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GAME MODEL AND ITS SOLVING METHOD FOR OPTIMAL SCALE OF POWER PLANTS ENTERING GENERATION POWER MARKET

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ABSTRACT. Based on social welfare maximum theory, the optimal scale of power plants entering generation power market being is researched. A static non-cooperative game model for short-term optimization of power plants with different cost is presented. And the equilibrium solutions and the total social welfare are obtained. According to principle of maximum social welfare selection, the optimization model is solved, optimal number of power plants entering the market is determined. The optimization results can not only increase the customer surplus and improve power production efficiency, but also sustain normal profits of power plants and scale economy of power production, and the waste of resource can also be avoided. At last, case results show that the proposed model is efficient.

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1. Introduction

For many decades, the vertically integrated monopolization utility is the major electricity operation mode, in which they generate, transmit, distribute and finally sell power to customers in their service territories. But vertically integrated monopolization utility could not provide services as efficiently as competitive companies. Recently, the electric power restructuring in China has created an environment where electricity trade is conducted in a market environment. In such environment, generators compete for the market share. But over-competition is also not good. Electricity generation industry takes on scale economy. Sharkey W. W. [1] and Ze Ye [2] had pointed out that barriers for withdrawal were very high because the equipment for generating electricity was

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uniqueness. Over-competition will make social welfare down. The paper [3-5] proposed that competition was efficient if and only if competition made electricity price fall, consumer over-plus increased and power plants' profits didn't decrease. So the regulator needs to control and optimize the total scale of electricity generation industry macroscopically after being deregulated.

Based on social welfare maximum theory, Lijun Yang et al [6] gave the optimal scale of new power plants entering power generation market deregulated. But that paper didn't consider the effect that the different cost of coal and the different installed capacity. This paper studies the effect coming from scale economy and the different cost of coal, gives the definition of differentia cost and constructs differentia cost function. With different fixed cost proportion coefficient of each power plant and uniform capacity price in power market the paper sets up a non-cooperative game model to optimize social welfare, and a method is presented for the problem.

2. Non-cooperative Game Model

Every power plant making output as a decision-making variable takes part in competition independently in order to maximize its profits in power market. The market-clearing price is determined by total output of all power plants, i.e. the equilibrium price is determined by all power plants together. This paper sets up non-cooperative game model of many power plants with asymmetry cost.

Since competition is introduced in electric power industry, the rule of competitive bidding has been put into practice, especially in the regions implementing two-part electricity rate. The deal regulation putting efficiency into first requires that power plants bid on changeable cost and chooses power plants to generate electricity from low to high price orderly. So the changeable cost impacts directly each power plant's profits. However, changeable cost of almost each power plant is different. We give the definition of differentia cost in order to analyze the problem. Differentia cost is the difference of the changeable cost to produce unit energy that each power plant compares with the plant possessing the highest producing efficiency. Coal is the primary produce goods for fossil fuel power plant. Yanpeng Wang et al [7] pointed the coal cost accounted for more than half of operation cost of power plant. So the differentia cost mainly comes from the difference of the coal price and coal-consuming rate. Pithead power plants can buy coal at pithead price. But the plant far from coal mine must pay transportation except for the pithead price. Then the differentia cost has the direct ratio with the distance. In addition, coal-consuming rate always relates to installed capacity. The paper [8] indicated that half of all coal-fired power plants were middle and small thermal power plants in China. These plants are almost high coal-consuming and low efficiency. So based on the current level of installed capacity, the more capacity, the lower coal-consuming rate. Let σ_i be differentia cost of power plant i. Then we give the expression of σ_i as following:

$$\sigma_i = d/r_i + fl_i, \tag{1}$$

where d and f are constant, got by fitting. l_i is distance from power plant i to coal mine. r_i is the installed capacity of power plant i.

F_i is fixed cost allocation of power plant i in a period of time. Fixed cost of power plant relates directly with installed capacity. μ_i is coefficient of fixed cost allocation of power plant i for unit installed capacity in a period of time. Then $F_i = \mu_i r_i$.

The paper sets up non-cooperative game model to optimize social welfare using non-cooperative theory of paper [9] and [10]. There are n coal-fired power plants in power market. q_i, q_i^{min} and q_i^{max} are the output, the minimum output and the maximum output from generators i in an hour, i.e. $q_i^{min} \le q_i \le q_i^{max}$, $i = 1, 2, \dots, n$. Q is the total output of n power plants in an hour, then $Q = \sum_{i=1}^{n} q_i$ and $\sum_{i=1}^{n} q_i^{min} \le Q \le \sum_{i=1}^{n} q_i^{max}$.

The demand function of market is:

$$P(Q) = a - bQ. \tag{2}$$

The cost function of power plant i is:

$$C_i(q_i) = (c + \sigma_i) q_i h_i + F_i, \tag{3}$$

where c is the least changeable cost of unit output in electricity industry.

Then the profit function of power plant i is:

$$\pi_i(q_i) = (a - bQ - c - \sigma_i) q_i h_i - (\mu_i + p_r h_i') r_i, \tag{4}$$

where p_r is average capability price. h_i denotes factual work time and h_i' is useable time of power plant i in a period time.

3. Short equilibrium

In the model, the object of power plant i ($i = 1, 2, \dots, n$) is to maximize its profits by choosing its energy output q_i under fixed q_j ($j \ne i, iandj = 1, 2, \dots, n$). The mathematical formulation of the gaming model can be expressed as follow:

$$\pi_i^* = \max \left\{ (a - bQ - c - \sigma_i) q_i h_i - (\mu_i + p_r h_i') r_i \right\} \tag{5}$$

$$s.t. \quad Q = \sum_{i=1}^{n} q_i, \quad q_i^{min} - q_i \le 0, \quad q_i - q_i^{max} \le 0.$$

Proposition 1. *The short equilibrium output of power plant i is:*

$$q_i^{sc}(n) = (a - c - \sigma_i)/b - Q^{sc}(n); \tag{6}$$

The short equilibrium profit of power plant i is:

$$\pi_i^{sc}(n) = b[q_i^{sc}(n)]^2 h_i - (\mu_i - p_r h_i') r_i; \tag{7}$$

In power market, the short equilibrium price is:

$$p^{sc}(n) = a - bQ^{sc}(n), \tag{8}$$

where

$$Q^{sc}(n) = \begin{cases} \sum_{i=1}^{n} q_i^{max} & if\ \frac{n(a-c)}{b(n+1)} - \frac{\sum_{i=1}^{n}\sigma_i}{b(n+1)} \geq \sum_{i=1}^{n} q_i^{max}, \\ \sum_{i=1}^{n} q_i^{min} & if\ \frac{n(a-c)}{b(n+1)} - \frac{\sum_{i=1}^{n}\sigma_i}{b(n+1)} \leq \sum_{i=1}^{n} q_i^{min}, \\ \frac{n(a-c)}{b(n+1)} - \frac{\sum_{i=1}^{n}\sigma_i}{b(n+1)} & else. \end{cases} \tag{9}$$

Proof. In case of no constrains, we can get the following equations according as marginal revenue equal to marginal cost.

$$a - bQ - bq_i - c - \sigma_i = 0, \qquad i = 1, 2, \ldots, n. \tag{10}$$

Sum Eq.(10) from 1 to n, then

$$(n+1)bQ = n(a-c) - \sum_{i=1}^{n} \sigma_i.$$

So under no constrains the total output of n power plants is:

$$Q^{sc}(n) = \frac{n(a-c)}{b(n+1)} - \frac{\sum_{i=1}^{n}\sigma_i}{b(n+1)}.$$

But the output of each power plants have to satisfy constrains. So in fact the total output of n power plants is Eq.(9). Then based on Eq.(2), (5), (9) we can get:
the short equilibrium output of power plant i is:

$$q_i^{sc}(n) = (a - c - \sigma_i)/b - Q^{sc}(n),$$

the short equilibrium profit of power plant i is:

$$\pi_i^{sc}(n) = b[q_i^{sc}(n)]^2 h_i - (\mu_i - p_r h_i')r_i,$$

and the short equilibrium price is:

$$p^{sc}(n) = a - bQ^{sc}(n),$$

where $Q^{sc}(n)$ is given by Eq.(9). □

Based on welfare economics theory [11], under equilibrium solution the total social welfare of n power plants is:

$$\begin{aligned} W_i^{sc}(n) &= b[Q_i^{sc}(n)]^2/2 + \sum_{i=1}^{n} \pi_i^{sc}(n) \\ &= b[Q_i^{sc}(n)]^2/2 + b\left[\sum_{i=1}^{n} (q_i^{sc})^2 h_i\right], \end{aligned} \tag{11}$$

where $b(Q^{sc})^2/2$ is consumer over-plus, $\sum_{i=1}^{n}\pi_i^{sc}(n)$ is producer over-plus.

We set up the optimization model of power plants scale to make social welfare maximum. Based on the optimal solution, the regulator can project the optimal power plants scale. The optimization model can be expressed as :

$$(P_1) \quad \max\left\{W^{sc}(n)\right\} \tag{12}$$

$$s.t. \quad q_i^{max} - q_i^{sc} \geq 0, \;\; q_i^{sc} - q_i^{min} \geq 0,$$

where $W^{sc}(n)$ is given by Eq.(11).

4. Solving method to optimization model

$b[Q^{sc}(n)]^2/2$, the first item of $W^{sc}(n)$, is a increasing function about n. $\sum_{i=1}^{n}\pi_i^{sc}(n)$, the second item of $W^{sc}(n)$, becomes no increasing when $\pi_j = 0$, $j \in 1,2,\ldots,n$. And in practice, plants will stop producing when their profits are zero or negative. So the maximum of $W^{sc}(n)$ is $W^{sc}(n)^*$, where $n^* = \max\{i \mid \pi_i^{sc}(n) \geq 0, i = 1,2,\ldots,n\}$. Then the optimum problem (P_1) can be rewritten to:

$$(P_2) \quad \max\left\{i \mid \pi_i^{sc}(n) \geq 0, i = 1,2,\ldots,n\right\} \tag{13}$$

$$s.t. \quad q_i^{max} - q_i^{sc} \geq 0, \;\; q_i^{sc} - q_i^{min} \geq 0,$$

where $\pi_i^{sc}(n) = b(q_i^{sc})^2 - (\mu_i - p_r h_i')r_i$, $q_i^{sc}(n) = (a - c - \sigma_i)/b - Q^{sc}(n)$.

We have developed an algorithm to the optimization model. The proposed algorithm calculates the numerical value of n based on optimal social welfare. The main algorithm is described as follows:

Step 1. Input initial data a, b, c, d, f, r_i, μ_i, p_r, h_i', q_i^{max}, q_i^{min}, where $i = 1,2,\ldots,n$.

Step 2. Compute $\sigma_i (i = 1,2,\ldots,n)$ using Eq.(1) and calculate $Q^{sc}(n)$ using Eq.(9). Appoint $n = k$ and based on the sequence of $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_i \geq \sigma_{i+1} \geq \cdots \geq \sigma_k$ solve accordingly q_i^{sc} and π_i^{sc} using Eq.(6), (7), (9).

Step 3. If $\pi_k^{sc} > 0$, appoint $n = k + 1$ and go to step 2. If $\pi_k^{sc} = 0$, output $n^* = k$ and stop. If $\pi_k^{sc} < 0$, output $n^* = k - 1$ and stop.

Then we can get the total output of n^* power plants Q^{sc}, the short equilibrium output q_i^{sc} and profits π_i^{sc} of power plant i, the short equilibrium price p^{sc} and the total social welfare $W^{sc}(n^*)$ using Eq.(6)-(9) and Eq.(11).

5. Numerical example

We assume $d = 200$ and $f = 0.005$. The least changeable cost of unit output c is 20(\$/MWh), average uniform capacity price p_r is 6.375(\$/MWh). The demand function of peak load, middle load and valley load can be described respectively as $P_p(Q) = 150 - 0.0668Q$, $P_m(Q) = 125 - 0.0585Q$, $P_v(Q) = 100 - 0.0516Q$.

And peak load period of time accounts for 300h, middle and valley load period of time are 200h and 220h in a month. The production information of producers can be shown in Tab.1. The solution process and the results can be seen in Tab.2. and Tab.3.

TABLE 1. The production information of producers

n	r_i(MW)	l_i(km)	μ_i(\$/MW)	h'_i(h)	σ_i(\$/MWh)
1	600	0	3981.07	560	0.33
2	300	220	4112.35	520	1.77
3	300	300	4120.36	520	2.17
4	300	430	4089.17	520	2.82
5	150	300	3865.33	500	2.83
6	150	310	3705.68	500	2.88
7	150	430	3898.75	500	3.48

As shown in Tab.3, the optimal scale of power plants n^* is 6. According Tab.2 and Tab.3 we can know:

(1) The power plants with smaller differentia cost can get more profits. So the competition scheme can impel power plant to decrease cost and improve efficiency.

(2) A certain extent, the last power plant selected with higher differentia cost elevates the average price, which brings abundant profits and less existence pressure to lower cost plants. That may augment the market power of lower cost plants and decreasing consumer over-plus. So the regulator can lead these power plants which are high energy consuming, low efficiency and high cost exit power market and lead some low energy consumption plants entry power market.

(3) Under the uniform capacity price, all power plants can compete equally in the power market, which makes for reducing the fixed cost of new power plants, but cannot ensure each power plant callback their fixed cost. And the uniform capacity price brings some additional profits to old power plants having got back all the fixed cost. But almost the old power plants are high energy consuming, which are not good for sustainable development.

The paper analyzes some influence factors using single factor analysis method. The results are illustrated in Fig.1-3. In these figures, abscissa is the number of power plant and ordinate is the total social welfare. Figure 1 shows that the total social welfare decreases and optimal number of power plant becomes 5 after increasing the differentia cost. In Fig.2, the fixed cost increases 20% on the basis of the initial data, which makes the total social welfare decrease and optimal number of power plant become 5. Fig.3 indicates that the total social welfare decreases rapidly and the optimal number of power plant is 5 after the capacity price reducing 20.8% on the basis of the initial data.

TABLE 2. Plants' quantities entering to electricity market and profits

total number n	power plant i	q_i ($/MWh) peak	midst	valley	π_i($)
1	1	600.00	600.00	600.00	1471.20
2	1	600.00	600.00	600.00	1471.20
	2	300.00	300.00	300.00	319.94
3	1	600.00	600.00	600.00	1471.20
	2	300.00	300.00	300.00	319.94
	3	300.00	300.00	210.33	265.52
4	1	600.00	600.00	600.00	1471.20
	2	300.00	300.00	300.00	319.94
	3	300.00	300.00	300.00	304.75
	4	300.00	212.54	0.00	164.01
5	1	600.00	600.00	600.00	1471.20
	2	300.00	300.00	300.00	319.94
	3	300.00	300.00	300.00	319.22
	4	300.00	269.61	0.00	195.29
	5	150.00	0.00	0.00	23.81
6	1	600.00	600.00	600.00	1471.20
	2	300.00	300.00	300.00	319.94
	3	300.00	300.00	300.00	319.22
	4	300.00	300.00	0.00	216.89
	5	150.00	0.00	0.00	23.81
	6	95.22	0.00	0.00	0.002
7	1	600.00	600.00	600.00	1471.20
	2	300.00	300.00	300.00	319.94
	3	300.00	300.00	300.00	319.22
	4	300.00	300.00	0.00	216.89
	5	150.00	0.00	0.00	23.81
	6	128.70	0.00	0.00	14.22
	7	0.00	0.00	0.00	-24.76

6. Special case

To getting generality conclusion, we let $r_i = r$, $\mu_i = \mu$, $h'_i = h$, $l_i = l$, then $\sigma_i = \sigma$, where $i = 1, 2, \ldots$. We suppose that the equilibrium output of every power plant can satisfies their capacity restrictions, i.e. $q^{min} \leq q_i^c \leq q^{max}$. In the case of symmetrical cost, the profits of each power plant are equal. Using

TABLE 3. The results under different scale of power plants

total number n	Q (MWh) peak	midst	valley	P (\$/MWh) peak	midst	valley	W(\$)
1	600.00	600.00	600.00	112.32	89.90	68.14	1474.34
2	900.00	900.00	900.00	93.48	72.35	52.21	1798.20
3	1200.00	1200.00	1109.86	74.64	54.80	41.07	2068.66
4	1500.00	1411.34	1178.22	55.80	42.44	37.44	2276.48
5	1650.00	1469.19	1200.00	46.39	39.17	36.28	2348.12
6	1745.00	1500.00	1200.00	40.41	37.25	36.28	2371.02
3	1778.70	1500.00	1200.00	38.30	37.25	36.28	2360.85

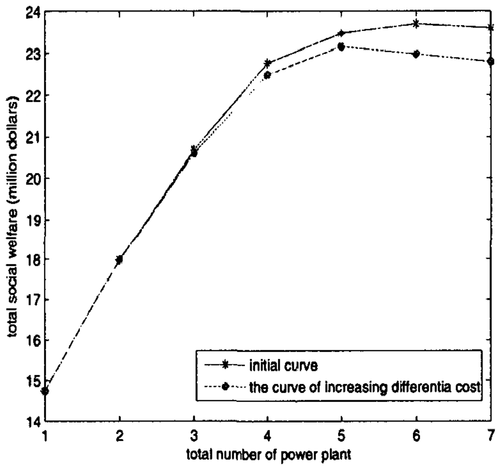

FIGURE 1. The results after increasing differentia cost

Eq. (6)-(9) and (11) we can get:

$$\begin{aligned}
\pi^c(n) &= [(a-c-\sigma)^2/b(n+1)^2] - (\mu - p_r h)r \\
Q^c(n) &= \frac{n(a-c-\sigma)}{b(n+1)} \\
q^c(n) &= \frac{(a-c-\sigma)}{b(n+1)} \\
P^c(n) &= \frac{a+n(c+\sigma)}{n+1} \\
W^c(n) &= \frac{n(n+2)(a-c-\sigma)^2}{2b(n+1)^2} - n(\mu - p_r h')r \qquad (14)
\end{aligned}$$

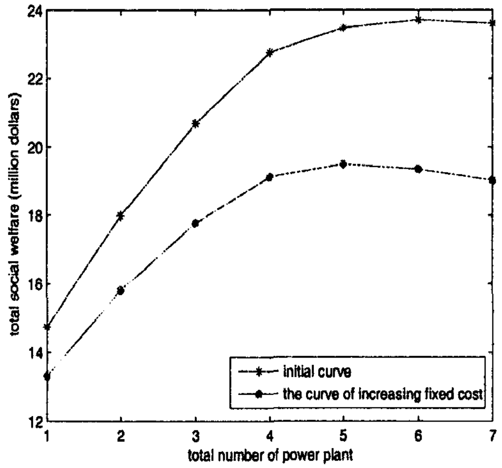

FIGURE 2. The results after increasing fixed cost

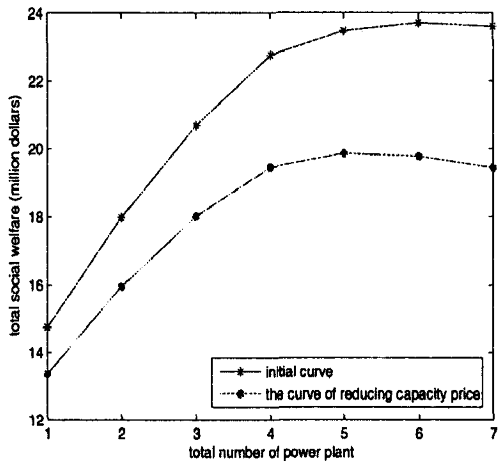

FIGURE 3. The results after reducing capacity price

Using the first-order condition of Eq.(14), we can get the optimal scale n^{c*}, which makes the social welfare achieve the maximum.

$$n^{c*} = \sqrt[3]{(a-c-\sigma)^2/b(\mu - p_r h')r} - 1 \quad (15)$$

Eq.(15) shows that the optimal scale is determined by multi-factor together. We discuss as follows:

(1) The optimal scale of power market relates with market demand (shown as a), elasticity of demand (shown as b), marginal cost c, differentia cost σ, fixed cost μr and capacity expense compensated $p_r h' r$. It can boost the scale of power plants that enlarging market demand, decreasing differentia cost and fixed cost, increasing capacity expense. So the regulator can lead power market develop healthily by adjusting that factors.

(2) The optimal scale decreases with the increasing fixed cost. But electricity generation industry needs justly a great lot fixed cost. That confirms electricity generation industry has the character of natural monopolization and natural barriers to entry. So the regulator needs to offer some favorable policy to entrant.

(3) In this section there is a hypothesis: the equilibrium output of every power plant can satisfy their capacity restrictions. If the restrictions operate, which means demand exceeds supply, we need to enlarge the scale of power plants.

7. Conclusion

(1) Monopolization will result to low efficiency. But excessive competition will result to wasting resource. Then the regulator needs to make sound overall planning when we introduce competitive mechanism in power market. In generation-side, the regulator can make some policy to prevent market power centralizing and excessive competing.

(2) We must insist on that the power plants with low differentia cost generate firstly in order to reducing cost, improving efficiency and maximizing social welfare. At the same time the regulator can lead these power plants which are high energy consuming, low efficiency and high cost to exit power market and help some low energy consumption plants entry power market.

(3) A certain extent, uniform capacity price brings criterion and make for reducing the fixed cost of new power plants. But it goes against old power plants exiting market. So we should make more policy to restrict the old plants under uniform capacity price.

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References

1. Ze Ye, *Competition in Electricity Supply Industry*, P.R. China Electric Power Press, Beijing, 2004.
2. Sharkey W. W., *TThe Theory of Natural Monoply*, Cambridge University Press, Cambridge, 1982.
3. Poitras Sutter, *The Efficiency Gains from Deregulation*, J. Regulation Economics **12**(1997), 81-89.
4. Zhiping Chang, Jiang Fu, *Analysis of Social Welfare Change Due to Deregulation*, Systems Engineering-theory Methodology Applications **4**(2001), 325-327.

5. Francisco D Galiana, Alexis L Motto, Francois Bouffard, *Reconciling Social Welfare, Agent Profit, and Consumer Payment in Electricity Pools*, IEEE Transaction on Power Systems, **2**(2003), 452-459.
6. Lijun Yang, Zhongfu Tan, et al, *Research on Scale of Optimal Power Plants Entering Electricity Generation Market Being Deregulated*, Proceedings of the Chinese Society for Electrical Engineering **10**(2005), 234-266.
7. Yanpeng Wang, Jie Zhou, et al, *Game Analysis of Gang Adjustment System of Coal and Electricity Prices*, Market Modernization **12**(2005), 141-142.
8. Wei Liu, Wu Yan, *Attention Paid to Some Issues during Assets Appraisal for Middle and Small Thermal Power Units*, Power Electricity Building **2**(2000), 49-50.
9. Paul A. Samuelson, *Some Game Theory Anecdotes*, Japan and the World Economy, **13** (2001), 299-302.
10. Tiaojun Xiao, *Game Theory and Its Applications*, Sanlian Publisher, Shanghai, 2004.
11. Qiguang An, QingChun Meng, *A Study about Relationship between Social Welfare Maximization and Consumer Utility Maximization*, Chinese Journal of Management Science **2**(2002), 67-70.

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