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EXISTENCE AND GLOBAL ATTRACTIVITY OF POSITIVE PERIODIC SOLUTIONS FOR A GENERALIZED PREDATOR-PREY MODEL WITH DIFFUSSION

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ABSTRACT. By employing the continuation theorem of coincidence degree theory, we derive a sufficient condition for the existence and attractricity of a positive periodic solution for a generalized predator-prey model with diffussion feedback controls.

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1. Introduction

In recent years, the existence of positive periodic solutions of the prey-predator model has been widely studied[1,2,3]. The qualitive analysis of predator-prey systems is an interesting mathematical problem and has attracted a great of attention from many mathematicians and biologists[4,5]. Recently, Xu and Chen [6] investigated persistence and stability for a two species ratio-dependent predator-prey model in a two-patch environment . Since realistic model require the inclusion of effect of Changing environment, recently, Shihua Chen and Feng Wang[7], considered the following model

$$\begin{cases} x_1'(t) = x_1(t)\left(a_1(t) - a_{11}(t)x_1(t) - \dfrac{a_{13}(t)x_3(t)}{m(t)x_3(t) + x_1(t)}\right) \\ \qquad +D_1(t)(x_2(t) - x_1(t)) \\ x_2'(t) = x_2(t)(a_2(t) - a_{22}(t)x_2(t)) + D_2(t)(x_1(t) - x_2(t)) \\ x_3'(t) = x_3(t)\left(-a_3(t) - a_4(t)x_3(t) + \dfrac{a_{31}(t)x_1(t-\tau)}{m(t)x_3(t-\tau) + x_1(t-\tau)}\right) \end{cases} \tag{1.1}$$

where $D_i(t)(i = 1,2), a_i(t)(i = 1,2,3), a_{11}(t), a_{13}(t),\ a_{22}(t), a_{31}(t)$ and $m(t)$ are strickly positive continuous $w-$periodic functions.

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In the paper, we will study the following model

$$\begin{cases} x_1'(t) = x_1(t)\left(g_1(t,x_1(t)) - \dfrac{a_{13}(t)x_3(t)}{m(t)x_3(t)+x_1(t)}\right) + D_1(t)(x_2(t)-x_1(t)) \\ x_2'(t) = x_2(t)(g_2(t,x_2(t)+D_2(t)(x_1(t)-x_2(t)) \\ x_3'(t) = x_3(t)\left(-a_3(t) - a_4(t)x_3(t) + \dfrac{a_{31}(t)x_1(t-\tau)}{m(t)x_3(t-\tau)+x_1(t-\tau)}\right) \end{cases} \tag{1.2}$$

where $D_i(t)(i=1,2), a_{13}(t), a_{31}(t), m(t)$ are the same as model (1.1). $g_i(t,x),(i=1,2)$ is differentiable on x and periodic on t.

In this paper ,we establish a sufficient condition for the existence and attractivity of at least a positive $w-$periodic solution of model (1.2),So far the result is new.

2. Existence of a positive periodic solution

To obtain the existence of positive periodic solutions of system (1.2), we summarize some concepts and results from[5] that will be basic for this section.

Let X, Z be Banach spaces, let $L : DomL \subset X \to X$ be a linear mapping, and let $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $dimKerL = codimImL < +\infty$ and ImL is closed in Z. If L is a Fredholm mapping of index zero, there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $ImP = KerL$ and $ImL = KerQ = Im(I-Q)$. It follows that $L|_{DomL\cap kerP}; (I-P)X \to ImL$ is invertibe, we denote the inverse of that map by Kp. If Ω is an open-bounded subset of X, the mapping N will be called $L-$compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $Kp(I-Q)N : \overline{\Omega} \to X$ compact. Since ImQ is isomorphic to $KerL$, there exist an isomorphism $J : ImQ \to KerL$.

In the proof of our existence theorem, we will use the continuation theorem of Gaines and Mawhin[8].

Lemma 2.1.[8] *Let L be a Fredholm mapping of index zero and let N be $L-$compact on $\overline{\Omega}$. Suppose:*

(i) *for each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ is such that $x\overline{\in}\partial\Omega$;*

(ii) *$QNx \neq 0$ for each $x \in \partial\Omega \cap KerL$;*

(iii) *$deg\{JQN, \Omega \cap kerL, 0\} \neq 0$.*

Then $Lx = Nx$ has at least one solution in $DomL \cap \overline{\Omega}$.

For convenience, we introduce the notations:

$$\overline{f} = \frac{1}{w}\int_0^w f(t)dt,\quad f^l = \min_{t\in[0,w]}|f(t)|,\quad f^M = \max_{t\in[0,w]}|f(t)|,$$

where f is a continuous $w-$periodic function.

Our main result on the global existence of a positive periodic solution of system (1.2) is stated in the following theorem.

Theorem 2.1. *Assume that:*

(H_1) *there exists a constant A such that for $\forall x \in R, t \in R$, when $x \geq A$,*

$$g_1(t, e^x) \leq 0;$$

(H_2) *there exists a constant B such that for $\forall x \in R$, when $x \geq B$,*

$$g_2(t, e^x) \leq 0;$$

(H_3) *there exists a constant $C(C < A)$ such that for $\forall x \in R, t \in R$, when $x \leq C$,*

$$g_1(t, e^x) \geq \left(\frac{a_{13}}{m}\right)^M.$$

(H_4) *there exists a constant $D(D < B)$ such that for $\forall x \in R, t \in R$ when $x \leq D$,*

$$g_2(t, e^x) \geq 0;$$

(H_5) $(a_{31}^M - a_3^M)e^{\rho_1} > a_3^M m^M e^{d_1}$. *where $\rho_1 = \min\{C, D\}$,$d_1 = \max\{A, B\}$. Then system (1.1) has at least one positive w-periodic solution.*

Proof. Consider the system

$$\begin{cases} u_1'(t) = g_1(t, e^{u_1(t)}) - \dfrac{a_{13}(t)e^{u_3(t)}}{m(t)e^{u_3(t)} + e^{u_1(t)}} + D_1(t)(e^{u_2(t)-u_1(t)} - 1) \\ u_2'(t) = g_2(t, e^{u_2(t)}) + D_2(t)(e^{u_1(t)-u_2(t)} - 1) \\ u_3'(t) = -a_3(t) - a_4(t)e^{u_3(t)} + \dfrac{a_3(t)e^{u_1(t-\tau)}}{m(t)e^{u_3(t)} + e^{u_1(t-\tau)}} \end{cases} \tag{2.1}$$

Let $x_i(t) = e^{u_i(t)}, i = 1, 2, 3$. Then system (1.2) changes into system (2.1). Hence it is easy to see that system (2.1) has an w-periodic solution $(u_1^*(t), u_2^*(t), u_3^*(t))^T$, and then $(e^{u_1^*(t)}, e^{u_2^*(t)}, e^{u_3^*(t)})^T$ is a positive w-periodic solution of system (1.2). Therefore, for (1.2) to have at least one positive w-periodic solution, it is sufficient that (2.1) have at least one w-periodic solution. In order to apply Lemma 2.1 to system (2.1), we take

$$X = Z = \{u(t) = (u_1(t), u_2(t), u_3(t))^T \in C(R, R^3), u(t + w) = u(t)\}$$

and

$$\|u\| = \|(u_1(t), u_2(t), u_3(t))^T\| = \sum_{i=1}^{3} \max_{t \in [0,w]} |u_i(t)|$$

for any $u \in X$(or Z). Then X and Z are Banach spaces with the norm $\|\cdot\|$. Let

$$Nu = \begin{bmatrix} g_1(t, e^{u_1(t)}) - \dfrac{a_{13}(t)e^{u_3(t)}}{m(t)e^{u_3(t)} + e^{u_1(t)}} + D_1(t)(e^{u_2(t)-u_1(t)} - 1) \\ g_2(t, e^{u_2(t)}) + D_2(t)(e^{u_1(t)-u_2(t)} - 1) \\ -a_3(t) - a_4(t)e^{u_3(t)} + \dfrac{a_3(t)e^{u_1(t-\tau)}}{m(t)e^{u_3(t) + e^{u_1(t-\tau)}}} \end{bmatrix}, \quad u \in X$$

$$Lu = u' = \frac{du(t)}{dt}, \quad Pu = \frac{1}{w}\int_0^w u(t)dt, \quad u \in X; \quad Qz = \frac{1}{w}\int_0^w z(t)dt, \quad z \in Z.$$

Then it follows that

$$KerL = R^3, \quad ImL = \left\{ z \in Z : \int_0^w z(t)dt = 0 \right\} \text{ is closed in } Z,$$
$$dimKerL = 3 = codimImL,$$

and P, Q are continuous projectors such that $ImP = KerL$, $KerQ = ImL = Im(I - Q)$. Therefore, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L), $Kp : ImL \to KerP \cap DomL$ is given by

$$Kp(z) = \int_0^t z(s)ds - \frac{1}{w}\int_0^w \int_0^t z(s)dsdt.$$

Thus

$$QNu = \begin{bmatrix} 1/w\int_0^w F_1(s)ds \\ 1/w\int_0^w F_2(s)ds \\ 1/w\int_0^w F_3(s)ds \end{bmatrix}$$

and

$$Kp(I-Q)Nu = \begin{bmatrix} \int_0^t F_1(s)ds - 1/w\int_0^w \int_0^t F_1(s)dsdt + (1/2 - t/w)\int_0^w F_1(s)ds \\ \int_0^t F_2(s)ds - 1/w\int_0^w \int_0^t F_2(s)dsdt + (1/2 - t/w)\int_0^w F_2(s)ds \\ \int_0^t F_3(s)ds - 1/w\int_0^w \int_0^t F_3(s)dsdt + (1/2 - t/w)\int_0^w F_3(s)ds \end{bmatrix},$$

where

$$F_1(s) = g_1(s, e^{u_1(s)}) - \frac{a_{13}(s)e^{u_3(s)}}{m(s)e^{u_3(s)} + e^{u_1(s)}} + D_1(s)(e^{u_2(s)-u_1(s)} - 1)$$
$$F_2(s) = g_2(s, e^{u_2(s)}) + D_2(s)(e^{u_1(s)-u_2(s)} - 1)$$

and

$$F_3(s) = -a_3(s) - a_4(s)e^{u_3(s)} + \frac{a_3(s)e^{u_1(s-\tau)}}{m(s)e^{u_3(s-\tau)} + e^{u_1(s-\tau)}}.$$

Obviously, QN and $Kp(I-Q)N$ are continuous. It is not difficult to show that $Kp(I-Q)N(\overline{\Omega})$ is compact for any open bounded $\Omega \subset X$ by using the Arzela-Ascoli theorem. Moreover, $QN(\overline{\Omega})$ is clearly bounded. Thus, N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

Now we reach the point where we search for an appropriate open bounded subset Ω for the application of the continuation theorem (Lemma 2.1). Corresponding to the operator equation $Lx = \lambda Nx, \lambda \in (0,1)$, we have

$$\begin{cases} u_1'(t) = \lambda \left[g_1(t, e^{u_1(t)}) - \dfrac{a_{13}(t)e^{u_3(t)}}{m(t)e^{u_3(t)} + e^{u_1(t)}} + D_1(t)(e^{u_2(t)-u_1(t)} - 1) \right] \\ u_2'(t) = \lambda \left[g_2(t, e^{u_2(t)}) + D_2(t)(e^{u_1(t)-u_2(t)} - 1) \right] \\ u_3'(t) = \lambda \left[-a_3(t) - a_4(t)e^{u_3(t)} + \dfrac{a_{31}(t)e^{u_1(t-\tau)}}{m(t)e^{u_3(t)} + e^{u_1(t-\tau)}} \right] \end{cases} \tag{2.2}$$

Assume that $u = u(t) \in X$ is a solution of system (2.2) for a certain $\lambda \in (0,1)$.

Because of $(u_1(t), u_2(t), u_3(t))^T \in X$, there exist $\xi_i, \eta_i \in [0, w]$ such that

$$u_i(\xi_i) = \max_{t\in[0,w]} u_i(t), \quad u_i(\eta_i) = \min_{t\in[0,w]} u_i(t), \quad i = 1, 2, 3.$$

It is clear that

$$u_i'(\xi_i) = 0, \quad u_i'(\eta_i) = 0, \quad i = 1, 2, 3.$$

From this and system (2.2), we obtain

$$g_1(\xi_1, e^{u_1(\xi_1)}) - \frac{a_{13}(\xi_1)e^{u_3(\xi_1)}}{m(\xi_1)e^{u_3(\xi_1)} + e^{u_1(\xi_1)}} + D_1(\xi_1)(e^{u_2(\xi_1)-u_1(\xi_1)} - 1) = 0, \quad (2.3)$$

$$g_2(\xi_2, e^{u_2(\xi_2)}) + D_2(\xi_2)(e^{u_1(\xi_2)-u_2(\xi_2)} - 1) = 0, \quad (2.4)$$

$$-a_3(\xi_3) - a_4(\xi_3)e^{u_3(\xi_3)} + \frac{a_{31}(\xi_3)e^{u_1(\xi_3-\tau)}}{m(\xi_3)e^{u_3(\xi_3-\tau)+e^{u_1(\xi_3-\tau)}}} = 0, \quad (2.5)$$

$$g_1(\eta_1, e^{u_1(\eta_1)}) - \frac{a_{13}(\eta_1)e^{u_3(\eta_1)}}{m(\eta_1)e^{u_3(\eta_1)} + e^{u_1(\eta_1)}} + D_1(\eta_1)(e^{u_2(\eta_1)-u_1(\eta_1)} - 1) = 0, \quad (2.6)$$

$$g_2(\eta_2, e^{u_2(\eta_2)}) + D_2(\eta_2)(e^{u_1(\eta_2)-u_2(\eta_2)} - 1) = 0. \quad (2.7)$$

$$-a_3(\eta_3) - a_4(\eta_3)e^{u_3(\eta_3)} + \frac{a_{31}(\eta_3)e^{u_1(\eta_3-\tau)}}{m(\eta_3)e^{u_3(\eta_3-\tau)} + e^{u_1(\eta_3-\tau)}} = 0. \quad (2.8)$$

There two cases to be considered for (2.3) and (2.4).

Case 1. Assume that $u_1(\xi_1) \geq u_2(\xi_2)$. Then $u_1(\xi_1) \geq u_2(\xi_1)$.

From this and (2.3), we have

$$g_1(\xi_1, e^{u_1(\xi_1)}) = \frac{a_{13}(\xi_1)e^{u_3(\xi_3)}}{m(\xi_1)e^{u_3(\xi_1)} + e^{u_1(\xi_1)}} - D_1(\xi_1)(e^{u_2(\xi_1)-u_1(\xi_1)} - 1) > 0$$

which, together with condition (H_1) in Theorem (2.1), gives

$$u_1(\xi_1) < A. \quad (2.9)$$

Thus

$$u_2(\xi_2) \leq u_1(\xi_1) < A. \quad (2.10)$$

Case 2. Assume that $u_1(\xi_1) \leq u_2(\xi_2)$. Then $u_1(\xi_2) \leq u_2(\xi_2)$.

From this and (2.4), we have

$$g_2(\xi_2, e^{u_2(\xi_2)}) = -D_2(\xi_2)(e^{u_1(\xi_2)-u_2(\xi_2)} - 1) > 0,$$

which, together with condition (H_2) in Theorem 2.1, gives

$$u_2(\xi_2) < B. \quad (2.11)$$

Thus

$$u_1(\xi_1) \leq u_2(\xi_2) < B. \quad (2.12)$$

From case 1 and case 2, we obtain

$$u_1(\xi_1) < \max\{A, B\} \stackrel{def}{=} d_1 \quad (2.13)$$

$$u_2(\xi_2) < \max\{A, B\} = d_1 \quad (2.14)$$

From (2.5), we get

$$a_4^l e^{u_3(\xi_3)} \le a_4(\xi_3)e^{u_3(\xi_3)} \le \frac{a_{31}(\xi_3)e^{u_1(\xi_3-\tau)}}{m(\xi_3)e^{u_3(\xi_3-\tau)} + e^{u_1(\xi_3-\tau)}} < a_{31}^M.$$

Thus

$$u_3(\xi_3) \le ln(\frac{a_{31}^M}{a_4^l}) \stackrel{def}{=} d_3. \tag{2.15}$$

There are two cases to consider for (2.6) and (2.7).

Case 1. Assume that $u_1(\eta_1) \le u_2(\eta_2)$. Then $u_1(\eta_1) \le u_2(\eta_1)$. From this and (2.5), we have

$$g_1(\eta_1, e^{u_1(\eta_1)}) = \frac{a_{13}(\eta_1)e^{u_3(\eta_1)}}{m(\eta_1)e^{u_3(\eta_1)} + e^{u_1(\eta_1)}} - D_1(\eta_1)(e^{u_2(\eta_1)-u_1(\eta_1)} - 1)$$
$$< \frac{a_{13}(\eta_1)e^{u_3(\eta_1)}}{m(\eta_1)e^{u_3(\eta_1)} + e^{u_1(\eta_1)}} < \left(\frac{a_{13}}{m}\right)^M$$

which, together with condition (H_3) in Theorem 2.1, gives

$$u_1(\eta_1) > C, \tag{2.16}$$

Hence

$$u_2(\eta_2) > u_1(\eta_1) > C. \tag{2.17}$$

Case 2. Assume that $u_1(\eta_1) \ge u_2(\eta_2)$. Then $u_1(\eta_2) \ge u_2(\eta_2)$. From this and (2.7), we have

$$g_2(\eta_2, e^{u_2(\eta_2)}) = -D_2(\eta_2)(e^{u_1(\eta_2)-u_2(\eta_2)} - 1) < 0$$

which together with condition (H_4) in Theorem 2.1, gives

$$u_2(\eta_2) > D. \tag{2.18}$$

Hence

$$u_1(\eta_1) > u_2(\eta_2) > D. \tag{2.19}$$

From case 1 and case 2, we have

$$u_1(\eta_1) > \min\{C, D\} \stackrel{def}{=} \rho_1 \tag{2.20}$$

$$u_2(\eta_2) > \min\{C, D\} = \rho_1 \tag{2.21}$$

From (2.8)and theorem1(H_5),noting that $\frac{a_{31}(t)e^x}{m(t)e^z + e^x}$ is increasing with x, we obtain

$$a_4^M e^{u_3(\eta_3)} > a_4(\eta_3)e^{u_3(\eta_3)} > \frac{a_{31}^l e^{\rho_1}}{m^M e^{d_1} + e(\rho_1)} - a_3^M,$$

and

$$u_3(\eta_3) > \ln \frac{(a_{31}^l - a_3^M)e^{\rho_1} - a_3^M m^M e^{d_1}}{a_4^M(m^M e^{d_1} + e^{\rho_1})} \stackrel{def}{=} \rho_3. \tag{2.22}$$

From (2.11)-(2.22), we obtain that for $\forall t \in R$,

$$|u_1(t)| \le \max\{|d_1|, |\rho_1|\} \stackrel{def}{=} R_1,$$

$$|u_2(t)| \leq \max\{|d_1|, |\rho_1|\} \overset{def}{=} R_2,$$

and

$$|u_3(t)| \leq \max\{|a_3|, |\rho_3|\} \overset{def}{=} R_3.$$

Clearly, $R_i (i = 1, 2, 3)$ are independent of λ. Denote $M = \sum_{i=1}^{3} R_i + R_0$, here R_0 is taken sufficiently large such that each solution $(\alpha^*, \beta^*, \gamma^*)^T$ of the following system:

$$\begin{aligned} & g_1(t_1, e^\alpha) - \frac{\overline{a}_{13}e^\gamma}{m(t_3)e^\gamma + e^\alpha} + \overline{D}_1(e^{\beta-\alpha} - 1) = 0, \\ & g_2(t_2, e^\beta) + \overline{D}_2(e^{\alpha-\beta} - 1) = 0, \\ & -\overline{a}_3 - a_4 e^r + \frac{\overline{a}_{31}e^\alpha}{m(t_4)e^\gamma + e^\alpha} = 0 \end{aligned} \tag{2.23}$$

satisfies $\|(\alpha^*, \beta^*, \gamma^*)^T\| = |\alpha^*| + |\beta^*| + |\gamma^*| < M$, provided that system (2.23) has a solution or a number of solutions, and that

$$\max\{|d_1|, |\rho_1|\} + \max\{|d_1|, |\rho_1|\} + \max\{|d_3|, |\rho_3|\} < M,$$

where $t_i \in (0, w)$ will appear in QNu below.

Now we take $\Omega = \{u = (u_1(t), u_2(t), u_3(t))^T \in X : \|u\| < M\}$. This satisfies condition (i) of lemma 2.1. When $u \in \partial\Omega \cap KerL = \partial\Omega \cap R^3$, u is a constant vector in R^3 with $\sum_{i=1}^{3} |u_i| = M$. If system (2.23) has one or more solutions, then

$$QNu = \begin{bmatrix} g_1(t_1, e^{u_1}) - \dfrac{\overline{a}_{13}e^{u_3}}{m(t_3)e^{u_3} + e^{u_1}} + \overline{D}_1(e^{u_2-u_1} - 1) \\ g_2(t_2, e^{u_2}) + \overline{D}_2(e^{u_1-u_2} - 1) \\ -\overline{a}_3 - \bar{a}_4 e^{u_3} + \dfrac{\overline{a}_{13}e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}} \end{bmatrix} \neq (0, 0, 0)^T$$

where $t_i \in (0, w)$ are one constant.

If system (2.23) does not have a solution, then naturally

$$QNu \neq (0, 0, 0)^T.$$

This shows that condition (ii) of Lemma 2.1 is satisfied finally. We will prove that condition (iii) of Lemma 2.1 is satisfied, we only prove that when $u \in \partial\Omega \cap KerL = \partial\Omega \cap R^3$, $\deg\{JQNu, \partial\Omega \cap KerL, (0, 0, 0)^T\} \neq 0$. When $u \in \partial\Omega \cap KerL = \partial\Omega \cap R^3, u$ is a constant vector in R^3 with $\sum_{i=1}^{3} |u_i| = M$. Before our proof is completed, we will prove several Lemmas at first.

Lemma 2.2. *Homotopic mapping and coincidence degree meet the following expressions*

$$deg\{JQNu, \Omega \cap KerL, (0, 0, 0)^T\} = deg\Big\{\Big(g(t_1, e^{u_1}), g(t_2, e^{u_2}),$$

$$-\bar{a}_3 - \bar{a}_4 e^{u_3} + \frac{\bar{a}_{31} e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}} - h_3 e^{-u_3}\Big)^T, \Omega \cap KerL, (0,0,0)^T\Big\}$$

Proof. we define mapping $\phi_1 : DomL \times [0,1] \to X$ by

$$\phi_1(u_1,u_2,u_3,\mu_1) = \begin{bmatrix} g_1(t_1,e^{u_1}) \\ g_2(t_2,e^{u_2}) \\ -\bar{a}_4 e^{u_3} + \dfrac{\bar{a}_{31} e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}} \end{bmatrix} + \mu_1 \begin{bmatrix} -\dfrac{\bar{a}_{13} e^{u_3}}{m(t_3)e^{u_3} + e^{u_1}} + \overline{D}_1(e^{u_2-u_1} - 1) \\ \overline{D}_2(e^{u_1-u_2} - 1) \\ -\bar{a}_3 \end{bmatrix}$$

where $\mu_1 \in [0,1]$ is a parameter, when $u = (u_1,u_2,u_3)^T \in \partial\Omega \cap KerL = \partial\Omega \cap R^3$, u is a constant vector in R^3 with $\sum_{i=1}^{3} |u_i| = M$. We will show that when $u \in \partial\Omega \cap KerL$, $\phi_1(u_1,u_2,u_3,\mu_1) \neq 0$.If the conclusion is not true, i.e., constant vector u with $\sum_{i=1}^{3} |u_i| = M$, satisfies $\phi_1(u_1,u_2,u_3,\mu_1) = 0$, then from

$$g_1(t_1,e^{u_1}) + \mu_1 \left(\frac{-\bar{a}_{13} e^{u_3}}{m(t_3)e^{u_3} + e^{u_1}} + \overline{D}_1(e^{u_2-u_1} - 1) \right) = 0,$$

$$g_2(t_2,e^{u_2}) + \mu_1 \overline{D}_2(e^{u_1-u_2} - 1) = 0 \text{ and } -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31} e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}} - \mu_1 \bar{a}_3 = 0$$

it follows the arguments of (2.11)-(2.22)that $|u_i| < R_i, \quad i = 1,2, \quad |u_3| < R_3$. Thus $\sum_{i=1}^{3} |u_i| < 2R_1 + R_3 < M$. which contradicts the fact that $\sum_{i=1}^{3} |u_i| = M$.

According to topological degree theory, we have

$$\begin{aligned} &deg\{(JQN, \Omega \cap KerL, (0,0,0)^T)\} \\ &= deg\{\phi_1(u_1,u_2,u_3,1)^T, \Omega \cap KerL, (0,0,0)^T)\} \\ &= deg\{\phi_1(u_1,u_2,u_3,0)^T, \Omega \cap KerL, (0,0,0)^T)\} \\ &= deg\Big\{\Big(g_1(t_1,e^{u_1}), g_2(t_2,e^{u_2}), -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31} e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}}\Big)^T, \\ &\qquad \Omega \cap KerL, (0,0,0)^T)\Big\} \end{aligned}$$ □

Lemma 2.3. *Homotopic mapping and coincidence degree meet the following expressions*

$$deg\left\{(g_1(t_1,e^{u_1}), g_2(t_2,e^{u_2}), -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31} e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}})^T, \Omega \cap KerL, (0,0,0)^T)\right\}$$

$$= deg\Big\{\Big(a_1 - a_{11}e^{u_1}, g(t_2, e^{u_2}), -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}}\Big)^T, \\ \Omega \cap KerL, (0,0,0)^T)\Big\}$$

where a_1, a_{11} are two chosen positive constants such that $C < \ln \frac{a_1}{a_{11}} < A$.

Proof. We define the mapping $\phi_2 : DomL \times [0,1] \to X$ by

$$\phi_2(u_1, u_2, u_3, \mu_2) = \mu_2 \begin{bmatrix} a_1 - a_{11}e^{u_1} \\ g_2(t_2, e^{u_2}) \\ -\bar{a}_4 e^{u_3} + \dfrac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}} \end{bmatrix} + (1-\mu_2) \begin{bmatrix} g_1(t_1, e^{u_1}) \\ g_2(t_2, e^{u_2}) \\ -\bar{a}_4 e^{u_3} + \dfrac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}}] \end{bmatrix}$$
$$= \begin{bmatrix} \mu_2(a_1 - a_{11}e^{u_1}) + (1-\mu_2)g_1(t_1, e^{u_1}) \\ g_2(t_2, e^{u_2}) \\ -\bar{a}_4 e^{u_3} + \dfrac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}} \end{bmatrix}$$

where $\mu_2 \in [0,1]$ is a parameter. We will prove that when $u \in \partial\Omega \cap KerL$, $\phi_2(u_1, u_2, u_3, \mu_2) \neq (0,0,0)^T$. When $u \in \partial\Omega \cap KerL = \partial\Omega \cap R^3$, u is a constant vector in R^3 with $\sum_{i=1}^{3} |u_i| = M$. Now we consider two possible cases:

(i) $u_1 \geq A$; (ii) $u_1 < A$.

(i) when $u_1 \geq A$, from condition (iii) in theorem 2.1, we have $g(t_1, e^{u_1}) \leq 0$. Moreover, $a_1 - a_{11}e^{u_1} \leq a_1 - a_{11}e^A < 0$, thus $\mu_2(a_1 - a_{11}e^{u_1}) + (1-\mu_2)g(t_1, e^{u_1}) < 0$. Therefore, $\phi_1(u_1, u_2, u_3, \mu_2) \neq (0,0,0)^T$.

(ii) when $u_1 < A$, if $u_1 \leq C$, then from condition (H_3) in theorem 2.1, we have $g(t_1, e^{u_1}) > 0$. At the same time, $a_1 - a_{11}e^{u_1} \geq a_1 - a_{11}e^C > 0$. Therefore, $\phi_1(u_1, u_2, u_3, \mu_2) \neq (0,0,0)^T$. If $u_1 > C$, we also consider two possible cases: (a) $u_2 \geq B$; (b) $u_2 < B$.

(a) when $u_2 \geq B$, from condition (H_2) in Theorem 2.1, we have $g_2(t_2, e^{u_2}) < 0$ Therefore $\phi_1(u_1, u_2, u_3, \mu_2) \neq (0,0,0)^T$.

(b) when $u_2 < B$, If $u_2 \leq D$, then from condition (H_4) in Theorem 2.1, we obtain $g_2(t_2, e^{u_2}) > 0$. Consequently $\phi_2(u_1, u_2, u_3, \mu_2) \neq (0,0,0)^T$. If $u_2 > D$, we can claim when $u \in \partial\Omega \cap KerL = \partial\Omega \cap R^3$, $\phi_2(u_1, u_2, u_3, \mu_2) \neq (0,0,0)^T$, otherwise from

$$-\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}} = 0,$$

we have $e^{u_3} < \dfrac{\bar{a}_{31}}{\bar{a}_4}$

and

$$e^{u_3} > \frac{-\bar{a}_4 e^{\rho_1} + \sqrt{(-\bar{a}_4 e^{\rho_1})^2 + 4 - \bar{a}_4 m(t_4)\bar{a}_{31} e^{\rho_1}}}{2\bar{a}_4 m(t_4)} > 0,$$

i.e.,

$$u_3 < \ln \bar{a}_{31} - \ln \bar{a}_4,$$

$$u_3 > \ln \frac{-\bar{a}_4 e^{\rho_1} + \sqrt{(-\bar{a}_4 e^{\rho_1})^2 + 4 - \bar{a}_4 m(t_4)\bar{a}_{31} e^{\rho_1}}}{2\bar{a}_4 m(t_4)}.$$

Thus

$$|u_1| < \max\{|d_1|, |\rho_1|\}, \quad |u_2| < \max\{|d_1|, |\rho_1|\}$$

and $|u_3| < \max\{|d_3|, |\rho_3|\}$. Therefore

$$\sum_{i=1}^{3} |u_i| < \max\{|d_1|, |\rho_1|\} + \max\{|d_1|, |\rho_1|\} + \max\{|d_3|, |\rho_3|\} < M,$$

which contradicts the fact that $\sum\limits_{i=1}^{3} |u_i| = M$. Based on the above discussion, for any $u \in \partial\Omega \cap KerL$, we have $\phi_2(u_1, u_2, u_3, \mu_2) \neq (0,0,0)^T$. According to topological degree theory, we obtain

$$\begin{aligned}
&deg\left\{\left(g_1(t_1, e^{u_1}), (g_2(t_2, e^{u_2}), -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31} e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}}\right)^T, \Omega \cap KerL, (0,0,0)^T\right\} \\
&= deg\{\phi_2(u_1, u_2, u_3, 1)^T, \Omega \cap KerL, (0,0,0)^T\} \\
&= deg\{\phi_2(u_1, u_2, u_3, 0)^T, \Omega \cap KerL, (0,0,0)^T\} \\
&= deg\{(a_1 - a_{11}e^{u_1}, g_2(t_2, e^{u_2}), -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31} e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}})^T, \Omega \cap KerL, (0,0,0)^T\}
\end{aligned}$$

Lemma 2.4. *Homotopic mapping and coincidence degree meet the following expressions*

$$\begin{aligned}
&deg\left\{(a_1 - a_{11}e^{u_1}), g_2(t_2, e^{u_2}), -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31} e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}})^T, \Omega \cap KerL, (0,0,0)^T)\right\} \\
&= deg\left\{\left(a_1 - a_{11}e^{u_1}, a_2 - a_{22}e^{u_2}, -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31} e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}}\right)^T, \right. \\
&\qquad\qquad \left. \Omega \cap KerL, (0,0,0)^T)\right\}
\end{aligned}$$

Proof. we define the mapping $\phi_3 : DomL \times [0,1] \to X$ by

$$\begin{aligned}\phi_3(u_1,u_2,u_3,\mu_3) &= \mu_3 \begin{bmatrix} a_1 - a_{11}e^{u_1} \\ a_2 - a_{22}e^{u_2}) \\ -\bar{a}_4 e^{u_3} + \dfrac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3}+e^{u_1}} \end{bmatrix} \\ &\quad +(1-\mu_3)\begin{bmatrix} a_1 - a_{11}e^{u_1}) \\ g_2(t_2,e^{u_2}) \\ -\bar{a}_4 e^{u_3} + \dfrac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3}+e^{u_1}} \end{bmatrix} \\ &= \begin{bmatrix} a_1 - a_{11}e^{u_1} \\ \mu_3(a_2 - a_{22}e^{u_2}) + (1-\mu_3)g(t_2,e^{u_2}) \\ -\bar{a}_4 e^{u_3} + \dfrac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3}+e^{u_1}}, \end{bmatrix}\end{aligned}$$

where $\mu_3 \in [0,1]$ is a parameter, and a_2, a_{22} are two closen positive constants such that $D < \ln \frac{a_2}{a_{22}} < B$. We will prove that when $u \in \partial\Omega \cap KerL$, $\phi_3(u_1, u_2, u_3, \mu_2) \neq (0,0,0)^T$. If is not true, then constant vector u satisfies $\phi_3(u_1, u_2, u_3, \mu_2) \neq (0,0,0)^T$ with $\sum_{i=1}^{3} |u_i| = M$. Thus we have

$$\begin{cases} a_1 - a_{11}e^{u_1} = 0, & (2.24) \\ \mu_3(a_2 - a_{22}e^{u_2}) + (1-\mu_3)g_2(t_2,e^{u_2}) = 0, & (2.25) \\ -\bar{a}_4 e^{u_3} + \dfrac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3}+e^{u_1}} = 0. & (2.26) \end{cases}$$

(2.24) implies

$$C < u_1 = \ln \frac{a_1}{a_{11}} < A. \tag{2.27}$$

We clain that $u_2 < B$; otherwise, if $u_2 \geq B$, then from condition (H_2) in Theorem 2.1, we have

$$(1-u_3)g_2(t_2,e^{u_2}) < 0,$$

and consequently

$$\mu_3(a_2 - a_{22}e^{u_2}) + (1-\mu_3)g_2(t_2,e^{u_2}) < 0,$$

which contradicts (2.23), We also claim that $u_2 > D$. If $u_2 \leq D$, then $g_2(t_2,e^{u_2}) > 0$. Moreover $a_2 - a_{22}e^{u_2} > a_2 - a_{22}e^D > 0$.

Thus $u_3(a_2 - a_{22}e^{u_2}) + (1-\mu_3)g_2(t_2,e^{u_2}) > 0$, which contradicts (2.24),(2.26) gives $-\bar{a}_4 e^{u_3} + \dfrac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3}+e^{u_1}} = 0$, that is, $u_3 < \ln \bar{a}_{31} - \ln \bar{a}_4$ and

$$u_3 > \ln \frac{-\bar{a}_4 e^{\rho_1} + \sqrt{(-\bar{a}_4 e^{\rho_1})^2 + 4 - \bar{a}_4 m(t_4)\bar{a}_{31}e^{\rho_1}}}{2\bar{a}_4 m(t_4)}.$$

Thus $|u_1| < \max\{|d_1|, |\rho_1|\}$, $|u_2| < \max\{|d_1|, |\rho_1|\}$ and $|u_3| < \max\{|d_3|, |\rho_3|\}$. Therefore

$$\sum_{i=1}^{3} |u_i| < \max\{|d_1|, |\rho_1|\} + \max\{|d_1|, |\rho_1|\} + \max\{|d_3|, |\rho_3|\} < M,$$

which leads to a contradiction. Therefore, by means of topological degree theory. We have

$$deg\left\{\left(a_1 - a_{11}e^{u_1}, g_2(t_2, e^{u_2}), -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}}\right), \Omega \cap KerL, (0,0,0)^T\right\}$$
$$= deg\{\phi_3(u_1, u_2, u_3, 1)^T, \Omega \cap KerL, (0,0,0)^T\}$$
$$= deg\{\phi_3(u_1, u_2, u_3, 0)^T, \Omega \cap KerL, (0,0,0)^T\}$$
$$= deg\{(a_1 - a_{11}e^{u_1}, a_2 - a_{22}e^{u_2}, -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}}), \Omega \cap KerL, (0,0,0)^T\}$$

□

From above Lemma , we have

Lemma 2.5. *Homotopic mapping and coincidence degree meet the following expressions*

$$deg\{JQNu, \Omega \cap KerL, (0,0,0)^T\}$$
$$= deg\left\{\left(a_1 - a_{11}e^{u_1}, a_2 - a_{22}e^{u_2}, -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}}\right), \Omega \cap KerL, (0,0,0)^T\right\} \neq 0.$$

Proof. Because of condition (H_5) in Theorem 2.1, the system of algebraic equations

$$\begin{cases} a_1 - a_{11}x = 0 \\ a_2 - a_{22}y = 0 \\ -\bar{a}_4 z + \dfrac{\bar{a}_{31}x}{m(t_4)z + x} = 0 \end{cases}$$

has a unique solution $(x^*, y^*, z^*)^T$ which satisfies:

$$x^* = \frac{a_1}{a_{11}} > 0, \quad y^* = \frac{a_2}{a_{22}} > 0,$$

$$z^* = \frac{a_4 x* + \sqrt{(a_4 x*)^2 + 4a_4 m(t_4)\bar{a}_{31}x*}}{2\bar{a}_4 m(t_4)} > 0.$$

Thus

$$deg\left\{\left(a_1 - a_{11}e^{u_1}, a_2 - a_{22}e^{u_2}, -\bar{a}_4 e^{u_3} + \frac{\bar{a}_{31}e^{u_1}}{m(t_4)e^{u_3} + e^{u_1}}\right), \Omega \cap KerL, (0,0,0)^T\right\}$$

$$= \begin{vmatrix} -a_{11}x^* & 0 & 0 \\ 0 & -a_{22}y^* & 0 \\ \cdots & 0 & -\bar{a}_4 z^* - \dfrac{\bar{a}_{31}m(t_4)x^*z^*}{[m(t_4)z^*+x^*]^2} \end{vmatrix} = -1.$$

Therefore from (2.20), we have

$$deg\{JQNu, \Omega \cap KerL, (0,0,0)^T\} = -1 \neq 0. \quad \square$$

From Lemma 2.1 to Lemma 2.5,Obviously we have the results of Theorem 2.1.

3. Global attractivity of positive periodic solution

In this section,by constructing a lyapunov function,we derive sufficient condition for the global attractivity of a postive solutions system (1.2).

Lemma 3.1(Barbalat's Lemma [9]). *Let f be nonegative function defined on $[0,+\infty)$ such that f is integrable and uniformly continuous on $[0,+\infty)$. Then*

$$\lim_{t\longrightarrow+\infty} f(t) = 0$$

.

Theorem 3.1. *For system (1.2), assume that the conditions in Theorem 2.1 and the following conditions hold:*

i) *There exists two positive constants G_1 and G_2 such that*

$$\frac{\partial g_i(t,x)}{\partial x} \leq -G_i \quad for \quad t \geq 0, i = 1,2;$$

ii) $G_1 > \dfrac{D_2^M}{e^{\rho_1}} + \dfrac{a_{13}^M + a_{31}^M}{m^L e^{\rho_3} + e^{\rho_1}}$;

iii) $G_2 > \frac{D_1^M}{e^{\rho_1}}$;

iv) $a_4^L > \dfrac{a_{13}^M + a_{31}^M m^M}{m^L e^{\rho_3} + e^{\rho_1}}$.

Then system (1.2) has a positive $\omega-$periodic solution which attracts all positive solutions.

Proof. According to Theorem 2.1, system (2.1) has at least one positive $\omega-$periodic solution $(x_1^*(t))^T, (x_2^*(t))^T, (x_3^*(t))^T$ such that for $t \geq 0$, $e^{\rho_1} < x_1^*(t) < e^{d_1}$, $e^{\rho_1} < x_2^*(t) < e^{d_1}$, $e^{\rho_3} < x_3^*(t) < e^{d_3}$.

Suppose that $(x_1(t), x_2(t), x_3(t))^T$ is an arbitrary positive solution of system (1.2) with the initial conditions $x_i(s) > 0$, $s \in [-\tau, 0)$, $i = 1,2,3$.

Consider the following Lyapunov function defined by

$$V(t) = \sum_{i=1}^{3} |\ln x_i(t) - \ln x_i^*(t)| + \frac{a_{31}^M m^M}{m^L e^{\rho_3} + e^{\rho_1}} \int_{t-\tau}^{t} |x_3(s) - x_3^*(s)| ds$$

$$+ \frac{a_{31}^M}{m^L e^{\rho_3} + e^{\rho_1}} \int_{t-\tau}^{t} |x_1(s) - x_1^*(s)| ds.$$

Calculating the upper right derivative of $V(t)$ along the solutions of system (2.1), we have

$$D^+V(t) = sgn(x_1(t)-x_1^*(t))\Big\{g_1(t,x_1(t))-g_1(t,x_1^*(t))-\Big[\frac{a_{13}(t)x_3(t)}{m(t)x_3(t)+x_1(t)}$$
$$-\frac{a_{13}(t)x_3^*(t)}{m(t)x_3^*(t)+x_1^*(t)}\Big]+D_1(t)\Big[\frac{x_2(t)}{x_1(t)}-\frac{x_2^*(t)}{x_1^*(t)}\Big]\Big\}$$
$$+ sgn(x_2(t)-x_2^*(t))\Big\{g_2(t,x_2(t))-g_2(t,x_2^*(t))+D_2(t)\Big[\frac{x_1(t)}{x_2(t)}-\frac{x_1^*(t)}{x_2^*(t)}\Big]\Big\}$$
$$+ sgn(x_3(t)-x_3^*(t))\Big\{-a_4(t)[x_3(t)-x_3^*(t)]+\frac{a_{31}(t)x_1(t-\tau)}{m(t)x_3(t-\tau)+x_1(t-\tau)}$$
$$-\frac{a_{31}(t)x_1^*(t-\tau)}{m(t)x_3^*(t-\tau)+x_1^*(t-\tau)}+\frac{a_{31}^M m^M}{m^L e^{\rho_3}+e^{\rho_1}}|x_3(t)-x_3^*(t)|$$
$$+\frac{a_{31}^M m^M}{m^L e^{\rho_3}+e^{\rho_1}}|x_1(t)-x_1^*(t)|-\frac{a_{31}^M m^M}{m^L e^{\rho_3}+e^{\rho_1}}|x_3(t-\tau)-x_3^*(t-\tau)|$$
$$-\frac{a_{31}^M m^M}{m^L e^{\rho_2}+e^{\rho_1}}|x_1(t-\tau)-x_1^*(t-\tau)|\Big\}$$
$$\le G_1|x_1(t)-x_1^*(t)|+\frac{D_1^M}{e^{\rho_1}}|x_2(t)-x_2^*(t)|-G_2|x_2(t)-x_2^*(t)|$$
$$+\frac{D_2^M}{e^{\rho_1}}|x_1(t)-x_1^*(t)|-a_4^L|x_3(t)-x_3^*(t)|$$
$$+\frac{a_{31}^M m^M}{m^L e^{\rho_2}+e^{\rho_1}}|x_3(t)-x_3^*(t)|+\frac{a_{31}^M}{m^L e^{\rho_2}+e^{\rho_1}}|x_1(t)-x_1^*(t)|$$
$$=-\left(G_1-\frac{D_2^M}{e^{\rho_1}}-\frac{a_{31}^M}{m^L e^{\rho_2}+e^{\rho_1}}\right)|x_1(t)-x_1^*(t)|-\left(G_2-\frac{D_1^M}{e^{\rho_1}}\right)$$
$$|x_2(t)-x_2^*(t)|-\left(a_4^L-\frac{a_{31}^M m^M}{m^L e^{\rho_2}+e^{\rho_1}}-\frac{a_{31}^M}{m^L e^{\rho_2}+e^{\rho_1}}\right)|x_3(t)-x_3^*(t)|.$$

It follows from the conditions $(ii)-(iv)$ in Theorem 3.1 that there exists a constant $\alpha>0$ such that

$$D^+V(t) < -\alpha\sum_{i=1}^{3}|x_i(t)-x_i^*(t)|,\quad t\ge 0.$$

Integrating both sides of the inequality above on $[0,t]$ leads to

$$V(t)+\alpha\int_0^t\sum_{i=1}^{3}|x_i(s)-x_i^*(s)|ds \le V(0) < +\infty,\quad t\ge 0,$$

which implies that $\sum_{i=1}^{3}|x_i(t)-x_i^*(t)|\in L^1[0,+\infty)$ and

$$\sum_{i=1}^{3}|\ln x_i(t)-\ln x_i^*(t)| < V(t)\le V(0) < +\infty,\quad t\ge 0.$$

From the boundedness of $x_i^*(t), (i = 1,2,3)$, it follows that $x_i(t), (i = 1,2,3)$ is bounded for $t \geq 0$. Thus $x_i - x_i^*(t), (i = 1,2,3)$ remains bounded on $[0, +\infty)$,that is $\sum_{i=1}^{3} |x_i(t) - x_i^*(t)|$ is bounded and uniformly continuous,Hence $\sum_{i=1}^{3} |x_i(t) - x_i^*(t)| \to 0, t \to 0$ due to Lemma 3.1.Therefore

$$\lim_{t\to+\infty} |x_i(t) - x_i^*(t)| = 0, \quad i = 1,2,3,$$

which implies that system(1.2) has a positive ω−periodic solution which attracts all the other positive solutions. □

4. Some examples

Consider the following system (4.1),

$$\begin{cases} x_1' = x_1(t)(a_1(t) - a_{11}(t)x_1(t) - \dfrac{a_{13}(t)x_3(t)}{m(t)x_3(t) + x_1(t)}) + D_1(t)(x_2(t) - x_1(t)) \\ x_2' = x_2(t)(a_2(t) - a_{22}(t)x_2(t) + D_2(t)(x_1(t) - x_2(t)) \\ x_3' = x_3(t)(-a_3(t) - a_4(t)x_3(t) + \dfrac{a_{31}(t)x_1(t-\tau)}{m(t)x_3(t-\tau) + x_1(t-\tau)}) \end{cases} \tag{4.1}$$

where $\tau > 0$ is a positive constant and all the parameters are positive continuous w-periodic functions with periodic $w > 0$.

In Theorem 2.1, $g_1(t, e^x) = a_1(t) - a_{11}(t)e^x$, $g_2(t, e^x) = a_2(t) - a_{22}(t)e^x$. It is easily shown that if $x \geq \ln\left(\dfrac{a_1^M}{a_{11}^l}\right)$, then $g_1(t, e^x) \leq 0$ and if $x \geq \ln\left(\dfrac{a_2^M}{a_{22}^l}\right)$, then $g_2(t, e^x) \leq 0$. We also can show if

$$x \leq \ln \frac{a_1^M - \left(\frac{a_{13}}{m}\right)^M}{a_{11}^l},$$

then $g_1(t, e^x) \geq \left(\dfrac{a_{11}}{m}\right)^M$ and if $x \leq \ln \dfrac{a_2^M}{a_{22}^l}$, then $g_2(t, e^x) > 0$.

Hence , corresponding to Theorem 2.1

$$A = \ln\left(\frac{a_2^M}{a_{22}^l}\right), \quad C = \frac{a_1^M - \left(\frac{a_{13}}{m}\right)^M}{a_{11}^l}, \quad D = \ln\frac{a_2^M}{a_{22}^l}$$

By Theorem 2.1, we have the following theorem:

Theorem 4.1. *If the following conditions hold:*

(H_1) $a_1^m > \left(\dfrac{a_{13}}{m}\right)^M$

(H_2) $\bar{a}_{31} > \bar{a}_3$

Then system (4.1) has at least one positive ω−periodic solution.

Below we will apply Theorem 3.1 to example 4.1. It is clear that

$$G_1 = a_{11}^L, \quad G_2 = a_{22}^L, \quad \rho_1 = \max\left\{ \ln \frac{a_1^M - \left(\frac{a_{13}}{m}\right)^M}{a_{11}^l}, \ln \frac{a_2^M}{a_{22}^l} \right\},$$

$$\rho_3 = \ln \frac{(a_{31}^l - a_3^M)e^{\rho_1} - a_3^M m^M e^{d_1}}{a_4^M(m^M e^{d_1} + e^{\rho_1})}, \quad d_1 = \ln \frac{a_2^M}{a_{22}^l}, \quad d_3 = \ln \frac{a_{31}^M}{a_4^l}.$$

Then we can obtain the theorem as follows.

Theorem 4.2. *Besides $(H_1) - (H_2)$ in Theorem 4.1 ,if system (4.1) satisfies the following conditions:*

i) $a_{11}^l > \frac{D_2^M}{e^{\rho_3}} + \frac{a_{13}^M + a_{31}^M}{m^l e^{\rho_3} + e^{\rho_1}}$, ii) $a_{22}^l > \frac{D_1^M}{e^{\rho_1}}$

iii) $a_4^l > \frac{a_{13}^M + a_{31}^M m^M}{m^l e^{\rho_3} + e^{\rho_1}}$

Then system (4.1) has an attractive positive $\omega-$periodic solution.

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