf{q}}) + P(\mathbf{q}) = F \quad (13)$$

With generalized coordinates, $\mathbf{q} = [q_1, q_2, q_3]$, $M(q)$ is inertia matrix, $N(q, \dot{q})$ is damping matrix adding viscous friction factor c_1 , c_2 and c_3 . Furthermore, $P(q)$ is stiffness matrix and F is just torque matrix acted on each axis.

$$M(\mathbf{q}) = \begin{bmatrix} I_3^{A/A^*} + (I_1^{B/B^*} + I_1^{C/C^*}) \sin^2 q_2 & 0 & -I_1^{C/C^*} \sin q_2 \\ + (I_3^{B/B^*} + I_3^{C/C^*}) \cos^2 q_2 & I_2^{B/B^*} + I_2^{C/C^*} & 0 \\ 0 & -I_1^{C/C^*} \sin q_2 & I_1^{C/C^*} \end{bmatrix} \quad (14)$$

$$N(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} (I_1^{B/B^*} + I_1^{C/C^*} - I_3^{B/B^*} - I_3^{C/C^*}) \sin 2q_2 \dot{q}_1 \dot{q}_2 - I_1^{C/C^*} \cos q_2 \dot{q}_2 \dot{q}_3 + c_1 \dot{q}_1 \\ -\frac{1}{2} (I_1^{B/B^*} + I_1^{C/C^*} - I_3^{B/B^*} - I_3^{C/C^*}) \sin 2q_2 \dot{q}_1^2 + I_1^{C/C^*} \cos q_2 \dot{q}_1 \dot{q}_3 + c_2 \dot{q}_2 \\ -I_1^{C/C^*} \cos q_2 \dot{q}_1 \dot{q}_2 + c_3 \dot{q}_3 \end{bmatrix} \quad (15)$$

$$P(\mathbf{q}) = \begin{bmatrix} 0 \\ k_2 q_2 \\ k_3 q_3 \end{bmatrix}, \quad F = \begin{bmatrix} M_A(t) \\ M_B(t) \\ M_C(t) \end{bmatrix} \quad (16)$$

Here, $M_A(t)$, $M_B(t)$ and $M_C(t)$ are torques exerted on the moving rotation axes.

3. Stability analysis

3.1 Equilibrium with small variations

If the dynamic characteristics analyzed in IRST, some equilibrium state of system like following conditions should be identified. The three equilibrium states are described with three small variations according to the variations of generalized coordinates. The azimuth direction rotates with constant equilibrium velocity and small fluctuation around this equilibrium velocity. Furthermore, angles q_2 , q_3 vary around the specific equilibrium angles. In the equilibrium state, whole angular accelerations become zero.

$$\ddot{q}_i = 0 \quad (i=1,2,3) \quad \dot{q}_2 = 0, \quad \dot{q}_3 = 0 \quad (17)$$

$$\begin{aligned} \dot{q}_1 &= \overline{\dot{q}_1} + \delta \dot{q}_1 \\ q_2 &= \overline{q_2} + \delta q_2 \\ q_3 &= \overline{q_3} + \delta q_3 \end{aligned} \quad (18)$$

The torques exerted externally on each axis are also expressed with small variations from equilibrium values and superscript “-” denotes the equilibrium values.

$$\begin{aligned} M_A(t) &= \overline{M}_A + \delta M_A \\ M_B(t) &= \overline{M}_B + \delta M_B \\ M_C(t) &= \overline{M}_C + \delta M_C \end{aligned} \quad (19)$$

Now, if we substitute the small variation of each equilibrium angle, angular velocity shown in (18) and variation of torque (19) into dynamic equation (13), it leads to following expressions under the assumption of small variation \dot{q}_1 and q_2, q_3 .

$$c_1 \dot{\delta q}_1 = \delta M_A \quad (20)$$

$$[(I_3^{B/B^*} + I_3^{C/C^*} - I_1^{B/B^*} - I_1^{C/C^*}) (\overline{q}_1)^2 + k_2] \delta q_2 = \delta M_B \quad (21)$$

$$k_3 \delta q_3 = \delta M_C \quad (22)$$

From the equilibrium values of equation (20)~(22), we can identify the fluctuation of angular velocity and angles. In equilibrium state, the variation of angular velocity $\dot{\delta q}_1$ is dependent on the viscous damping friction coefficient, c_1 and its control torque, δM_A . Similarly, Equation (22) reveals the variation of δq_3 has relation with stiffness factor, k_3 which is denoted with rotational bearing friction and control torque, δM_C . Furthermore, from the equation (21), we can identify that the variation of motor torque of body B is dependent on the bearing friction factor, k_2 and constant equilibrium angular velocity, \overline{q}_1 . However, if following condition of inertia of body B and C from the first term of equation (21) is satisfied,

$$I_3^{B/B^*} + I_3^{C/C^*} = I_1^{B/B^*} + I_1^{C/C^*} \quad (23)$$

we can fairly reduce the needed control torque, δM_B and also make the additional torque fluctuation dependent only on bearing friction, k_2 similar to equation (22) without consideration of equilibrium velocity of azimuth direction. Furthermore, if the angle q_2 has so small value from zero equilibrium point and let it zero, we can neglect the off-diagonal term in the mass matrix $M(q)$ and each value of element of mass matrix becomes constant. In addition to that, by neglecting viscous friction and stiffness variables, totally independent and uncoupled equation motion can be derived into equation (24). This means that if SISO(Single input Single output) control scheme is applied, the q_2 should be zero or sufficiently small.

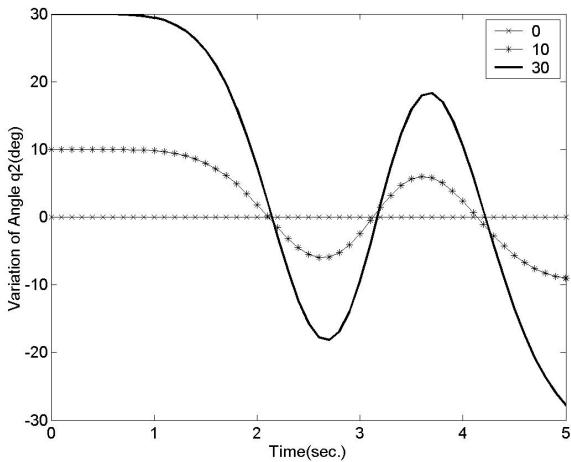
$$\tilde{M}(\mathbf{q})\ddot{\mathbf{q}} = F \quad (24)$$

$$\tilde{M}(\mathbf{q}) = \begin{bmatrix} I_3^{A/A^*} + I_3^{B/B^*} + I_3^{C/C^*} & 0 & 0 \\ 0 & I_2^{B/B^*} + I_2^{C/C^*} & 0 \\ 0 & 0 & I_1^{C/C^*} \end{bmatrix} \quad (25)$$

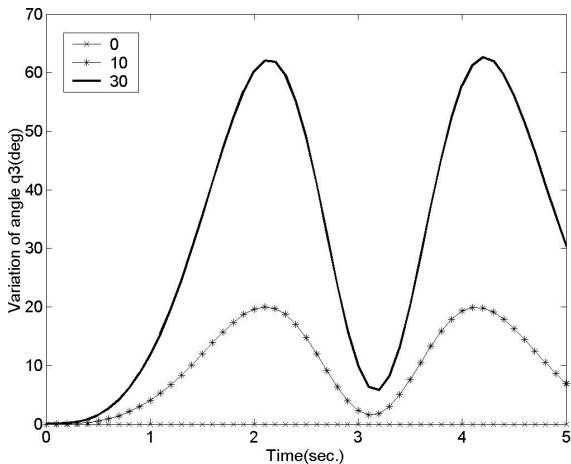
Figure 2 represents the nonlinear response of the angle q_2 using equation (13) with different initial angle q_2 , 0° , 10° and 30° with constant input torque, $M_A = 10(N.m)$. In this case, we neglect the viscous friction and stiffness friction factors. From this figure, we identified that the angle q_2 becomes independent with torque M_A when initial angle of is 0° . Figure 3 shows the fluctuation of angle q_3 in a same condition of q_2 and we can conclude that larger initial angle q_2 derives the more fluctuation of angles q_2 and q_3 .

In conclusion, the control torque of IRST can be reduced under the specific condition of inertia and the coupling effect also does not need to

consider constructing three independent control from the coupled one equation even though it is required more complex algorithm for the dynamically coupled controller design.



[Figure 2] Fluctuations of q_2 with different initial angle q_2



[Figure 3] Fluctuation of q_3 with different initial angle of q_2

3.2 Stability with Lyapunov

In this section, the stability of motion of system is described by Lyapunov method^[9]. If we rewrite the nonlinear equation (13) into linear state

equation, it becomes following expression, (27), by putting the equations into state form using following state variables, x_i , ($i=1,\dots,6$)

$$x_i = q_i \text{ and } x_{i+3} = \dot{x}_i \quad (i=1,2,3) \quad (26)$$

Now, an equilibrium point at the initially zero is considered and the following equation is derived by using Taylor series expansion^[11] neglecting higher order terms.

$$\begin{aligned} \dot{x}_2 &= x_5, \quad \dot{x}_3 = x_6 \\ \dot{x}_4 &= \frac{-c_1 x_4 + M_A(t)}{I_3^{A/A^*} + I_3^{B/B^*} + I_3^{C/C^*}}, \quad \dot{x}_5 = \frac{-c_2 x_5 - k_2 x_2 + M_B(t)}{I_2^{B/B^*} + I_2^{C/C^*}} \\ \dot{x}_6 &= \frac{M_A(t)x_2}{I_3^{A/A^*} + I_3^{B/B^*} + I_3^{C/C^*}} + \frac{1}{I_1^{C/C^*}}(-c_3 x_6 - k_3 x_3 + M_C(t)) \end{aligned} \quad (27)$$

Neglecting $x_1 (= q_1)$ since it does not appear on the right hand side of equation (13), the first equilibrium point can be determined as,

$$\mathbf{x}_l(0,0,0,0,0,0), \text{ with } \mathbf{x}_l = [x_2, x_3, x_4, x_5, x_6]^T \quad (28)$$

Next, the Lyapunov function is introduced into following expression.

$$Ly = \frac{1}{2} \sum_{i=2}^6 x_{l,i}^2 \quad (29)$$

To understand the feedback control of IRST, we set up the torque with angular velocities and gains as the expression (30). Now, the time derivatives of Lyapunov function with velocity feedback control torque is (31).

$$M_a = k_a x_4, M_b = k_b x_5, M_c = k_c x_6 \quad (30)$$

$$\frac{dLy}{dt} = -x^T Q x \quad (31)$$

Here,

$$Q = I, \quad A^T P + PA + Q = 0 \quad (32)$$

Here, the matrix A is,

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-c_1 + k_a}{In_3} & 0 & 0 \\ \frac{-k_2}{In_2} & 0 & 0 & \frac{-c_2 + k_b}{In_2} & 0 \\ 0 & \frac{-k_3}{I_1^{c/c^*}} & 0 & 0 & \frac{-c_3 + k_c}{I_1^{c/c^*}} \end{bmatrix} \quad (33)$$

with

$$\begin{aligned} In_3 &= I_3^{A/A^*} + I_3^{B/B^*} + I_3^{C/C^*} \\ In_2 &= I_2^{B/B^*} + I_2^{C/C^*} \end{aligned}$$

k_a, k_b, k_c are feedback gains and P, Q are positive definite matrix. According the Lyapunov stability theory, the matrix Q should be positive definite. Therefore, we can investigate the property of positive definite by calculating the eigenvalues of the expression, (31) and obtained these values with (34).

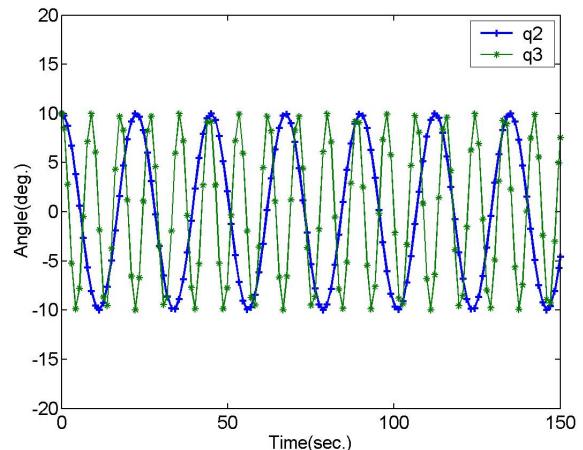
$$\begin{aligned} \lambda_1 &= -\frac{c_1 - k_a}{I_3^{B/B^*} + I_3^{C/C^*}} \\ \lambda_{2,3} &= -\frac{1}{2I_1^{c/c^*}}((c_3 - k_c) \mp \sqrt{(c_3 - k_c)^2 - 4k_3 I_1^{C/C^*}}) \\ \lambda_{4,5} &= -\frac{1}{2(I_2^{B/B^*} + I_2^{C/C^*})}((c_2 - k_b) \mp \sqrt{(c_2 - k_b)^2 - 4k_2(I_2^{B/B^*} + I_2^{C/C^*})}) \end{aligned} \quad (34)$$

Now, we investigate the stability with three different cases having different values of eigen values.

CASE I. Pure imaginary eigen values

- Condition : $c_i = k_j, i = 1,2,3, j = a,b,c$
with $k_1, k_2, k_3 \neq 0$

In condition of CASE I, the eigenvalues of expression (34) have imaginary values. Next, if we investigate the nonlinear velocity feedback response, we can get Figure 4. From the figure, the fluctuation of angle q_2 and q_3 can be identified. The magnitude of amplitude does not decay with time because the energy dissipation by viscous friction is exactly compensated by velocity feedback torque given by expression (30).

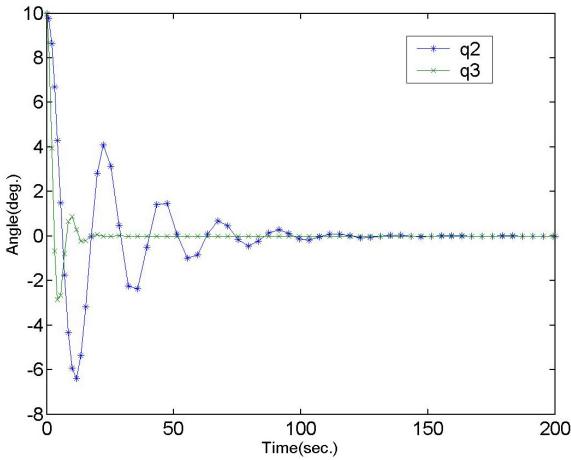


[Figure 4] Variation of q_2 and q_3 of CASE I

CASE II. Negative real eigen values

- Condition : $c_i > k_j, i = 1,2,3, j = a,b,c$
with $k_1, k_2, k_3 \neq 0$

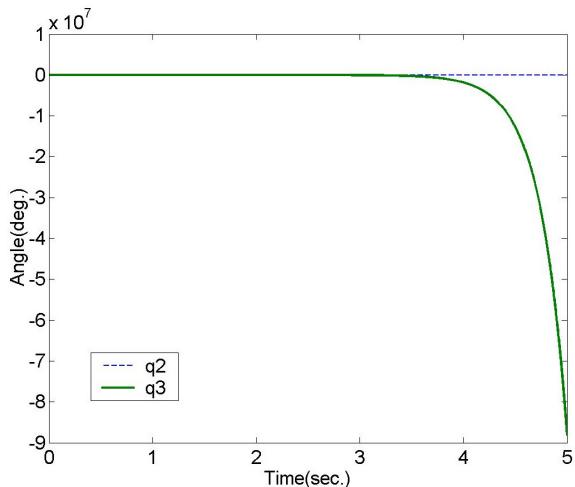
This is the case the eigenvalues have negative real values. In this case, the dissipation by viscous friction exists with time not to totally compensated by velocity feedback torque. We can easily check the converge of motions.

[Figure 5] Variation of q_2 and q_3 of CASE II

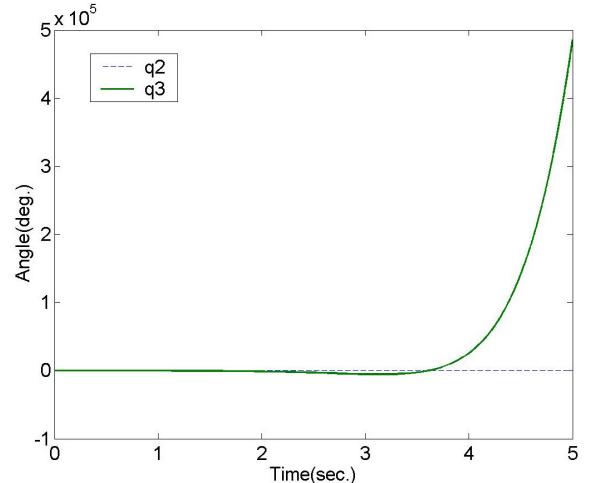
CASE III. Positive real values

- Condition : $c_i < k_j$, $i = 1, 2, 3$, $j = a, b, c$
with $k_1, k_2, k_3 \neq 0$

In case III, the eigen values have positive real values and the divergent characteristics with time is expected and is identified as in the figure 6.

[Figure 6] Variation of q_2 and q_3 of CASE III
(Positive in imaginary part)

From the result of above Figure 6, we can also identify that the angle q_3 is divergent into negative values of magnitude with positive eigenvalues. In addition to that, if the eigenvalues have negative real values in imaginary parts, then the q_3 also diverges having positive magnitude as in figure 7.

[Figure 7] Variation of q_2 and q_3 of CASE III
(Negative in imaginary part)

Now, if we consider the another equilibrium point $x_l(0, 0, \bar{x}_1, 0, 0)$ as the second example with following assumption,

$$k_a = c_1 \quad (35)$$

In this case, we use equilibrium velocity of $\dot{\bar{x}}_1$ instead of zero value. To meet the condition of equilibrium point of Lyapunov, we introduce the equation (35) and used same Lyapunov function, (29). Now the linearized form from nonlinear dynamic equation of motion (13) becomes same expression (27). The derivative of Lyapunov also has same equation and finally the eigen values to determine the stability of system can be obtained into (34). From the above examination, we can

conclude that the stability boundary of IRST which rotates with constant angular velocity at the azimuth direction is same as the equilibrium point explained first.

4. Conclusion

The motion of IRST is characterized by three angle variables and has different motion characteristics with different inertias. To keep the dynamic stability of this system is very important issue to get a good quality of infrared image. To get the dynamic equation of motion of IRST, Euler-Lagrange equation was used and the dynamic coupling effect among the axes was examined. Furthermore, the specific condition of inertia independent of rotational velocity in the azimuth direction is derived with small variation from the equilibrium point. We also identified the specific condition which the coupled dynamic system can be derived into the uncoupled system with simple assumption and manipulation. Finally, to investigate the system stability, Lyapunov function is introduced and stability condition with different eigen values are obtained at the two equilibrium points.

References

- [1] George R. Ax et al., Navy EO Sensor Testbed Development for Infrared Search and Track, SPIE Vol. 2552, pp. 235~246.
- [2] <http://www.global-defence.com/2006>
- [3] Anthoney E. Bentley, Jeffrey L. Wilcoxen, Pointing Control System For a High precision Flight Telescope, Sandia Report, Sandia National Laboratories, December, 2000.
- [4] Chetan Nichkawde, P. M. Harish, Stability analysis of a multibody system model for coupled slosh- vehicle dynamics, Journal of Sound and Vibration 275, pp. 1069~1083, 2004.
- [5] Takashi Nagata et al. Multibody Effect on Nutational Dynamics of Spin-Stabilized Satellites with Fuel Sloshing, Journal of Guidance, Control and Dynamics, Vol. 21, No. 4, July-August, 1998.
- [6] Raymond C.Montgomery et al., Using Bond Graphs for Articulated, Flexible Multi-bodies, Sensors, Actuators and Controllers with Application to the International Space Station, NASA Langley Research Center.
- [7] Z.-M. Ge and P.-C. TGsen, Non-linear dynamic analysis and control of chaos for a two-degrees of freedom rigid body with vibrating support, Journal of Sound and Vibration, 240(2), pp. 323~349, 2001.
- [8] 박영웅, 방효중, Roll/Yaw Controller Design Using Double Gimbaled Momentum Wheel, Proceedings of the 11th KACC, October, 1996.
- [9] H. K. Khalil, Nonlinear System, England Chiffs, Prentice-Hall, 1996.
- [10] W. E. Wiesel, Spaceflight Dynamics, McGraw -Hill, 1989.