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제품군 디자인에서 공통속성의 활용이 사회적 효용에 미치는 영향

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Effects of Commonality Strategy in Product Line Design on Social Welfare

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■ Abstract ■

Commonality strategy is a popular design practice in designing a product line as it enables the firm cost saving and simplification in design, manufacturing, and distribution processes. However the issue of commonality has been mostly analyzed from a profit maximizing firm's perspective and, to our knowledge, there is no literature that deals with the issue from a different perspective. In this paper, we consider the issue of commonality strategy from a social welfare maximization perspective, and argue that commonality strategy used in designing of public goods can bring certain benefits not just for a firm but also for consumers, i.e., for society as a whole. While we assume certain cost saving in production process due to economies of scale under commonality strategy, we conceptualize two different effects of commonality strategy, utility effect due to cost saving and weighted-averaging effect, and show how these two effects interplay to determine the design of common attribute and desirability of commonality strategy. We also discuss how the implementation of commonality strategy differs under different objectives of a product line designer : social welfare and firm's profit maximization.

Keyword : Commonality Strategy, Social Welfare, Modeling, Product Line Design

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1. Introduction

The notion of commonality and modularity has been used broadly in many product categories, and its primary benefits include cost savings in design, manufacturing and logistics whereas its primary disadvantage includes reduced product differentiation among the products in a line. While the basic tradeoff relationship has been modeled in different papers in literature, its effects have been analyzed mostly from a profit maximizing firms' perspective ([2, 4, 5, 8]). However the notion of commonality needs to be analyzed from a social welfare perspective as it can also be applied in designing of public goods where social welfare is a primary objective. Examples of public sectors include public education, railroad, electricity, and public insurance, to name a few. In these instances, the goods and services are designed with multiple attributes for which consumers typically exhibit heterogeneous preference for different attributes, and the social planner may benefit from segmenting consumers into a few distinct segments in order to better serve their diverse needs and different levels of willingness to pay for different attributes. For example, basic and premium public insurance packages with different price structure can be designed to serve two segments with different insurance needs and as a result the social welfare can be improved compared with a situation where only one type of insurance package is offered to the public. The notion of commonality can also be judiciously used in these instances to provide certain cost advantage for society as a whole through economies of scale along with different dimensions of the goods and services. At the same time, however, it can reduce the amount of differentiation among the goods and can cause undesirable cannibalization or switching between goods by public.

In this paper, we analyze the issue of commonality from a social welfare maximization perspective, and ask the following questions ; 1) What are the effects of commonality strategy on social welfare, consumer surplus, and firm surplus? 2) How the commonality affects the allocation of goods to different segments in market through pricing scheme? 3) How to select common attribute?

The paper is organized as follows. A literature review is provided in the next section. In §3, we introduce two models of product line design problem under non-commonality and commonality strategies, respectively, and these models are analyzed and compared in §4. The commonality model under social welfare maximization is compared with a commonality model under firm's profit maximization in commonality strategy in §5. Implications and conclusions are in the last section.

2. Literature Review

The issue of commonality has been analyzed in literature from many different perspectives, and one approach is addressing the issue from a perspective of product line design and analytic modeling. (Two recent reviews of the papers on the topic of modularity and commonality are provided in [3] and [11]). Among the papers that adopt the analytic modeling approach in a context of product line design, firm's profit maximization is a common objective function ([2, 4, 5, 8]) and one common theme across the papers is that there is a tradeoff relationship in adopting commonality strategy : cost saving due to economies of scale versus cost of product indifferentiability. While a number of papers discuss different types of benefits that are associated with commonality strategy, relatively small number of academic papers addresses the (implicit) cost of commonality strategy.

Ulrich and Tung are the first who conceptually discussed the cost of product similarity in the context of product line design that is arising due to commonality strategy [13]. While there are number of papers in marketing and psychology areas that address the issue of similarity (e.g., [1, 12]), the notion of similarity has rarely been extended and tied with the notion of modularity and commonality in the context of product line design. Kim and Chhajed introduce the notion of valuation premium and discount that is arising due to product similarity and consider the valuation discount of high-end product as a cost of commonality strategy [5]. They later demonstrate through an experimental study the effect of product similarity due to commonality strategy ([6, 8]). Krishnan and Gupta model the tradeoff relationship associated with commonality strategy using an ideal-point type of customer preference structure, and they define a cost of commonality strategy in terms of 'cost of not providing an exact product specification to individual customer's need' which limits firm's revenue [8]. Desai et al. consider a quality-type of customer preference structure, and model 'cost of not differentiating an attribute for individual segments' as an implicit cost of commonality strategy which limits firm's revenue [2]. Heese and Swaminathan consider two different, additive and multiplicative, cost saving functions associated with commonality strategy, and they model 'not being able to differentiate an attribute' as a cost of commonality strategy that limits firm's revenue [4]. They also identified 'revenue effect' of commonality strategy that is arising due to heightened quality level that is enabled by cost saving effect of commonality strategy. However, Desai et al. and Heese and Swaminathan didn't explain the mechanism through which the cost of commonality takes place, how to quantify it, and to what extent the costs depend on the model assumptions ([2, 4]). Furthermore, all of the models we reviewed considered firm's profit as an objective to maximize.

3. Model

In this section we introduce notation and develop the model. A list of major notation used in the paper is provided in <Table 1>. In this paper, we assume that there are two consumer segments in a market with different quality valuations. Size of the market is normalized at 1 and r_i represents the relative size of each segment, i = 1, 2. We also assume that there are two quality-type attributes ($j = u, k$), for which more is always better, and each segment's valuation of the attribute is denoted by w_{ij}. We refer to an attribute that is offered at different levels in two products as custom attribute, and an attribute that is offered at the same level in two products as common attribute, respectively. We assume strictly vertical preference structure between segments, $w_{1j} > w_{2j}$, for both attributes so that the first segment values each of the two attributes more than the second segment does. We also term the first and the second segments as high-and low-end segments, respectively. The level of an attribute j offered in product x is denoted by q_{xj}, and the cost of offering an attribute is convex increasing and is denoted by $c_j q_{xj}^2$ where c_j is a constant [9].

〈Table 1〉 A list of major notation

r_i	Relative size of segment i, i = 1, 2.
c_j	Unit cost of attribute j, j = u, k.
α_j	Cost saving associated with attribute j if is common, $0 \le \alpha_j < 1$.
w_{ij}	Part worth of customer segment i for attribute j.
q_{ij}	Amount of attribute j offered in a product for segment i.
P_i	Price of product offered to segment i.

In our model, the social planner has a goal of maximizing social welfare and it designs and prices the products. We assume that the social planner doesn't have resource or capacity constraint in serving consumers. It can designate a firm, whose cost structure is known to the social planner, to produce products for the two segments. Social welfare is the sum of consumer surplus and firm's profit; consumer surplus is given by the difference between consumers' utility from products and product prices, P_i, they have to pay whereas firm's profit is given by the difference between its revenue based on prices set by the social planner and its own costs of producing the products. Thus the total welfare can simply be expressed by the difference between consumer utility and total cost of the products.

While the product prices are irrelevant to the amount of total welfare in this social welfare maximization problem, they are necessary and important for the social planner to properly assign different goods to different segments : high-end product to high-end segment and low-end product to low-end segment. Hence, our model considers quality differentiation and third-degree price discrimination for the best allocation of goods to consumer segments. Consumers make their purchasing decision based on self-selection mechanism whereby they evaluate products based on utility from products and prices they have to pay for the products and choose the one that gives largest amount of surplus.

In order to obtain insights on commonality strategy through a comparison with non-commonality strategy, we develop two models, one for each strategy, and derive an optimal solution under each strategy. Under non-commonality strategy where both attributes are offered as custom attribute, a product line design problem to maximize social welfare (henceforth referred to as problem NCPD) can be stated as follows.

(Program NCPD)

$$Max\ \Sigma_{i=1,2}\, r_i\{w_{iu}q_{iu} + w_{ik}q_{ik}\} - \Sigma_{i=1,2}\, r_i\{c_u q_{iu}^2 + c_k q_{ik}^2\}$$

$$w_{1u}q_{1u} + w_{1k}q_{1k} - P_1 \ge w_{1u}q_{2u} + w_{1k}q_{2k} - P_2 \quad (1)$$

$$w_{2u}q_{2u} + w_{2k}q_{2k} - P_2 \ge w_{2u}q_{1u} + w_{2k}q_{1k} - P_1 \quad (2)$$

$$w_{1u}q_{1u} + w_{1k}q_{1k} \ge P_1 \quad (3)$$

$$w_{2u}q_{2u} + w_{2k}q_{2k} \ge P_2 \quad (4)$$

$$q_{1u},\ q_{2u},\ q_{1k},\ q_{2k},\ P_1,\ P_2 \ge 0$$

The first constraint is self-selection constraint for the first segment. In order for the first segment to buy the product assigned to it, product 1, it should receive the same or larger amount of surplus from the assigned product. The second constraint is similar to the first one but is for the second segment. The last two constraints are participation constraints for the two segments.

In order to participate in the market, each segment should get non-negative amount of surplus from the product assigned to it.

On the other hand, under commonality strategy, we assume that attribute k is common attribute, and attribute u is custom attribute. For common attribute, $j=k$, the cost coefficient decreases to $c_k(1-\alpha_k)$ where $0 \le \alpha_k < 1$. The cost saving parameter, α_k, represents the benefits in design, production and operations that are due to commonality and includes savings from simpler product design and development process; and subsequent simplification in manufacturing, inventory management and material outsourcing process. Thus the total cost of offering product x using a combination of custom and common attributes is $c_t q_{xt}^2 + c_k(1-\alpha_k) q_{xk}^2$. Throughout the paper, we also denote the offered level of common attribute by $q_{1k} = q_{2k} = q_k$. The social welfare maximization problem under commonality strategy (henceforth referred to as problem CPD) can be stated as :

(Program CPD)

$$Max\ \Sigma_{i=1,2}\, r_i\{w_{iu} q_{iu} + w_{ik} q_k\} - \Sigma_{i=1,2}\, r_i\{c_u q_{iu}^2 + c_k(1-\alpha_k) q_k^2\}$$

$$w_{1u} q_{1u} + w_{1k} q_k - P_1 \ge w_{1u} q_{2u} + w_{1k} q_k - P_2 \quad (5)$$

$$w_{2u} q_{2u} + w_{2k} q_k - P_2 \ge w_{2u} q_{1u} + w_{2k} q_k - P_1 \quad (6)$$

$$w_{1u} q_{1u} + w_{1k} q_k \ge P_1 \quad (7)$$

$$w_{2u} q_{2u} + w_{2k} q_k \ge P_2 \quad (8)$$

$$q_{1u},\ q_{2u},\ q_k,\ P_1,\ P_2 \ge 0$$

The only difference between programs (NCPD) and (CPD) is that in CPD, we use $q_{1k} = q_{2k} = q_k$ to indicate that attribute k is used as common in both products.

4. Analysis

4.1 Analysis of non-commonality strategy

Using the first order condition with the objective function, the optimal product line design to problem (NCPD) is

$$q_{ij}^{NC} = \frac{w_{ij}}{2c_j},\ i=1,\ 2,\ j=k,\ u$$

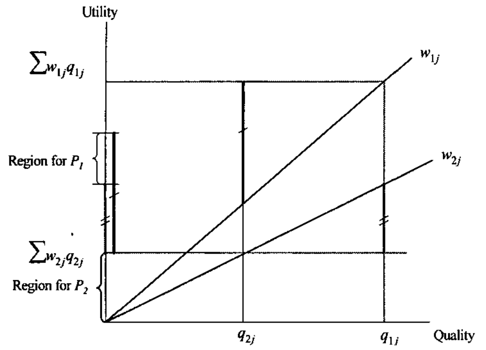
and they are socially efficient qualities. Superscript NC denotes non-commonality strategy. Although the pricing is irrelevant to the optimal product design in social welfare maximization problem, it is important to confirm that there is a feasible pricing scheme that doesn't cause segments' switching behavior. From constraints (1) and (2), a range of feasible price gap that doesn't cause segments' switching to the other product is

$$w_{2u}(q_{1u}-q_{2u}) + w_{2k}(q_{1k}-q_{2k}) < P_1 - P_2 < w_{1u}(q_{1u}-q_{2u}) + w_{1k}(q_{1k}-q_{2k}).$$

Therefore, for a given $P_2 \le w_{2u} q_{2u} + w_{2k} q_{2k}$ in (4), the price of the first product should be set in the region given by

$$P_2 + w_{2u}(q_{1u}-q_{2u}) + w_{2k}(q_{1k}-q_{2k}) < P_1 < P_2 + w_{1u}(q_{1u}-q_{2u}) + w_{1k}(q_{1k}-q_{2k}).$$

The actual pricing may depend on the social planner's discretion, e.g., marginal cost based pricing or maximum possible pricing in favor of the firm, but it is always feasible to find the prices that satisfy consumers' self-selection constraints (see [Figure 1]).



[Figure 1] Feasible regions for product prices, P_1 and P_2

In Figure 1, suppose that the product prices are set in the marked regions, respectively. The second segment will get non-negative amount of surplus from the second product as the amount of utility from the second product is more than or equal to the price of the second product. However it won't switch to the first product since the price of the first product is higher than the amount of utility it can get from the first product. On the other hand, since $P_1 < P_2 + \Sigma_j w_{1j}(q_{1j} - q_{2j})$ the first segment will buy the first product in order to get better amount of surplus.

In order to obtain more insights from the analysis, we from now on assume that the social planner sets the product prices at their respective upper bounds to allow the firm maximum possible profits so that $P_1 = w_{1u}q_{1u} + w_{1k}q_{1k} - (w_{1u} - w_{2u})q_{2u} - (w_{1k} - w_{2k})q_{2k}$ and $P_2 = w_{2u}q_{2u} + w_{2k}q_{2k}$ without violating constraints (1)-(4). This price schedule allows the maximum possible firm's profit from the second segment and it makes the first segment indifferent to either product in terms of surplus. With this pricing, constraints (1) and (4) are binding and the amount of total surplus available to the first segment is expressed by $r_1\{(w_{1u} - w_{2u})q_{2u} + (w_{1k} - w_{2k})q_{2k}\}$ ([7, 10]).

Using the expressions for q_{ij}^{NC}, P_1 and P_2, social welfare (SW), consumer surplus (CS), and firm's profit (FP) under non-commonality strategy can be expressed as follows.

$$SW^{NC} = r_1\sum_j \frac{w_{1j}^2}{4c_j} + r_2\sum_j \frac{w_{2j}^2}{4c_j};$$

$$CS^{NC} = r_1\sum_j \frac{(w_{1j} - w_{2j})w_{2j}}{2c_j};$$

$$FP^{NC} = r_1\sum_j \frac{(w_{1j} - w_{2j})^2}{4c_j} + \sum_j \frac{w_{2j}^2}{4c_j}.$$

We note that the efficient qualities maximize social welfare which is increasing in segments' part worth and is decreasing in cost of providing products. Furthermore, the amount of consumer surplus and firm's profit are increasing in w_{1j} whereas the value of w_{2j} has mixed effects on consumer surplus and firm's profit as the price of the second product is increasing in w_{2j} and gap in two segments' valuation of the second product is decreasing in w_{2j}.

4.2 Analysis of commonality strategy

Using the first order condition to the objective function, the optimal solution to problem (CPD) is

$$q_{1u}^C = \frac{w_{1u}}{2c_u},\ q_{2u}^C = \frac{w_{2u}}{2c_u},\ q_k^C = \frac{r_1 w_{1k} + r_2 w_{2k}}{2c_k(1-\alpha_k)},$$

where superscript C denotes commonality strategy. The custom attribute is offered at each segment's efficient level whereas the offered level of common attribute is based on weighted average of part worth with weights being equal to relative size of each segment.

From constraints (5) and (6), a range of price gap without causing segments' switching to the other product is now reduced to

$$w_{2u}(q_{1u}-q_{2u})<P_1-P_2<w_{1u}(q_{1u}-q_{2u}),$$

and for a given $P_2 \le w_{2u}q_{2u}+w_{2k}q_k$, a feasible price range for the first product is

$$P_2+w_{2u}(q_{1u}-q_{2u})<P_1<P_2+w_{1u}(q_{1k}-q_{2u}).$$

Using the same assumption we made under non-commonality strategy, pricing schedule that enables the firm maximum possible profit without violating self-selection constraints can be set as $P_1=w_{1u}q_{1u}-(w_{1u}-w_{2u})q_{2u}+w_{2k}q_k$ and $P_2=w_{2u}q_{2u}+w_{2k}q_k$, and the amount of total consumer surplus received by the first segment is $r_1\{(w_{1u}-w_{2u})q_{2u}+(w_{1k}-w_{2k})q_k\}$. Using the expressions for q^C_{ij}, P_1 and P_2, social welfare, consumer surplus, and firm's profit under commonality strategy can be expressed as follows.

$$SW^C=\frac{r_1w_{1u}^2+r_2w_{2u}^2}{4c_u}+\frac{(r_1w_{1k}+r_2w_{2k})^2}{4c_k(1-\alpha_k)};$$

$$CS^C=r_1\left\{\begin{aligned}&\frac{(1_{1u}-w_{2u})w_{2u}}{2c_u}\\&+\frac{(w_{1k}-w_{2k})(r_1w_{1k}+r_2w_{2k})}{2c_k(1-\alpha_k)}\end{aligned}\right\};$$

$$FP^C=r_1\frac{(1_{1u}-w_{2u})^2}{4c_u}+\frac{w_{2u}^2}{4c_u}+\frac{(r_1w_{1k}+r_2w_{2k})}{4c_k(1-\alpha_k)}\{w_{2k}-r_1(w_{1k}-w_{2k})\}.$$

4.3 Comparison of non-commonality and commonality strategies

We first analyze the effect of commonality strategy on the design of attribute *k*. Let $w_{1k}=w_{2k}\beta_k$, $\beta_k>1$, then β_k represents the relative gap in part worth for attribute *k* between segments.

Proposition 1 : Followings are the comparison of quality levels under commonality and non-commonality strategies.

i) $q_k^C>q_{2k}^{NC}$

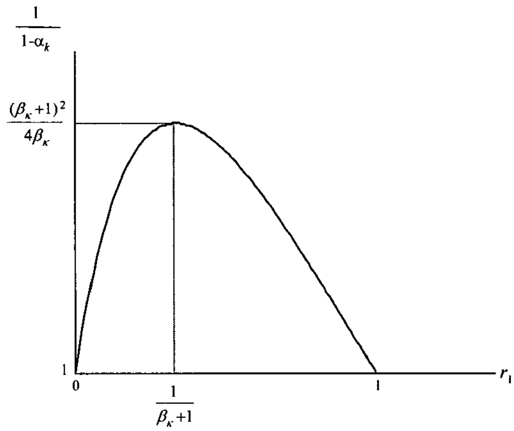
ii) $q_k^C>q_{1k}^{NC}$ if $1/(1-\alpha_k)>\beta_k/(1+r_1(\beta_k-1))$, otherwise $q_k^C>q_{1k}^{NC}$.

Proposition 1 states that the second segment always gets higher level of attribute *k* under commonality strategy. The first segment also gets higher level of attribute *k* if the associated cost saving is significant enough. Higher level of attribute *k* in these cases is due to *cost saving effect* under commonality strategy. This heightened level of common attribute due to cost saving factor needs to be conceptually differentiated from savings in production cost of the products which is our model assumption. On the other hand, if relative size of the first segment is small and/or the gap in part worth is large compared to the magnitude of cost saving, then commonality strategy will result in lower level of attribute *k* in the first product. This is because the optimal level of common attribute is based on weighted average of part worth with weights being equal to segment sizes so that small value of r_1 or large value of β_k lowers the relative level of common attribute in the first product. Note that without cost saving, the level of attribute *k* under commonality strategy will be in between of efficient levels of the attribute under non-commonality strategy. We term it as *weighted-averaging effect* of commonality strategy under strictly vertical preference structure between segments. The effect implies that commonality strategy should take the preference characteristics of *both* segments into consideration and it in fact is a burden to the social planner since the firm is not able to use the attribute to differentiate the products to meet each segment's exact preference. Thus the optimal level of common attribute is decided through a tradeoff between the cost saving effect and the weighted-averaging effect. We next examine the change in social welfare due to the commonality strategy.

Proposition 2 : Commonality strategy improves social welfare if the following condition is satisfied.

$$\triangle SW = \frac{w_{2k}^2 (1+r_1(\beta_k - 1))^2}{4c_k} \left\{ \frac{1}{(1-\alpha_k)} - \frac{1+r_1(\beta_k^2-1)}{(1+r_1(\beta_k-1))^2} \right\} > 0. \tag{9}$$

Note that the first and second terms in the bracket represent relative benefits of commonality and non-commonality strategies, cost saving and quality differentiation, respectively. In [Figure 2], the value of r_1 and $1/(1-\alpha_k)$ are plotted on x-and y-axis, respectively, and the value of r_1 at which the value of the second term is highest is obtained by setting the first derivative of the second term equal to zero. Note also that the value of the second term in the above bracket is greater than one and is increasing in β_k. The area above the parabola represents the region where commonality strategy is preferred to non-commonality strategy and we will say cost saving is significant if $1/(1-\alpha_k) > (\beta_k+1)^2/4\beta_k$ since then commonality strategy improves social welfare regardless of the size of the first segment whereas we say it is moderate if $1/(1-\alpha_k) < (\beta_k+1)^2/4\beta_k$.



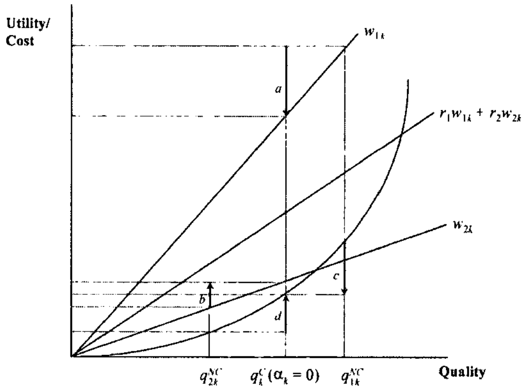
[Figure 2] Conditions where commonality is preferred when $\beta_k = 2$

Compared with non-commonality strategy, commonality strategy always improves the social welfare when cost saving is significant, $1/(1-\alpha_k) > (\beta_k+1)^2/4\beta_k$ (see [Figure 2]), as it makes products considerably cheaper to offer through economies of scale. When cost saving is moderate, $1/(1-\alpha_k) < (\beta_k+1)^2/4\beta_k$, commonality strategy is more likely to be preferred when i) cost saving is greater, ii) the valuation gap is smaller, and iii) relative sizes of the segments are dissimilar from each other. The cost saving factor helps commonality strategy in two different ways : economies of scale and utility effect. The lowered production cost due to economies of scale is a model assumption in our paper but the lower cost makes higher level of common attribute affordable for the social planner, which in turn improves consumer utility. The second and third observations are related to the weighted-averaging effect of commonality strategy. In our model, the weighted-averaging effect is causing an implicit cost of commonality strategy that is the cost of offering an attribute away from socially efficient qualities for the two segments. This cost can also be viewed as cost of diluting products or not differentiating an attribute. In order to quantify the cost of weighted-averaging effect in terms of social welfare, let $\alpha_k = 0$ in (9) to obtain

$$-\frac{w_{2k}^2(\beta_k-1)^2 r_1 r_2}{4c_k}.$$

With fixed values of weights, the cost of weighted-averaging effect gets prominent when part worths are farther from each other whereas with fixed values of part worths, the cost of weighted-averaging effect gets prominent when segment sizes are similar to each other. Thus commonality strategy is more likely to be preferred when the weighted-averaging effects is not prominent, i.e., small valuation gap or dissimilar segment sizes.

In order to visualize the cost of weighted-averaging effect, a comparison of efficient qualities under non-commonality and commonality strategies without cost saving is shown in [Figure 3]. Using three levels of part worth, w_{1k}, $r_1 w_{1k} + r_2 w_{2k}$, w_{2k}, and a quadratic cost function, three different efficient qualities q_{1k}^{NC}, q_{2k}^{NC}, q_k^C with $\alpha_k = 0$ are identified on *x*-axis. Lines *a* and *b* represent the change in the first and second segment's utility, respectively, whereas lines *c* and *d* represent the changes in cost of producing the efficient qualities. It can be shown that for the first segment, the decrease in utility (length of line *a*) is more than the decrease in cost (length of line *c*) whereas for the second segment, the increase in utility (length of line *b*) is less than the increase in cost (length of line *d*). Thus, for each segment, the amount of social welfare is decreased due to common attribute that is based on the weighed-averaging of segments' part worths.



[Figure 3] Cost of the weighted-averaging effect in commonality strategy

On the other hand, in order to understand the changes in amount of social welfare due to commonality strategy, the following lemma is in order.

Lemma 1

(i) $\frac{\partial \Delta SW}{\partial w_{2k}} > 0$;

(ii) $\frac{\partial \Delta SW}{\partial \beta_k} = \frac{2w_{2k}^2 r_1\{1+r_1(\beta_k-1)\}}{4c_k}$; $\left\{\frac{1}{1-\alpha_k} - \frac{\beta_k}{1+r_1(\beta_k-1)}\right\}$

(iii) $\frac{\partial \Delta SW}{\partial r_1} = \frac{2w_{2k}^2(\beta_k-1)\{1+r_1(\beta_k-1)\}}{4c_k}$. If $\left\{\frac{1}{1-\alpha_k} - \frac{\beta_k+1}{21+r_1(\beta_k-1)}\right\}$ $r_1 > 0.5$, then $\frac{\partial \Delta SW}{\partial r_1} > 0$.

Part (i) in Lemma 1 indicates that the amount of social welfare increase due to commonality strategy is strictly increasing in the second segment's valuation of quality. This is because as w_{2k} increases, valuation gap decreases so that weighted-averaging effect of commonality reduces whereas the level of common attribute increases so that utility effect prevails.

On the other hand, part (ii) in Lemma 2 indicates that the effect of valuation gap, β_k, on the amount of social welfare increase differs under different conditions. With relatively small cost saving, the amount of social welfare increase is decreasing in valuation gap, implying that the weighted-averaging effect of commonality is still significant compared to the utility and cost saving effect of commonality. However, if cost saving is significant, $1/(1-\alpha_k) > \beta_k/(1+r_1(\beta_k-1))$, then the offered level of common attribute is high due to cost saving effect so that utility effect due to cost saving outweighs the averaging effect. As a result, the amount of social welfare increase gets greater as the valuation gap increases.

Combining the observations from parts (i) and (ii) in Lemma 1 and Proposition 2, we can conclude that with small enough cost saving, high value of w_{2k} and small gap in valuation helps improving social welfare so that desirable attribute for commonality is the one that both segments value highly and similarly. However, with large cost saving, desirable attribute for commonality is the one that both segments value highly but not necessarily similarly. These observations are summarized in the following proposition.

Proposition 3 : When commonality strategy improves social welfare and cost saving is moderate, $\frac{\beta_k}{1+r_1(\beta_k-1)} > \frac{1}{(1-\alpha_k)} > \frac{1+r_1(\beta_k^2-1)}{(1+r_1(\beta_k-1))^2}$, desirable attribute for commonality is the one that both segments value highly and similarly. When commonality strategy improves social welfare and cost saving is significant $\frac{1}{(1-\alpha_k)} > \frac{\beta_k}{1+r_1(\beta_k-1)}$, desirable attribute for commonality is the one that both segments value highly but not necessarily similarly.

It is interesting that the characteristics of desirable common attribute differ under different conditions so that one has to compare utility effect with the weighted-averaging effect in order to choose better attribute for commonality.

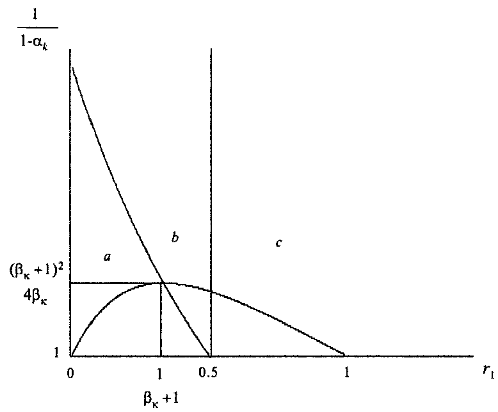
We next examine the effect of relative size of the first segment on the improvement of social welfare. Part (iii) in Lemma 1 indicates that the effect of r_1 also differs under different conditions. When r_1 is relatively large, greater than 0.5, it enhances the utility effect of commonality strategy so that social welfare improvement is increasing in the size of the first segment. On the other hand, when r_1 is relatively small, less than 0.5, it enhances the utility effect only if cost saving is significant. With small value of r_1 and moderate cost saving, the weighed-averaging effect outweighs the utility effect so that amount of social welfare increase is decreasing in the relative size of the first segment. These observations are visualized in [Figure 4]. In [Figure 4], commonality strategy improves social welfare in the region above the parabola (Proposition 2). Within the region, the amount of social welfare increase is decreasing in r_1 in region *a* whereas it is increasing in r_1 in regions *b* and *c*. For a fixed value of cost saving, the role of r_1 in social welfare improvement is summarized in following proposition.

Proposition 4 : When commonality strategy improves social welfare, for a given cost saving, the amount of social welfare improvement is at first decreasing in r_1 and then is increasing in r_1.
PROOF : When commonality strategy improves social welfare and when $r_1 > 0.5$, the amount of improvement in social welfare is increasing in r_1. When $r_1 > 0.5$ and commonality strategy improves social welfare, the amount of improvement in social welfare is decreasing in r_1 if $\frac{1+r_1(\beta_k^2-1)}{(1+r_1(\beta_k-1))^2} < \frac{1}{(1-\alpha_k)} < \frac{\beta_k+1}{2\{1+r_1(\beta_k-1)\}}$, and it is increasing in r_1 if $\frac{1}{(1-\alpha_k)} < \frac{\beta_k+1}{2\{1+r_1(\beta_k-1)\}}$.

We next examine the effect of commonality strategy on consumer surplus and firm's profit. A key issue is to understand who, consumer or firm, will be the beneficiary of the improved social welfare due to commonality strategy.

Proposition 5 : If commonality strategy results in improved social welfare, then it also improves consumer surplus even if the social planner allows the firm maximum possible profit.
PROOF : $SW^C > SW^{NC}$ is equivalent to $\frac{1}{(1-\alpha_k)} > \frac{1+r_1(\beta_k^2-1)}{(1+r_1(\beta_k-1))^2}$ and $CS^C > CS^{NC}$ is equivalent to $\frac{1}{1-\alpha_k} > \frac{1}{(1+r_1(\beta_k-1))}$.
Since $\frac{1+r_1(\beta_k^2-1)}{(1+r_1(\beta_k-1))^2} > \frac{1}{(1+r_1(\beta_k-1))}$,
$SW^C > SW^{NC} \Rightarrow CS^C > CS^{NC}$.

Proposition 5 indicates that a decision to adopt commonality strategy to improve social welfare



[Figure 4] Effect of r_1 on the amount of social welfare improvement, $\beta_k = 2$

always results in improvement of consumer surplus, and more importantly, this result is even with the pricing assumption that guarantees the maximum profit for the firm. Thus, if the social planner employs a different pricing scheme which is less favorable to the firm, then amount of increase in consumer surplus would be even more.

Proposition 6 : When commonality strategy results in improved social welfare, consumer surplus and firms' profit can be improved simultaneously when $r_1(\beta_k - 1) < 1$ and $\frac{1}{(1-\alpha_k)} > \frac{1+r_1(\beta_k-1)^2}{1-\{r_1(\beta_k-1)\}^2}$.
PROOF : Show that $SW^C > SW^{NC} \Rightarrow CS^C > CS^{NC}$ and $FP^C > FP^{NC}$ under certain condition.
$SW^C > SW^{NC}$ is equivalent to
$\frac{1}{(1-\alpha_k)} > \frac{1+r_1(\beta_k^2-1)}{(1+r_1(\beta_k-1))^2}$ and $CS^C > CS^{NC}$ is equivalent to $\frac{1}{1-\alpha_k} > \frac{1}{(1+r_1(\beta_k-1))}$. Since $\frac{1+r_1(\beta_k^2-1)}{(1+r_1(\beta_k-1))^2} > \frac{1}{(1+r_1(\beta_k-1))}$,
$SW^C > SW^{NC} \Rightarrow CS^C > CS^{NC}$ · $FP^C > FP^{NC}$ is

equivalent to

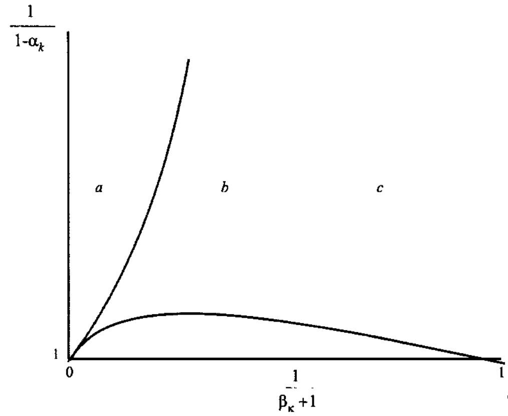
$$\frac{w_{2k}^2\{1-r_1(\beta_k-1)^2\}}{4c_k}\left\{\frac{1}{(1-\alpha_k}-\frac{1+r_1(\beta_k-1)^2}{1-r_1(\beta_k-1)^2}\right\}.$$

It can be shown that if $FP^C > FP^{NC}$, then $SW^C > SW^{NC}$.

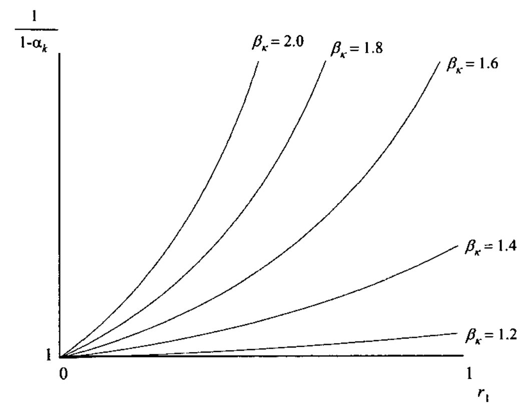
Proposition 6 indicates that when a social planner decides to adopt commonality strategy to improve social welfare, it can simultaneously improves consumer surplus and firm's profit under certain conditions. In other words, commonality strategy targeting to maximize social welfare does not necessarily require the sacrifice of the firm. The increased amount of social welfare due to cost saving associated with commonality strategy can be allocated between consumers and the firm so that both parties can be better off under certain conditions. It is interesting that commonality strategy can result in such a win-win result under social welfare maximization.

In [Figure 5], the regions where different parties benefit from commonality strategy are indicated. In region *a* of the area above the parabola, both consumer and firm benefit from commonality strategy whereas in regions *b* and *c* only consumers benefits from the commonality strategy. In the region *a*, as the relative size of the first segment increases it is increasingly difficult to have a favorable pricing scheme for the firm due to the first segment's switching to the second product so that more cost saving is necessary for the firm to benefit from commonality strategy. This trend continues until $r_1 = 1/(\beta_k - 1)$ beyond which firm can not benefit from commonality strategy.

In order to show that the condition in Proposition 6 is fairly general one, we provide in [Figure 6] different conditions using five different values of valuation gap, β_k= 1.2, 1.4, 1.6, 1.8 and 2. Note that for all values of β_k, the first condition in Proposition 6, $r_1(\beta_k - 1) < 1$, is satisfied. Thus for each value of β_k, the area above the plotted line indicates the region where both the firm and consumers benefit from the commonality strategy. As one can see, the size of the regions are fairly significant and as the value of β_k increases the size of the region shrinks, indicating



[Figure 5] Regions where both consumer and firm benefit from commonality strategy, $\beta_k = 3$



[Figure 6] Regions where both consumer and firm benefit from commonality strategy for various values of β_k

more cost saving is required in order to benefit both parties in the market. As one can see, as the value of β_k increases the size of the region shrinks, indicating that more cost saving is required in order to benefit both parties in the market.

As a numerical example for Proposition 6, consider $r_1 = 0.2$, $r_1 = 0.8$, $w_{1u} = 1.6$, $w_{2u} = 1$, $w_{1u} = 1.6$, $w_{2u} = 1$, $w_{1k} = 1.4$, $w_{2k} = 1$, $\alpha_k = 0.1$, and $c_u = c_k = 1$. Thus the first segment is a niche segment with relatively small size and high part worth, and the common attribute k exhibits relatively small part worth gap and reasonable cost saving factor. With these parameter values, $SW^C = 0.652 > SW^{NC} = 0.626$, $CS^C = 0.108 > CS^{NC} = 0.1$ and $FP^C = 0.544 > FP^{NC} = 0.526$, thus the commonality results in improved consumer surplus and firm's profit simultaneously.

5. Comparison with Commonality Strategy under Firm's Profit Maximization

Effects of commonality strategy under different objective function, firm's profit maximization, have been extensively analyzed in literature (Heese and Swaminathan 2006, Krishnan and Gupta 2001 Desai et al. 2001, Kim and Chhajed 2000). In order to understand the implication of commonality strategy under different objective function and compare it with commonality strategy under social welfare maximization objective, a counterpart formulation of (CPD) with profit maximization objective (with one custom and one common attributes) can be formulated as follows.

(Program CPD_FP)

$$Max\ \Sigma_{i=1,2} r_i P_i - \Sigma_{i=1,2} r_1\{c_u q_{iu}^2 + c_k(1-\alpha_k)q_k^2\}$$

$$w_{1u}q_{1u} + w_{1k}q_k - P_1 \geq w_{1u}q_{2u} + w_{1k}q_k - P_2$$

$$w_{2u}q_{2u} + w_{2k}q_k - P_2 \geq w_{2u}q_{1u} + w_{2k}q_k - P_1$$

$$w_{1u}q_{1u} + w_{1k}q_k \geq P_1$$

$$w_{2u}q_{2u} + w_{2k}q_k \geq P_2$$

$$q_{1u},\ q_{2u},\ q_k,\ P_1,\ P_2 \geq 0$$

In the objective function in (CPD_FP), the objective function is stated as difference between product price and cost from two segments. Profit maximizing pricing schedule without violating self selection and participation constraints is $P_1 = w_{1u}q_{1u} + w_{1k}q_{1k} - (w_{1u} - w_{2u})q_{2u} - (w_{1k} - w_{2k})q_{2k}$ and $P_2 = w_{2u}q_{2u} + w_{2k}q_{2k}$, and optimal product design, profit, consumer surplus, and resulting social welfare are provided in <Table 2>.

<Table 2> Optimal product design, firm's profit, consumer surplus, and social welfare under commonality strategy with profit maximizing objective function

$$q_{1u}^{CP} = \frac{w_{1u}}{2c_u}\ ;\ q_{2u}^{CP} = \frac{w_{2u}(1-R_u)}{2c_u}\ ;\ q_k^{CP} = \frac{w_{2k}}{2c_k(1-\alpha_k)}\ ;$$

$$FP^{CP} = \frac{w_{2u}^2(1+R_u^{\prime})}{4c_u} + \frac{w_{2k}^2}{4c_k(1-\alpha_k)}\ ;$$

$$CS^{CP} = \frac{2w_{2u}^2\left\{\frac{(w_{1u}-w_{2u})}{w_{2u}} - R_u^{\prime}\right\}}{4c_u} + \frac{2w_{2k}^2\left(\frac{w_{1k}-w_{2k}}{w_{2k}}\right)}{4c_k(1-\alpha_k)}\ ;$$

$$SW^{CP} = \frac{w_{2u}^2\left\{1+2\left(\frac{w_{1u}-w_{2u}}{w_{2u}}\right) - R_u^{\prime}\right\}}{4c_k} + \frac{w_{2k}^2\left(1+2\left(\frac{w_{1k}-w_{2k}}{w_{2k}}\right)\right)}{4c_k(1-\alpha_k)}$$

where $R_u = \dfrac{r_1(w_{1u}-w_{2u})}{r_2 w_{2u}}$ and

$$R_u^{\prime} = \frac{r_1}{r_2}\left(\frac{(w_{1u}-w_{2u})}{w_{2u}}\right)^2.$$

Note) Superscript CP indicates commonality strategy under profit maximizing objective function.

Proposition 6 : Commonality strategy under social welfare maximization results in higher level of custom attribute in the second product and in higher level of common attribute, i.e., $q_{1u}^{C} = q_{1u}^{CP}, q_{2u}^{C} > q_{2u}^{CP}, q_{k}^{C} > q_{k}^{CP}$.

In profit maximization problem such as (CPD_FP), it is known that in an optimal solution the firm reduces the level of custom attribute in the second product below efficient level in order to avoid cannibalization whereas the level of common attribute is set at the second segment's part worth for that attribute in order to extract all surplus from the second segment. On the other hand, in the social welfare maximization problem, the issue of cannibalization still exists but implicit cost of cannibalization is reflected in maximum product prices that a firm can charge rather than in product design so that all attributes are offered at their respective efficient levels. Note that the optimal level of common attribute is higher under the objective of social welfare maximization, implying that the notion of commonality, if used in design of public goods, is utilized much more by a social planner than by a monopolistic firm.

Proposition 7 : With maximum possible pricing for the firm, commonality strategy under social welfare maximization results in larger amount of consumer surplus than under firm's profit maximization.

PROOF : By definition, $SW^{C} > SW^{CP}$ and $FP^{C} < FP^{CP}$. Since $SW^{C} > SW^{CP} \Leftrightarrow CS^{C} - CS^{CP} > FP^{CP} - FP^{C}$ and $FP^{CP} - FP^{C} > 0$, $CS^{C} > CS^{CP}$ must hold.

Proposition 7 indicates that under social welfare maximization, a social planner can ensure larger amount of consumer surplus even if it allows maximum possible product prices to the firm. If it uses other pricing schemes such as marginal cost based pricing, then the amount of consumer surplus will be even greater.

6. Discussion and conclusion

The current paper addresses the issue of commonality strategy in product line design from a perspective of social planner who wants to maximize social welfare. While the issue of commonality strategy has been mostly addressed from a profit maximizing firm's perspective in literature, the main motivation of the paper is to understand similarities and differences in the effects of commonality strategy under different objective functions.

One of the interesting issues in commonality strategy is about selection of common attributes. Out result (Proposition 2) indicates that an attribute is considered to be desirable as a common attribute as i) cost saving is greater, ii) valuation gap is smaller, and iii) relative sizes of the segments are vastly different from each other. The first two characteristics are similar to the ones identified under firm's profit maximizing objective function, but the last characteristic is not. Under the objective of firm's profit maximization, the products are designed in a way the first segments gets its efficient quality and the second segment gets quality level that is less than its efficient level to prevent the first segment's switching to the second product. Thus the degree of cannibalization is larger as the size of the first segment gets larger. Since the adoption of commonality strategy makes quality differentiation impossible, the firm has to bear the cost of cannibalization internally and as a result it would prefer to adopt commonality strategy when relative size of the first segment is small. (Desai et al. 2001, Heese and Swaminathan 2006) However, under the objective of social welfare maximization, the implicit cost of commonality is due to the weighted-averaging effect, i.e., cost of not providing efficient qualities for both segments, and its magnitude gets larger as relative sizes of the segments get similar (see [Figure 2]). Thus, commonality strategy would be preferred when the segment sizes are dissimilar.

When it comes to the amount of social welfare improvement due to commonality strategy, our results show that the amount is increasing in the second segment's part worth, it is at first decreasing and then increasing in valuation gap, and it is at first decreasing and then increasing in the relative size of the first segment. All of these factors are related to the utility effect and cost saving effect of commonality strategy. Given our model assumption of $w_{1j} > w_{2j}$, it is always desirable to have higher value w_{2j} in maximizing utility and thereby social welfare. On the other hand, with moderate cost saving, small valuation gap and small relative size of the first segment are beneficial in improving social welfare as the weighted-averaging effect outweighs the cost saving effect. However, with significant cost saving, utility effect that is invoked by the cost saving outweighs the weighted-averaging effect so that large value of part worths and large relative size of the first segment are beneficial in improving social welfare. These observations imply that if both attributes are qualified as common attribute, then better attribute for commonality is different under different conditions, depending on cost saving and part worth structure; for moderate cost saving, better attribute for commonality is the one that is equally important to both segments whereas for large cost saving, it is the one that is important (but not necessarily equally important) to both segments.

Our result in Proposition 5 and 6 indicates that commonality strategy can lead to improved consumer surplus and firm's profit simultaneously. In other words, cost saving and utility effect brought by commonality strategy can be shared by both parties in society and this is feasible even when the social planner allows maximum profit to the firm. Thus, it doesn't require sacrifice of the firm.

The current paper addresses the issue of commonality strategy under social welfare maximization, and analyzes the effects that are caused by the commonality strategy. Cost saving effect is the model assumption, but the utility effect and the weighted-averaging effect is the ones that are caused by the commonality strategy. We have conceptualized and quantified the weighted-averaging effect, an implicit cost of commonality strategy, which is only conceptually discussed in terms of dilution of products or cost of not differentiating attribute. We showed that the weighted-averaging effect in our model can be viewed as cost of not offering customers their efficient qualities so that the implicit cost of commonality is always levied from both segments. We note that in models of profit maximization problem in strictly vertical preference structure (Kim and Chhajed 2000, Desai et al 2001, Heese and Swaminathan 2006), the implicit cost of commonality is only levied from the low-end segment.

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