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NON LINEAR VARIABLE VISCOSITY ON MHD MIXED CONVECTION HEAT TRANSFER ALONG HIEMENZ FLOW OVER A THERMALLY STRATIFIED POROUS WEDGE

R.KANDASAMY*, I.HASHIM AND K.RUHAILA

ABSTRACT. The effect of variable viscosity on MHD mixed convection Hiemenz flow over a thermally stratified porous wedge plate has been studied in the presence of suction or injection. The wall of the wedge is embedded in a uniform Darcian porous medium in order to allow for possible fluid wall suction or injection and has a power-law variation of the wall temperature. An approximate numerical solution for the steady laminar boundary-layer flow over a wall of the wedge in the presence of thermal diffusion has been obtained by solving the governing equations using numerical technique. The fluid is assumed to be viscous and incompressible. Numerical calculations are carried out for different values of dimensionless parameters and an analysis of the results obtained shows that the flow field is influenced appreciably by the magnetic effect, variable viscosity, thermal stratification and suction / injection at wall surface. Effects of these major parameters on the transport behaviors are investigated methodically and typical results are illustrated to reveal the tendency of the solutions. Comparisons with previously published works are performed and excellent agreement between the results is obtained.

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1. Introduction

In many mixed flows of practical importance in nature as well as in many engineering devices, the environment is thermally stratified. The discharge of hot fluid into enclosed regions often results in a stable thermal stratification with lighter fluid overlying denser fluid. The present trend in the field of thermal stratification and magnetic effect analysis are to give a mathematical model for

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the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular, the study of heat transfer with thermal stratification is of considerable importance in chemical and hydrometallurgical industries. Frequently the transformations proceed in a moving fluid, a situation encountered in a number of technological fields. A common area of interest in the field of aerodynamics is the analysis of thermal boundary layer problems for two dimensional steady and incompressible laminar flow passing a wedge.

Heat transfer from different geometrics embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and under ground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection / conduction transport processes. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in no conventional energy sources, such as the use of salt-gradient solar ponds for energy collection and storage. In particular, mixed convection induced by the simultaneous action of buoyancy forces resulting from thermal diffusion is of considerable interest in nature and in many industrial applications such as geophysics, oceanography, drying processes, solidification of binary alloy and chemical engineering. Stagnation flows are found in many applications such as flows over the tips of rockets, aircrafts, submarines and oil ships. The study of stagnation flow problem was started by Hiemenz [17] who developed an exact solution to the Navier-Stokes governing equations for the forced convection case. Later, Eckert [13] studied a similar solution for both the momentum and energy governing equations excluding the effect of buoyancy forces. Ariel [2] has considered the stagnation-point flow of electrically-conducting fluids in the presence of large transverse magnetic field strengths. Yih [26] has reported on the effects of uniform suction or blowing and magnetic field on the heat transfer characteristics of the Hiemenz problem in porous media.

Frequently the transformations proceed in a moving fluid, a situation encountered in a number of technological fields. In previous investigations, Chambre and Acrivos [7], analyzed catalytic surface reactions in hydrodynamic flows. The paper was concerned with its counterpart, namely, an investigation of a certain special class of homogeneous volume reactions in flow systems. Chambre et al. [6] had studied the diffusion of a chemically reactive species in a laminar boundary layer flow. Acrivos [1] analyzed the laminar forced convection mass transfer with homogeneous chemical reaction. A unified boundary layer analysis was applied to the problem of steady state mass transfer of a chemical species, diffusing from a surface and reacting isothermally in a linear fluid stream.

Stagnation flows are found in many applications such as flows over the tips of rockets, aircrafts, submarines and oil ships. In these types of problems, the well known Falkner-Skan transformation is used to reduce boundary-layer equations into ordinary differential equations for similar flows. It can also be used for non-similar flows for convenience in numerical work because it reduces, even if it does not eliminate, dependence on the x-coordinate. The solutions of the Falkner-Skan equations are sometimes referred to as wedge-flow solutions with only two of the wedge flows being common in practice. The dimensionless parameter, plays an important role in such type of problems because it denotes the shape factor of the velocity profiles. It has been shown [?] that when $\beta_1 < 0$ (increasing pressure), the velocity profiles have point of inflexion whereas when $\beta_1 > 0$ (decreasing pressure), there is no point of inflexion. This fact is of great importance in the analysis of the stability of laminar flows with a pressure gradient.

Yih [27] presented an analysis of the forced convection boundary layer flow over a wedge with uniform suction / blowing, whereas Watanabe [25] investigated the behavior of the boundary layer over a wedge with suction or injection in forced flow. Recently, MHD laminar boundary layer flow over a wedge with suction or injection had been discussed by Kafoussias and Nanousis [20] and Kumari [21] discussed the effect of large blowing rates on the steady laminar incompressible electrically conducting fluid over an infinite wedge with a magnetic field applied parallel to the wedge. Anjali Devi and Kandasamy [11] have studied the effects of heat and mass transfer on nonlinear boundary layer flow over a wedge with suction or injection. The effect of induced magnetic field is included in the analysis. Chamkha and Khaled [9] investigated the problem of coupled heat and mass transfer by MHD free convection from an inclined plate in the presence of internal heat generation or absorption. For the problem of coupled heat and mass transfer in MHD free convection, the effects of viscous dissipation and Ohmic heating with chemical reaction are not studied in the above investigation. However, it is more realistic to include these effects to explore the impact of the magnetic field on the thermal transport in the buoyancy layer. With this awareness, the effect of Ohmic heating on the MHD free convection heat transfer has been examined for a Newtonian fluid [18] and for a micropolar fluid [16]. Kuo Bor-Lih [5] studied the effect of heat transfer analysis for the Fakner-Skan wedge flow by the differential transformation method. Cheng and Lin [10] analyzed the non-similarity solution and correlation of transient heat transfer in laminar boundary layer flow over a wedge. Pantokratoras [23] discussed the Falkner - Skan flow with constant wall temperature and variable viscosity. Hossain et al.[19] analyzed the effects of radiation on free convection from a porous plate. These effects on combined heat transfer in MHD mixed convection flow past a thermally stratified porous wedge in the presence of suction or injection have not yet been studied. The similarity solutions for hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media as explained by Chamkha and Khaled [8]. This study is therefore initiated to investigate the problem of MHD mixed convection Hiemenz flow over a thermally stratified porous wedge, taking in to consideration the effects of variable viscosity and thermal stratification effect.

Since no attempt has been made to analyze the effects of nonlinear variable viscosity on MHD mixed convection heat transfer along Hiemenz flow over a thermally stratified porous wedge in the presence of suction or injection. We have investigated it in this article. The similarity transformation has been utilized to convert the governing partial differential equations into ordinary differential equations and then the numerical solution of the problem is drawn using Runge Kutta Gill method,[15]. Numerical calculations up to third level of truncation were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically. Examination of such flow models reveals the influence of magnetic effect, variable viscosity and thermal stratification on velocity and temperature profiles. The analysis of the results obtained shows that the flow field is influenced appreciably by the presence of variable viscosity, thermal stratification and magnetic effect in the presence of suction or injection at the wall of the wedge. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2. Mathematical Analysis

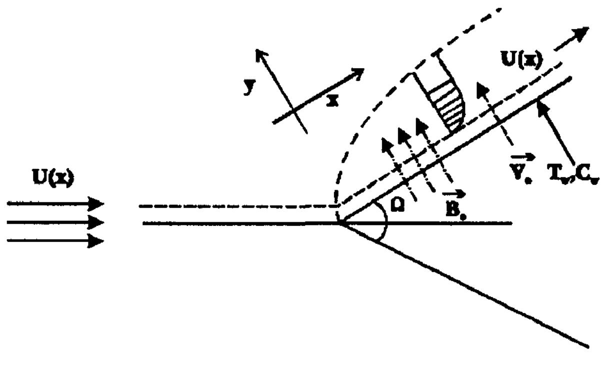

FIGURE 1. Flow analysis along the wall of the wedge

Two dimensional MHD laminar boundary layer flow of an incompressible, viscous, electrically conducting double diffusive and Boussinesq fluid over a wall of the wedge with suction or injection is considered. Let the x-axis is taken parallel to the wedge and y-axis is taken normal to it as cited in Figure 1. An uniform transverse magnetic field of strength B_o is applied parallel to the y-axis. The fluid is assumed to be Newtonian, electrically conducting, nonlinear variable viscosity and its property variations due to temperature are limited to density and viscosity. The density variation and the effects of the buoyancy are taken into account in the momentum equation. Since the magnetic Reynolds number is very small for most used in industrial applications, we assume that the induced magnetic field is negligible. Now the governing boundary layer equations of momentum and energy for the flow under Boussinesq's approximation, [5] and [10] are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + U\frac{dU}{dx} - \frac{\sigma B_0^2}{\rho}(u-U) + g\beta(T-T_\infty)\sin\frac{\Omega}{2} - \frac{\nu}{K}(u-U) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho c_p}u^2 \tag{3}$$

where
B_0^2 -Strength of the magnetic effect.
u,v - velocity components in x and y direction.
U - Flow velocity of the fluid away from the wedge.
g - Acceleration due to gravity.
β - Coefficient of volume expansion.
K- Permeability of the porous medium
T- Temperature of the fluid.
T_w - Temperature of the wall.
T_∞ - Temperature of the fluid far away from the wall.
ρ - Density of the fluid.
σ - Electric conductivity of the fluid.
α - Thermal diffusivity.
The boundary conditions are

$$u = 0, \quad v = v_0, \quad T = T_w \quad \text{at} \quad y = 0 \tag{4}$$

$$u = U(x), \quad T = T_\infty(x) = (1-n)T_0 + nT_w(x) \quad \text{at} \quad y \to \infty \tag{5}$$

where n is a constant which is the thermal stratification parameter and is such that $0 \leq n < 1$. The n defined above as thermal stratification parameter, is equal to $m_1/(1+m_1)$ of Nakayama and Koyama [22] and Anjali Devi and Kandasamy [12], where m_1 is a constant. T_0 is constant reference temperature say, $T_\infty(0)$. The suffixes w and ∞ denote surface and ambient conditions. Following the lines of Yih [26] and Bansal [3], the following change of variables are introduced

$$\psi(x,y) = \sqrt{\frac{2U\alpha_e x}{1+m}}f(x,\eta) \tag{6}$$

$$\eta(x,y) = y\sqrt{\frac{(1+m)U}{2\alpha_e x}} \tag{7}$$

where α_e is the effective thermal diffusivity of the porous medium ($\alpha_e = \frac{k_e}{\rho c_p}$), where k_e is the porous medium effective thermal conductivity. The variations of viscosity are written in the form of [14]:$\frac{\mu}{\mu_0} = e^{-\alpha\theta}$, where μ_0 is the viscosity at temperature T_w and α is the viscosity parameter.
Under this consideration, the potential flow velocity can be written as

$$U(x) = Ax^m, \qquad \beta_1 = \frac{2m}{1+m} \tag{8}$$

where A is a constant and β_1 is the Hartree pressure gradient parameter that corresponds to $\beta_1 = \frac{\Omega}{\pi}$ for a total angle Ω of the wedge. The wall temperature is assumed to have power-law variation forms as shown by the following equations:

$$T_w = T_\infty + c_1 x^n$$

where c_1 is constant and n is the power of index of the wall temperature. The wall temperature is assumed to have the power index n. The continuity equation (1) is satisfied by the stream function $\psi(x, y)$ defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{9}$$

To transform (2) and (3) into a set of ordinary differential equations, the following dimensionless variables are introduced [10]:

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{10}$$

$$Gr_1 = \frac{\nu g \beta (T - T_\infty)}{U^3} \quad \text{(Grashof number)} \tag{11}$$

$$Re_x = \frac{Ux}{\nu} \quad \text{(Reynolds number)} \tag{12}$$

$$Pr = \frac{\mu c_p}{K} \quad \text{(Prandtl number)} \tag{13}$$

$$M^2 = \frac{\sigma B_0^2}{\rho c k^2} \quad \text{(magnetic parameter)} \tag{14}$$

$$S = -v_0 \sqrt{\frac{(1+m)x}{2\nu U}} \quad \text{(suction or injection parameter)} \tag{15}$$

$$Ec = \frac{c^2}{c_p (T_w - T_\infty)(k^2)^{\frac{2m}{1-m}}} \quad \text{(Eckert number)} \tag{16}$$

$$\lambda = \frac{\alpha_e}{KA} \quad \text{(Dimensionless porous medium parameter)} \tag{17}$$

Now the equations (2) and (4) become

$$\frac{Pr}{e^{\alpha\theta}}\frac{\partial^3 f}{\partial\eta^3} = -f\frac{\partial^2 f}{\partial\eta^2} - \frac{2m}{1+m}\Big(1-(\frac{\partial f}{\partial\eta})^2\Big) - \frac{2}{1+m}Gr_1 Re_x\theta\sin\frac{\Omega}{2} +$$
$$\frac{2x}{1+m}\Big(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial x\partial\eta} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial\eta^2}\Big) + \frac{2x}{m+1}\frac{\sigma B_0^2}{\rho U}(\frac{\partial f}{\partial\eta}-1) + \frac{2}{m+1}\lambda(\frac{\partial f}{\partial\eta}-1) \quad (18)$$

$$\frac{\partial^2\theta}{\partial\eta^2} = -Pr\frac{\partial\theta}{\partial\eta} + \frac{2Pr}{1+m}\Big(\theta+\frac{n}{1-n}\Big)\frac{\partial f}{\partial\eta} + Pr\frac{2x}{1+m}\Big(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial\theta}{\partial\eta}\Big)$$
$$-\frac{\sigma B_0^2}{\rho U}\frac{U^2}{c_p(T_w-T_\infty)}(\frac{\partial f}{\partial\eta})^2 \quad (19)$$

The boundary conditions can be written as

$$\eta=0: \quad \frac{\partial f}{\partial\eta}=0, \quad \frac{f}{2}\Big(\frac{x}{U}\frac{dU}{dx}+1\Big) + x\frac{\partial f}{\partial x} = -v_0\sqrt{\frac{(1+m)x}{2\nu U}}, \quad \theta=1$$
$$\eta\to\infty: \quad \frac{\partial f}{\partial\eta}=1, \quad \theta=0 \quad (20)$$

where v_0 is the velocity of suction if $v_0 < 0$ and injection if $v_0 > 0$.
The equations (18) and (19) and boundary conditions (20) can be written as

$$\frac{Pr}{e^{\alpha\theta}}\frac{\partial^3 f}{\partial\eta^3} + \Big(f+\frac{1-m}{1+m}\xi\frac{\partial f}{\partial\xi}\Big)\frac{\partial^2 f}{\partial\eta^2} - \frac{1-m}{1+m}\xi\frac{\partial^2 f}{\partial\xi\partial\eta}\frac{\partial f}{\partial\eta} - \frac{2}{1+m}M^2\xi^2(\frac{\partial f}{\partial\eta}-1)$$
$$+\frac{2m}{1+m}\Big(1-(\frac{\partial f}{\partial\eta})^2\Big) + \frac{2}{1+m}Gr_1 Re_x\theta\sin\frac{\Omega}{2} - \frac{2}{1+m}\lambda(\frac{\partial f}{\partial\eta}-1) = 0 \quad (21)$$

$$\frac{\partial^2\theta}{\partial\eta^2} + Pr\Big(f+\frac{1-m}{1+m}\xi\frac{\partial f}{\partial\xi}\Big)\frac{\partial\theta}{\partial\eta} - \frac{2Pr}{1+m}\Big(\theta+\frac{n}{1-n}\Big)\frac{\partial f}{\partial\eta} - \frac{1-m}{1+m}\xi\frac{\partial\theta}{\partial\xi}\frac{\partial f}{\partial\eta}$$
$$+Pr\frac{2}{1+m}M^2Ec\xi^{\frac{2(1+m)}{1-m}}(\frac{\partial f}{\partial\eta})^2 = 0 \quad (22)$$

$$\eta=0: \quad \frac{\partial f}{\partial\eta}=0, \quad \frac{(1+m)f}{2} + \frac{1-m}{2}\xi\frac{\partial f}{\partial\xi} = S, \quad \theta=1$$
$$\eta\to\infty: \quad \frac{\partial f}{\partial\eta}=1, \quad \theta=0 \quad (23)$$

where S is the suction parameter if $S > 0$ and injection if $S < 0$ and $\xi = kx^{\frac{1-m}{2}}$ [6],is the dimensionless distance along the wedge ($\xi > 0$). In this system of equations $f(\xi,\eta)$ is the dimensionless stream function; $\theta(\xi,\eta)$ be the dimensionless temperature; Pr, the Prandtl number, Re_x ,Reynolds number etc. which are defined in (10) to (17). The parameter ξ indicates the dimensionless distance along the wedge ($\xi > 0$). It is obvious that to retain the ξ-derivative terms, it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. In addition, owing to the coupling between adjacent stream wise location through the ξ-derivatives, a locally autonomous solution, at any given stream wise location can not be obtained. In such a case, an implicit marching numerical solution scheme is usually applied proceeding the solution in the ξ-direction, i.e., calculating unknown profiles at $\xi_{\iota+1}$ when the same profiles at ξ_ι are known. The process starts at $\xi = 0$ and the solution proceeds from ξ_ι to $\xi_{\iota+1}$ but such a procedure is time consuming.

However, when the terms involving $\frac{\partial f}{\partial \xi}$ and $\frac{\partial \theta}{\partial \xi}$ their η derivatives are deleted, the resulting system of equations resembles, in effect, a system of ordinary differential equations for the functions f and θ with ξ as a parameter and the computational task is simplified. Furthermore a locally autonomous solution for any given ξ can be obtained because the stream wise coupling is severed. So, following the lines of [20], R.K. Gill, [15] and Shooting numerical solution scheme are utilized for obtaining the solution of the problem. Now, due to the above mentioned factors, the equations (21) and (22) are changed to

$$f''' + \frac{e^{\alpha\theta}}{Pr} f f'' + \frac{2m}{1+m}\frac{e^{\alpha\theta}}{Pr}(1-(f')^2) + \frac{2}{1+m}\frac{e^{\alpha\theta}}{Pr} Gr_1 Re_x \theta \sin\frac{\Omega}{2}$$

$$- \frac{e^{\alpha\theta}}{Pr}\frac{2}{1+m}(M^2\xi^2 + \lambda)(f'-1) = 0 \qquad (24)$$

$$\theta'' + Pr f\theta' - \frac{2Pr}{1+m}\left(\theta + \frac{n}{1-n}\right) f' + Pr\frac{2}{1+m} M^2 Ec \xi^{\frac{2(1+m)}{1-m}} (f')^2 = 0 \qquad (25)$$

with boundary conditions

$$\eta = 0: \quad f(0) = \frac{2}{1+m} S, \quad f'(0) = 0, \quad \theta(0) = 1,$$

$$\eta \to \infty: \quad f'(\infty) = 1, \quad \theta(\infty) = 0 \qquad (26)$$

3. Numerical solutions

The set of non-linear ordinary differential equations (24) and (25) with boundary conditions (26) have been solved by using the R .K Gill method, [15] along with Shooting technique with α, Ω, M^2 and n as prescribed parameters. The computational were done by a program which uses a symbolic and computational computer language Matlab. A step size of $\Delta\eta = 0.001$ was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The value of η_∞ was found to each iteration loop by assignment statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞, to each group of parameters α, Ω, M^2 and n determined when the values of unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-7}. Effects of thermal stratification of heat transfer are studied for different values of suction / injection at the wall of the wedge and the strength of applied magnetic field. In the following section, the results are discussed in detail.

4. Results and Discussion

The computations have been carried out for various values of variables of variable viscosity ($0 \leq \alpha \leq 1$), magnetic parameter (M^2), porous medium (λ) and thermal stratification parameter,($0 \leq n < 1$). The edge of the boundary layer $\eta_\infty = 8$ depending on the values of parameters.

In order to validate our method, we have compared steady state results of velocity and temperature for various values of Pr with those of [20], [21], [25] and [27] and found them in excellent agreement.

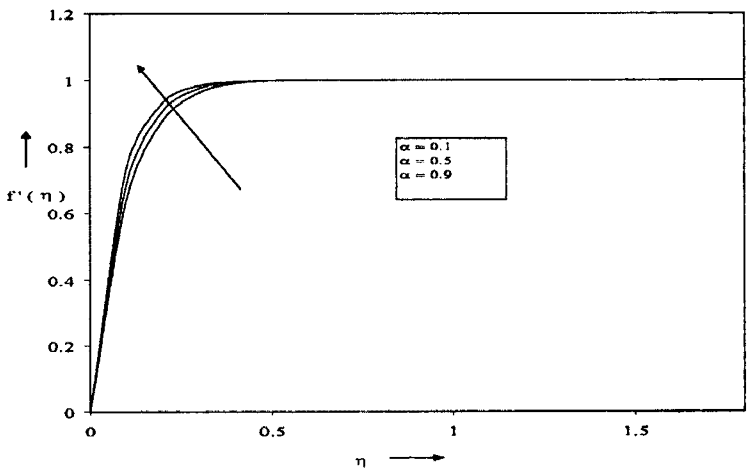

Figure 2. Effects of viscosity over the velocity profiles with $Gr_1 = 1.0$, $Ec = 0.001$, $m = 0.0909$, $M^2 = 1.0$, $Re_x = 3.0$, $S = 3.0$, $\Omega = 30^o$, $\lambda = 0.1$, $n = 0.5$

Figure 2 represents the dimensionless velocity profiles for different values of viscosity. In the presence of uniform strength of the magnetic field, it is clear that the velocity increases and the temperature of the fluid decreases with increase of viscosity and these are shown in the Figures 2 and 3 respectively. The results presented demonstrate quite clearly that a, which is an indicator of the variation of viscosity with temperature, has a substantial effect on the drag and heat transfer characteristics. In particular, the temperature of the fluid gradually changes from higher value to the lower value only when the strength of the viscosity is higher than the porous medium parameter. All these physical behavior are due to the combined effects of the strength of the magnetic field and suction at the wall of the wedge.

The dimensionless velocity profiles for different values of the strength of magnetic field are plotted in Figure 4. In the case of uniform thermal stratification, it is clear that the velocity of the fluid increases with increase of the strength of magnetic effects, while the temperature of the fluid decreases with increase of magnetic effects and these are shown in Figures 4 and 5. As magnetic strength

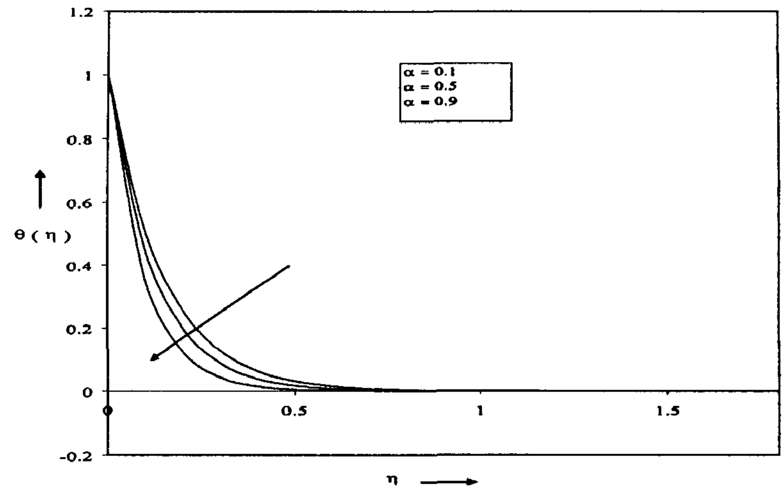

Figure 3. Effects of viscosity over the temperature profiles with $Gr_1 = 1.0$, $Ec = 0.001$, $m = 0.0909$, $M^2 = 1.0$, $Re_x = 3.0$,$S = 3.0$, $\Omega = 30^o$, $\lambda = 0.1$, $n = 0.5$.

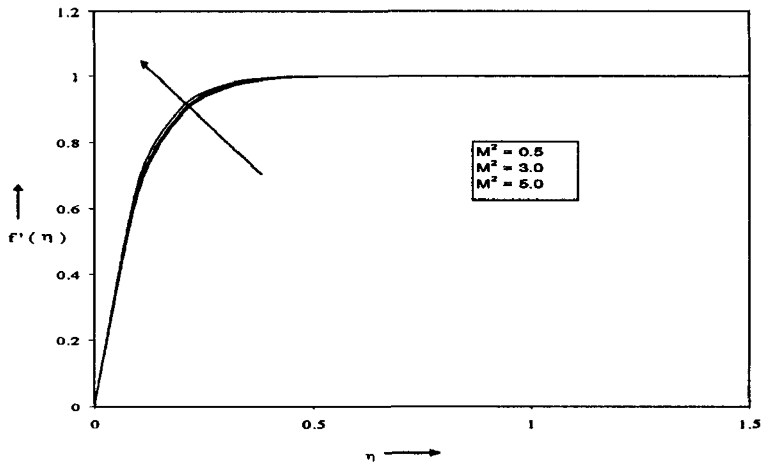

Figure 4. Influence of magnetic field over the velocity profiles with $Gr_1 = 1.0$, $Ec = 0.001$, $m = 0.0909$, $\alpha = 0.1$, $Re_x = 3.0$, $S = 3.0$, $\Omega = 30^o$, $\lambda = 1.0$, $n = 0.5$.

increases, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow. This results qualitatively agrees with the expectations, since the magnetic field exerts retarding force on the mixed convection flow.

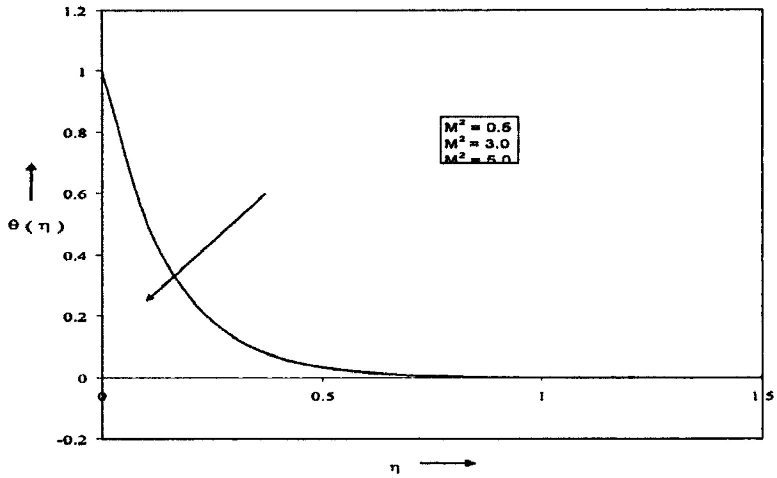

Figure 5. Effects of magnetic field over the temperature profiles with $Gr_1 = 1.0$, $Ec = 0.001$, $m = 0.0909$, $\alpha = 0.1$, $Re_x = 3.0$, $S = 3.0$, $\Omega = 30^o$, $\lambda = 1.0$, $n = 0.5$.

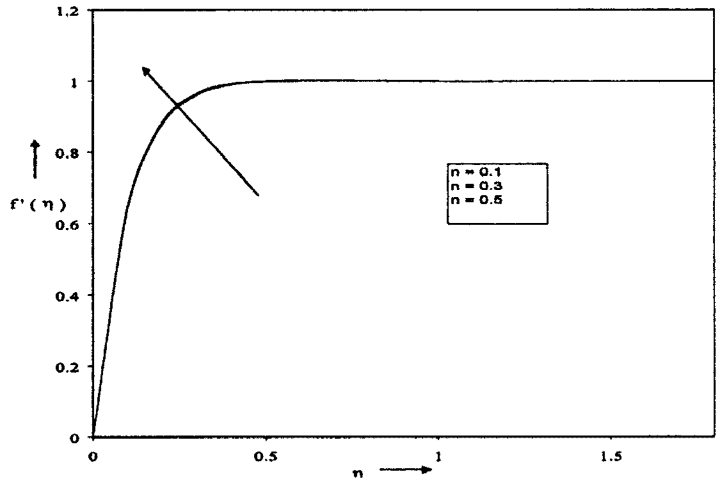

Figure 6. Effects of thermal stratification over the velocity profiles with $Gr_1 = 1.0$, $Ec = 0.001$, $m = 0.0909$, $\alpha = 0.1$, $Re_x = 3.0$, $S = 3.0$, $\Omega = 30^o$, $\lambda = 1.0$, $M^2 = 0.5$.

Figure 6 represents the dimensionless velocity profiles for different values of thermal stratification parameter. In the presence of uniform strength of the magnetic field, it is clear that the velocity and temperature of the fluid increase with increase of thermal stratification parameter and these are shown in the

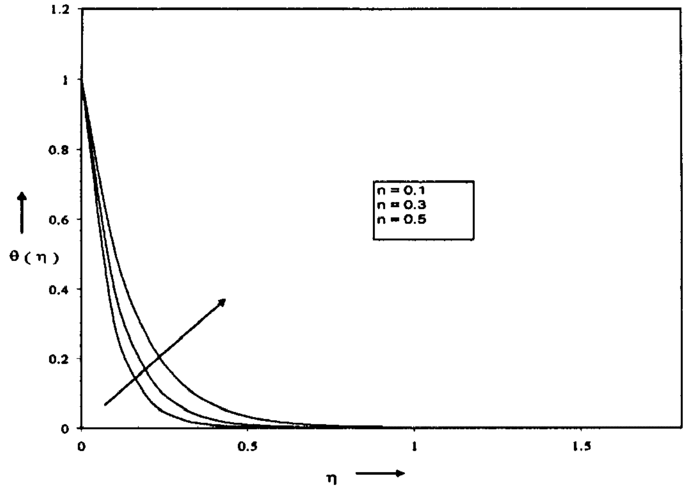

Figure 7. Influence of thermal stratification over the temperature profiles with $Gr_1 = 1.0$, $Ec = 0.001$, $m = 0.0909$, $\alpha = 0.1$, $Re_x = 3.0$, $S = 3.0$, $\Omega = 30^o$, $\lambda = 1.0$, $M^2 = 0.5$.

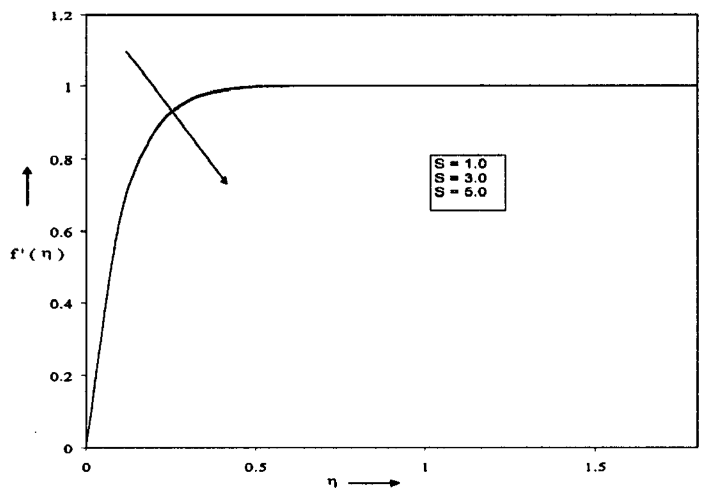

Figure 8. Influence of suction over the velocity profiles with $Gr_1 = 1.0$, $Ec = 0.001$, $m = 0.0909$, $\alpha = 0.1$, $Re_x = 3.0$, $n = 0.5$, $\Omega = 30^o$, $\lambda = 1.0$, $M^2 = 0.5$.

Figures 6 and 7 respectively. All these physical behavior are due to the combined effects of the strength of the magnetic field and the thermal conductivity of the porous medium.

Figure 8 demonstrates the dimensionless velocity profiles for different values of suction parameter. In the presence of uniform thermal stratification, it is seen

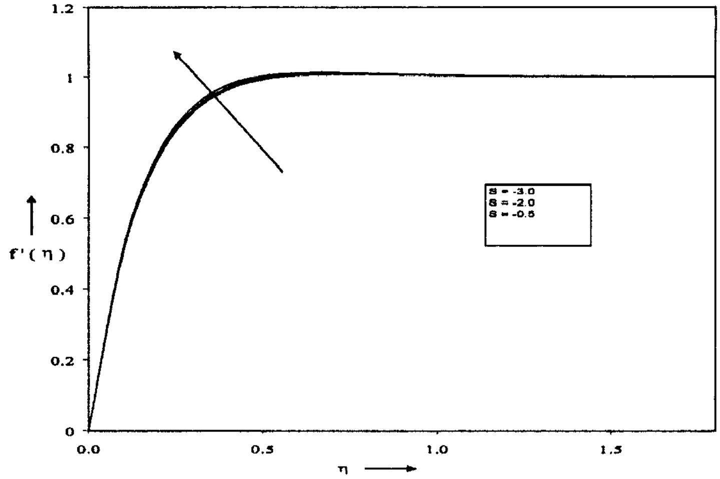

Figure 9. Influence of injection over the velocity profiles with $Gr_1 = 1.0$, $Ec = 0.001$, $m = 0.0909$, $\alpha = 0.1$, $Re_x = 3.0$, $n = 0.5$, $\Omega = 30^o$, $\lambda = 1.0$, $M^2 = 0.5$.

that the velocity of the fluid decreases with increase of suction and increases with increases of injection and these are shown in the Figures 8 and 9 respectively. As might be expected, suction thins the momentum boundary layer whereas injection thickens it.

4. Conclusions

This article studied the effect of variable viscosity on MHD mixed convection heat transfer along Hiemenz flow over a thermally stratified porous wedge plate in the presence of suction or injection. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. Comparisons with previously published works are performed and excellent agreement between the results is obtained. The results are presented graphically and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters.

We conclude the following from the results and discussions:

- In the presence of uniform strength of the magnetic field, it is clear that the velocity increases and the temperature of the fluid decreases with increase of viscosity. The results presented demonstrate quite clearly that a, which is an indicator of the variation of viscosity with temperature, has a substantial effect on the drag and heat transfer characteristics. In particular, the temperature of the fluid gradually changes from higher value to the lower value only when the strength of the viscosity is higher than the porous medium parameter. All these physical behavior are due to the combined effects of the strength of the magnetic field and suction at the wall of the wedge.

- Due to the uniform thermal stratification effects, it is seen that the velocity of the fluid increases with increase of the strength of magnetic effects, while the temperature of the fluid decreases with increase of magnetic effects. As magnetic strength increases, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow. This results qualitatively agrees with the expectations, since the magnetic field exerts retarding force on the mixed convection flow.
- In the presence of uniform strength of the magnetic field, it is observed that the velocity and temperature of the fluid increase with increase of thermal stratification parameter. All these physical behavior are due to the combined effects of the strength of the magnetic field and the thermal conductivity of porous medium.
- In the presence of uniform viscosity, it is to note that the velocity of the fluid decreases with increase of suction and increases with increases of injection. As might be expected, suction thins the momentum boundary layer whereas injection thickens it.
- Comparison of velocity profiles shows that the velocity increases near the plate and thereafter remains uniform. It is interesting to note that due to increase in thermal stratification, the temperature increases at a faster rate in comparison to variation in the parameters in the case of cooling of plate ($Gr > 0$).

The analysis of the present investigation of flow through a wedge medium are playing a predominant role in the applications of Science and Engineering. The flow of this kind has enormous importance in technical problems such as flow through packed beds, sedimentation, environmental problem, centrifugal separation of particles, blood rheology and in many transport processes in nature and engineering devices, nuclear reaction, electronic equipments etc., in which the effect of buoyancy forces on the forced flow, or the effect of forced flow on the buoyant flow is significant. Particularly the findings may be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field, in the migration of underground water and in the filtration and water purification processes. It is hoped that the present work will serve as a vehicle for understanding more complex problems involving the various physical effects investigated in the present problem.

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