한국해양공학회지 제22권 4호, pp 14-19, 2008년 8월 (ISSN 1225-0767)

Application to Speed Control of Brushless DC Motor Using Mixed H_2/H_∞ PID Controller with Genetic Algorithm

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KEY WORDS: Linear quadratic regulator(LQR), Robust control, Genetic algorithm, Brushless direct current(BLDC) motor, H_2/H_∞ proportional integral derivative(PID) controller

ABSTRACT: *This paper proposes a mixed $H_2/H\infty$ optimal PID controller with a genetic algorithm based on the dynamic model of a brushless direct current (BLDC) motor and applies it to speed control. In the dynamic model of the BLDC motor with perturbation, the proposed controller guarantees arobust and optimal tracking performance to the desired speed of the BLDC motor. A genetic algorithm was used to obtain parameters for the PID controller that satisfy the mixed H2/H∞ constraint. To implement the proposed controller, a control system based on PIC18F4431 was developed. Numerical and experimental results are shown to prove that the performance of the proposed controller was better than that of the optimal PID controller.*

1. Introduction

The disadvantages of brushed DC motors emerge due to the employment of mechanical commutator since the life expectancy of the brush construction is restricted. Furthermore, the mechanical commutators lead to losses and contact uncertainties at small voltages and can cause electrical disturbances (sparking). Therefore, brushless direct current (BLDC) motors without brushes for commutation are electronically commutated, and are a type of synchronous motor. This BLDC motor do not experience the "slip" that is normally seen in induction motors. In addition, the BLDC motor has the better heat dissipation characteristic and the ability to operate at higher speed (Hemati, 1992). Because of theses advantages of the BLDC motor, the BLDC motor can be used in DC motor or induction motor in various industrial fields such as in the hard disk of computer and in air plane. Especially, in the field of ocean engineering, the BLDC motor can be used as propeller motor instead of the DC motor or induction motor for underwater vehicle, surface ship, submarine, etc. However, the BLDC motor constitutes more difficult problems in modeling and control system design due to its multi-input nature and coupled nonlinear dynamics than the brushed DC motor.

Therefore, a compact representation of the BLDC model was obtained by Ong (1998). Based on this model, a lot of PID controllers have been easily applied to control the BLDC motors. So there have been a lot of approaches to search the parameters of optimal PID controllers to control the BLDC motors, including using iterative learning control (Lim et al., 2004), using LQR approach (Yu and Hwang, 2004) and H_∞ approach (Lin and Jan, 2002). However, most of them only satisfy one of two criterions: optimal performance or robust performance.

Mixed H_2/H_∞ control design methods have received a great deal of attention from the viewpoint of theoretical design. A mixed H_2/H_∞ PID controller is to find an internally stabilizing PID controller that minimizes an H_2 performance index using genetic algorithm (GA) subject to an inequality constraint on the H_∞ criterion (Chen et al., 1995).

In this paper, a mixed H_2/H_∞ PID controller with the genetic algorithm is used to achieve an optimal robust PID controller for controlling speed of BLDC motor with perturbation. Numerical and experimental results are shown to prove the good performance in the proposed controller.

2. Brushless DC Motors

Unlike a permanent magnet DC motor, the commutation of a BLDC motor is controlled electronically. To rotate the BLDC motor, the stator windings should be energized in a sequence. It is important to know the rotor position in order to understand which winding will be energized following the energizing sequence. The rotor position is sensed using hall

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effect sensors embedded into the stator or optical position sensors (encoder). The dynamic characteristics of the BLDC motors are similar to brushed DC motors. The nominal transfer function of the BLDC motor $P(s)$ can be represented as (Ong, 1998).

$$P(s) = \frac{\omega(s)}{v_{app}(s)} = \frac{K_t}{(Ls+R)(Js+D)+K_tK_b} \quad (1)$$

where $v_{app}(t)$ is the applied voltage, $\omega(t)$ is the motor speed, L is the inductance of the stator, R is the resistance of the stator, D is the viscous coefficient, J is the moment of inertia, K_t is the motor torque constant, K_b is the back electromotive force constant, and s is Laplace variable.

3. Mixed H_2/H_∞ Control Problem Description

Consider the PID control system with plant perturbation $\triangle P(s)$ in Fig. 1, where $r(t)$, $e(t)$, $u(t)$, $y(t)$ and $P(s)$ are the reference, error, the control signal and the output, and plant, respectively. The system output $y(t)$ is defined as the motor speed $w(t)$. The PID controller $C(s)$ is of the following form:

$$C(s) = K_p + K_i/s + K_d s \quad (2)$$

The plant $P(s)$ is assumed to be stable but have the following bounded perturbation.

$$|\triangle P(j\omega)| < |\xi(j\omega)| \qquad \forall\, \omega \in [0, \infty) \quad (3)$$

where the function $\xi(s)$ is stable and known.

The tracking error $e(t)$, the control signal $u(t)$ and the load $l(t)$ of motor are defined as

$$e(t) = r(t) - y(t) \quad (4)$$

$$u(t) = k_p e(t) + k_i \int e(t)dt + k_d \frac{de(t)}{dt} \quad (5)$$

$$l(t) = k_{pl} r_l + k_{dl} \frac{dr_l}{dt} \quad (6)$$

where $k_{pl} = \dfrac{R}{K_e K_t}$, $k_{dl} = \dfrac{L}{K_e K_t}$ are and r_l unit step reference input of load.

The control objective is to make the perturbed system with plant perturbation $\triangle P(s)$ in (6) be robustly and asymptotically stable. To do this, firstly, a controller $C(s)$ is chosen so that the nominal closed loop system with $\triangle P(s) = 0$ and $l(t) = 0$ in Fig. 1 is asymptotically stable. Secondly, the following inequality (6) holds

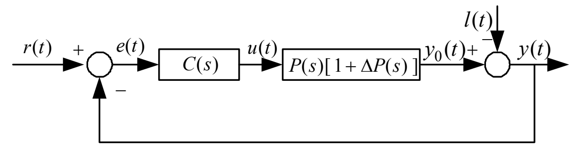

Fig. 1 PID control system with plant perturbation

$$\left\| \frac{P(s)C(s)\xi(s)}{1+P(s)C(s)} \right\|_\infty \le 1 \quad (6)$$

If the two conditions are satisfied, the perturbed closed loop system in Fig. 1 is also asymptotically stable under plant perturbation $\triangle P(s)$ in (6).

The constraint in (6) is equivalent to the following:

$$\left\| \frac{P(s)C(s)\xi(s)}{1+P(s)C(s)} \right\|_\infty = \underbrace{sup}_{\omega\in[0,\infty)} \sqrt{\frac{P(-j\omega)P(j\omega)C(-j\omega)C(j\omega)\xi(-j\omega)\xi(j\omega)}{[1+P(-j\omega)C(-j\omega)][(1+P(j\omega)C(j\omega)]}}$$

$$= \underbrace{sup}_{\omega\in[0,\infty)} \sqrt{\frac{\beta(\omega)}{\alpha(\omega)}} = \sqrt{\underbrace{sup}_{\omega\in[0,\infty)} \frac{\beta(\omega)}{\alpha(\omega)}} \le 1 \quad (7)$$

In general, scanning ω in [0, ∞] to find the peaks of $\beta(\omega)/\alpha(\omega)$ in (7) is not an easy task. The peaks of $\beta(\omega)/\alpha(\omega)$ occur at the points which must satisfy the following equation:

$$\frac{d}{d\,\omega}\frac{\beta(\omega)}{\alpha(\omega)} = \frac{\alpha(\omega)\dfrac{d\beta(\omega)}{d\omega} - \beta(\omega)\dfrac{d\alpha(\omega)}{s\omega}}{\alpha^2(\omega)} = 0 \quad (8)$$

Therefore, only the real root λ_i of the following equation needs be found:

$$\alpha(\omega)\frac{d\beta(\omega)}{d\omega} - \beta(\omega)\frac{d\alpha(\omega)}{d\omega} = \prod_{i=1}^{n}(\omega - \lambda_i) = 0 \quad (9)$$

The robust stability constraint in (6) is equivalent to

$$\sqrt{\underbrace{\max}_{\lambda_i} \frac{\beta(\lambda_i)}{\alpha(\lambda_i)}} \le 1 \quad (10)$$

However, the control system designed with only robustness stability is not good enough. Optimal tracking performance is also appealing in many practical control engineering applications. Therefore, the mixed H_2/H_∞ control problem is formulated as follows (Chen et al., 1995):

$$J = \underbrace{\min}_{C} \int_0^\infty e^2(t)dt \tag{11}$$

where $e(t)$ is the tracking error in nominal state of Fig. 1:

$$e(s) = \frac{r(s)}{1+P(s)C(s)} \tag{12}$$

By Parseval's theorem (Vidyassagar, 1985), the quality index function can be obtained:

$$\begin{aligned} J &= \underbrace{\min}_{C} \int_0^\infty e^2(t)dt = \underbrace{\min}_{k_p,k_i,k_d} \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} e(-s)e(s)ds \\ &= \underbrace{\min}_{k_p,k_i,k_d} \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{r(-s)r(s)}{[1+P(-s)C(-s)][1+P(s)C(s)]}ds \\ &= \underbrace{\min}_{k_p,k_i,k_d} \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{B(-s)B(s)}{A(-s)A(s)}ds \end{aligned} \tag{13}$$

where $A(s)$ and $B(s)$ are Hurwitz polynomials of s with appropriate degree.

Let $A(s) = \sum_{k=0}^{m} a_k s^k$ and $B(s) = \sum_{k=0}^{m-1} b_k s^k$, then (13) can be rewritten as follows:

$$J_m(k_p,k_i,k_d) = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{\left(\sum_{k=0}^{m-1} b_k s^k\right)\left(\sum_{k=0}^{m-1} b_k (-s)^k\right)}{\left(\sum_{k=0}^{m} a_k s^k\right)\left(\sum_{k=0}^{m} a_k (-s)^k\right)}ds \tag{14}$$

The minimization problem in the above equation can be solved via the aid of the residue theorem. The value of $J_m(k_p,k_i,k_d)$ can be found from the formulation of Jury and Dewey (1965).

Then, the H_2 performance in (11) must be of the following form:

$$J_m = \underbrace{\min}_{k_p,k_i,k_d} J_m(k_p,k_i,k_d) \tag{15}$$

where J_m is a function of PID parameters (k_p,k_i,k_d) with appropriate m.

From the above analysis, our mixed H_2/H_∞ PID control design problem is the minimization problem (15) under the inequality constraint (10).

4. Mixed H_2/H_∞ PID Controller Design

In this chapter, the genetic algorithm(GA) is applied to tune the parameters (k_p,k_i,k_d) of the PID controller easily. A genetic algorithm is an optimization method that manipulates a string of numbers in a manner similar to how chromosomes are changed in biological evolution. It does not need to assume that the search space is differentiable or continuous. An initial population made up of strings of numbers is chosen at random or is specified by the user. Each string of numbers is called as a "chromosome" or an "individual", and each number slot is called a "gene". A set of chromosomes forms a population. Each chromosome represents a given number of traits which are the actual parameters that are being varied to optimize the "fitness function". The fitness function is a performance index that seeks to be maximized.

4.1 Chromosome coding and decoding

GA works with chromosome (numerical string), not with the parameters themselves. Each chromosome represents a given number of traits. For example, with the binary coding method, the three traits (k_p,k_i,k_d) are coded as binary strings.

In this paper, decimal coding is used (Passino and Yurkovich, 1998). A chromosome would be decoded as follows: The first gene of each trait determines the sign of the trait. For our base-10 algorithm, if this gene is 0-4, the trait is negative, and if this gene is 5-9, the trait is positive. The remaining genes determine the size of the trait. The second gene is the most significant digit, while the last gene is the least significant digit. To determine the relative magnitude of a trait, the decimal constant is used. This number determines how many digits to the left of the decimal place the first digit of the trait.

In case of trait $k_i = 746997$, k_i is decoded as Table 1.

4.2 Fitness and cost function

Fitness function is an objective function to be optimized which provides the mechanism for evaluating each chromosome.

The cost function is defined as follows:

Table 1 Coded values of decimal constants and k_i

Decimal constant	k_i
-2	0.0046997
-1	0.046997
0	0.46997
1	4.6997

$$E(k_p,k_i,k_d)=J_m(k_p,k_i,k_d),\qquad k_p,k_i,k_d\in D \tag{16}$$

The cost function is only defined for parameter sets of (k_p,k_i,k_d) in D in which the system is stabilized. Our objective is to search parameter sets of (k_p,k_i,k_d) in D to achieve the optimization problem of (14). The relation between the cost function and the fitness function can be expressed as follows:

$$F(k_p,k_i,k_d)=\begin{cases}-E(k_p,k_i,k_d), & \sqrt{\underbrace{\max}_{\lambda_i}\dfrac{\beta(\lambda_i)}{\alpha(\lambda_i)}}\le 1\\ 0, & \sqrt{\underbrace{\max}_{\lambda_i}\dfrac{\beta(\lambda_i)}{\alpha(\lambda_i)}}>1\end{cases} \tag{17}$$

The minimum of the quality index in (16) is equivalent with the maximum fitness value in (17). A chromosome that has a lower quadratic quality index should be assigning a larger fitness value. Then the genetic algorithm tries to generate better offsprings to improve the fitness. Therefore, a better PID controller can be obtained via better fitness in genetic algorithms.

The flowchart of the proposed control design procedure is shown in Fig. 2. based on the procedure of a simple genetic algorithm (Chen et al., 1995).

Step 1: Specify the condition of (k_p,k_i,k_d) to guarantee the stability of the nominal closed loop system via the Routh-Hurwitz criterion.

Step 2: Specify the stability domain D of (k_p,k_i,k_d).

Step 3: Establish the parameters of GA: crossover probability, mutation probability, population size, maximum generation and termination criterion.

Step 4: Make the initial population and chromosome coding.

Step 5: Compute λ_i from (9).

Step 6: Compute the fitness function from (17).

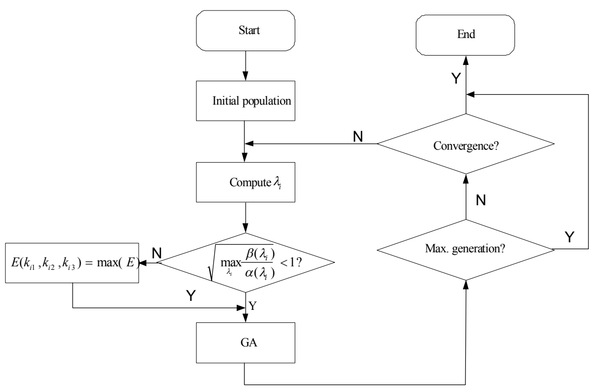

Fig. 2 Flowchart of the proposed control design procedure

Step 7: Create the next generation

Step 8: Check the stop condition. If it is not satisfactory, repeat procedure step 5 to step 8.

5. Numerical and Experimental Results

In this chapter, some simulation results are used to explore the proposed controller, and compare its performance with that of the traditional PID controller and the optimal PID controller using LQR approach (Yu and Hwang, 2004).

Figure 1 shows the block diagram of the proposed PID control system for speed control of the BLDC motor. From the specification of the BLDC motor used in this paper as shown in Table 2, the transfer function of the BLDC motor is obtained.

$$P(s)=\frac{275577.36}{s^2+417.7s+43567.5} \tag{18}$$

The plant perturbation is bounded as follows:

$$\left|\triangle P(s)\le\right|\frac{d}{s^2+0.1s+10}\left|\le 0.1|d|\right. \tag{19}$$

where d is defined as parameter of plant perturbation.

In nominal state with $\triangle P=0$ and $l(t)=0$, closed loop transfer function and output error with unit step reference are as follows:

$$G_n(s)=\frac{275577.36(k_ds^2+k_ps+k_i)}{s^3+a_2s^2+a_1s+a_0} \tag{20}$$

$$e(s)=\frac{1000(s^2+417.7s+43567.5)}{s^3+a_2s^2+a_1s+a_0} \tag{21}$$

where $a_2=417.7+275577.36k_d$,
$a_1=43567.5+275577.36k_p$,
$a_0=275577.36k_i$

Table 2 Specification of the BLDC motor

Parameters	Values	Units
R	21.2	Ω
K_b	0.1433	V • s/rad
D	1x10^{-4}	kg-m • s/rad
L	0.052	H
K_t	0.1433	kg-m/A
J	1x10^{-5}	kg-m • s^2/rad

In this case, $m=3$, the cost function is J_3

$$J_3=\frac{b_2^2\,a_0\,a_1+(b_1^2-2b_0b_2)\,a_0\,a_3+b_0^2\,a_2\,a_3}{2\,a_0\,a_3\,(-a_0a_3+a_1a_2)} \tag{22}$$

The GA begins by randomly generating a population of 200 chromosomes. After 20 generations, the proper PID controller parameters are obtained, and the corresponding PID control parameters are $k_p=180.1755$, $k_i=4.6997$, $k_d=0.0353$.

The speed response of the traditional PID controller with PID control parameters of $k_p=30, k_i=15, k_d=0.001$, the optimal PID controller with PID control parameters of $k_p=70.556, k_i=10, k_d=0.0212$(Yu and Hwang, 2004), and the proposed PID controller are shown in Fig. 3. Table 3 lists the performance of the two different PID controllers.

It is obvious that the transient response characteristic of the proposed PID controller is better than the optimal PID controller proposed by Yu and Hwang (2004).

In perturbed state with $\triangle P\neq 0$ and $l(t)=0$, open loop transfer function $P_b(s)$, closed loop transfer function $G_b(s)$ and output error $e_b(s)$ with unit step reference are as follows:

$$\begin{aligned}P_b(s)&\equiv P(s)[1+\triangle P(s)]\\&=\frac{275577.36[s^2+0.1s+(10+d)]}{s^4+417.8s^3+43619.27s^2+8533.75s+435675}\end{aligned} \tag{23}$$

$$G_b(s)=\frac{b_4s^4+b_3s^3+b_2s^2+b_1s+b_0}{s^5+c_4s^4+c_3s^3+c_2s^2+c_1s+c_0} \tag{24}$$

$$e_b(s)=\frac{s^4+417.8s^3+43619s^2+8533.8s+435675}{s^5+c_4s^4+c_3s^3+c_2s^2+c_1s+c_0} \tag{25}$$

where

$b_4=275577.36k_d$, $b_3=27557.736(k_d+10k_p)$,
$b_2=275577.36[(10+d)k_d+0.1k_p+k_i]$,
$b_1=275577.36[(10+d)k_p+0.1k_i]$, $b_0=275577.36(10+d)k_i$
$c_4=417.8+275577.36k_d$,
$c_3=43619+27558k_d+275577.36k_p$,
$c_2=8533.8+275577.36(10+d)k_d+27558k_p+275577.36k_i$,
$c_1=435675+275577.36(10+d)k_p+27558k_i$,
$c_0=275577.36(10+d)k_i$

In case of load with the parameter gains of $k_{pl}=1032$, $k_{dl}=2.53$ and plant perturbation, output is given in Fig. 1 as

$$y(s)=\frac{P(s)[1+\triangle P(s)]C(s)\,r(s)-l(s)}{1+P(s)[1+\triangle P(s)]C(s)} \tag{26}$$

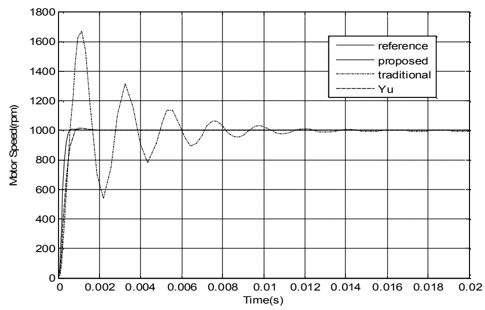

Fig. 3 Speed responses of the PID controllers in nominal state

Table 3 Simulated performances of different PID controllers

Items	Yu and Hwang	Proposed	Units
Rising time	0.84	0.59	ms
Settling time	1.58	1.07	ms
Overshoot	1.48	0.58	%

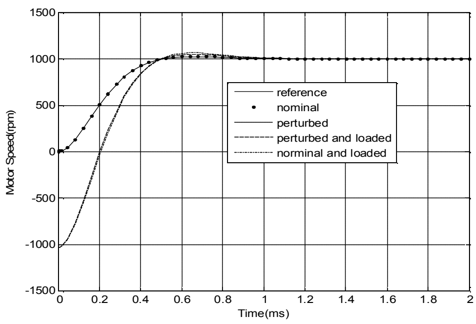

Fig. 4 Speed responses of the proposed controller corresponding to nominal, perturbed$(d=0.1)$ and loaded states

where $y(s), r(s), l(s)$ are Laplace transformations of output $y(t)$, step reference $r(t)$ and load $l(t)$.

Speed responses of the proposed controller corresponding to nominal, perturbed$(d=0.1)$ and loaded states are shown in Fig. 4. Although d is changed, d does not have much influence on the output of the system is because the transfer functions of $P(s)$ and $\triangle P(s)$ are stable. Therefore, it is shown that the proposed controller has robust performance to control the system with plant perturbation and load successfully.

To illustrate the effectiveness, a speed control scheme of the BLDC motor is implemented and shown in Fig. 5. The experimental set up is shown in Fig. 6. A BLDC motor driver is built using Hex MOSFET IRF540, IR2101 as a gate driver, and encoder as a speed feedback sensor. The main controller is PIC18F4431 microchip. Figure 7 shows hall sensor signals versus motor phase voltages in Fig. 8. Figure 9 shows the motor speed response for step reference change. It shows that the motor speed tracks the step reference change well.

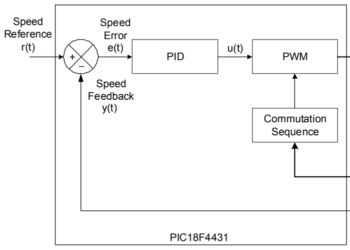

Fig. 5 Block diagram of speed control scheme by the proposed controller

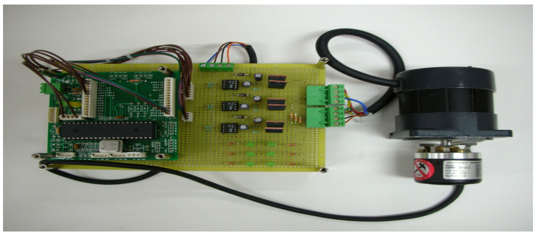

Fig. 6 Developed speed control of the BLDC motor system

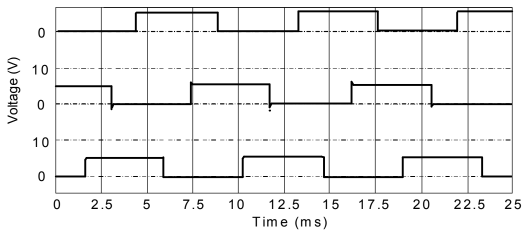

Fig. 7 Hall sensor signals

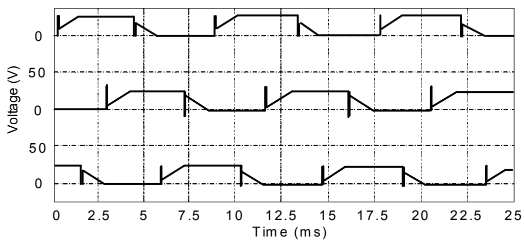

Fig. 8 Motor phase voltages

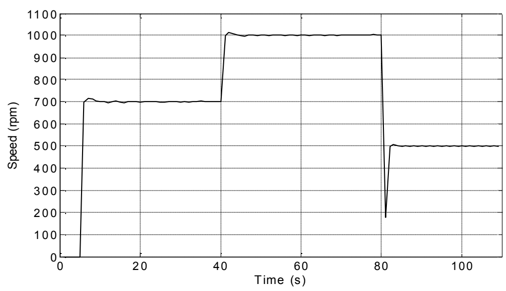

Fig. 9 Motor response for step reference change

6. Conclusions

This paper proposes a mixed H_2/H_∞ PID controller with GA and applies it to control the speed of the BLDC motor with plant perturbation. The proposed controller shows the robustness to a model with perturbation and load. The proposed controller is compared with the optimal PID controller using LQR approach suggested by Yu and Hwang for the speed tracking of the BLDC motor. To implement the proposed controller, the control system based on PIC18F4431 is developed. The simulation results show that the proposed controller has better effectiveness than the optimal PID controller suggested by Yu and Hwang. Experimental Results are shown to prove the good performance of the proposed controller.

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2008년 1월 21일 원고 접수
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