Commun. Korean Math. Soc. **23** (2008), No. 1, pp. 81–93

A REMARK ON INVARIANCE OF QUANTUM MARKOV SEMIGROUPS

Veni Choi and Chul Ki Ko

Abstract. In [3, 9], using the theory of noncommutative Dirichlet forms in the sense of Cipriani [6] and the symmetric embedding map, authors constructed the KMS-symmetric Markovian semigroup $\{S_t\}_{t\geq 0}$ on a von Neumann algebra $\mathcal{M}$ with an admissible function f and an operator $x \in \mathcal{M}$. We give a sufficient and necessary condition for x so that the semigroup $\{S_t\}_{t\geq 0}$ acts separately on diagonal and off-diagonal operators with respect to a basis and study some results.

1. Introduction

A quantum Markov semigroup on the algebra $L(\mathfrak{h})$ of all bounded operators on a complex separable Hilbert space $\mathfrak{h}$ is a semigroup $\{S_t\}_{t\geq 0}$ of completely positive, identity preserving and normal maps on $L(\mathfrak{h})$ [6]. Quantum Markov semigroups are the natural generalization of classical Markov semigroups and were introduced in physics to model the decay equilibrium of quantum open systems [1, 2, 3, 6, 7, 9, 10]. Many mathematicians and physicists have been interested to the problems whether quantum Markov semigroups on the subalgebra have their extensions on the full algebra [1, 5, 8]. These semigroups acts separately on diagonal and off-diagonal bounded operators with respect to the chosen basis.

Let $\mathcal{M}$ be a von Neumann algebra acting on a complex Hilbert space $\mathcal{H}$ and ξ_0 be a fixed cyclic and separating vector for $\mathcal{M}$. Let Δ and σ_t be the modular operator and the modular group associated with the pair $(\mathcal{M}, \xi_0)$, respectively [4]. Consider the symmetric embedding map:

$$\begin{aligned} &i_0 : \mathcal{M} \longrightarrow \mathcal{H} \\ &i_0(A) = \Delta^{1/4} A\xi_0. \end{aligned}$$

In [3, 9], using the theory of noncommutative Dirichlet forms in the sense of Cipriani [6], authors constructed the symmetric Markovian semigroup $\{T_t\}_{t\geq 0}$ on the standard forms of the von Neumann algebra $\mathcal{M}$, and by the symmetric

Received August 11, 2007.
2000 *Mathematics Subject Classification.* 46L55, 37A60.
Key words and phrases. quantum Markov semigroups, diagonal operators, invariant subspaces.

embedding map i_0, the KMS-symmetric Markovian semigroup $\{S_t\}_{t\geq 0}$ on $\mathcal{M}$: $i_0 \circ S_t = T_t \circ i_0$. Concretely, for a fixed admissible function f and $x \in \mathcal{M}$, $x = x_1 + ix_2, x_k^* = x_k, k = 1, 2$ satisfying suitable conditions, they constructed the KMS-symmetric Markovian semigroup $\{S_t\}$ with the generator G on $\mathcal{M}$:

$$G(A) = \int [G_1(A,t) + G_2(A,t)]f(t)dt, \tag{1.1}$$

where

$$\begin{aligned} G_1(A,t) &= \sigma_{t+i/2}(x_1)\sigma_t(x_1)A + A\sigma_t(x_1)\sigma_{t-i/2}(x_1) \\ &\quad -\sigma_{t+i/2}(x_1)A\sigma_t(x_1) - \sigma_t(x_1)A\sigma_{t-i/2}(x_1) \end{aligned}$$

and

$$\begin{aligned} G_2(A,t) &= \sigma_{t+i/2}(x_2)\sigma_t(x_2)A + A\sigma_t(x_2)\sigma_{t-i/2}(x_2) \\ &\quad -\sigma_{t+i/2}(x_2)A\sigma_t(x_2) - \sigma_t(x_2)A\sigma_{t-i/2}(x_2). \end{aligned}$$

In this paper, we consider the faithful, normal, semifinite trace Tr on $L(\mathfrak{h})$. The Hilbert-Schmidt class $L^2(\mathfrak{h})$ is a Hilbert space with the inner product, $\langle \xi, \eta \rangle = \mathrm{Tr}(\xi^*\eta)$. π_L is the faithful, normal representation of $L(\mathfrak{h})$ given by $\pi_L(x) = L_x$, $L_x\xi = x\xi$ for $\xi \in L^2(\mathfrak{h})$. Put $\mathcal{M} = \pi_L(L(\mathfrak{h}))$.

Let $\{e_k\}_{k=0}^{\infty}$ be an orthonormal basis of $\mathfrak{h}$. Let ρ, ω and ξ_0 be a strictly positive density matrix, the corresponding normal state and the corresponding cyclic and separating vector, respectively:

$$\begin{aligned} &\rho = \sum_{k=0}^{\infty} r_k |e_k\rangle\langle e_k|, \text{ and } r_k > 0, \sum_{k=0}^{\infty} r_k = 1, \\ &\omega(L_x) = \mathrm{Tr}(\rho x) = \langle \xi_0, x\xi_0 \rangle, \\ &\xi_0 = \rho^{1/2} = \sum_{k=0}^{\infty} r_k^{1/2} |e_k\rangle\langle e_k|. \end{aligned}$$

Since π_L is a monomorphism, we identify $\mathcal{M} = \pi_L(L(\mathfrak{h}))$ with $L(\mathfrak{h})$.

The purpose of this paper is to find a sufficient and necessary condition for $x \in \mathcal{M}$ used to construct the semigroup $\{S_t\}_{t\geq 0}$ on $\mathcal{M}$ so that the semigroup acts separately on diagonal and off-diagonal operators with respect to this basis and, as an application, study the KMS-symmetric Markovian semigroup $\{S_t\}_{t\geq 0}$ which is $\rho-$invariant.

This paper is organized as follows. In Section 2, we introduce a necessary terminologies and the generator of KMS-symmetric Markovian semigroup on $\mathcal{M}$ constructed in [3, 9]. In Section 3, we give our main results and proofs.

2. The quantum Markov semigroup

In this section, we introduce some terminologies and the generator of KMS-symmetric Markovian semigroup on $\mathcal{M}$ constructed in [3, 9].

Let $\mathcal{M}$ be a σ-finite von Neumann algebra acting on a complex Hilbert space $\mathcal{H}$ with an inner product $\langle \cdot, \cdot \rangle$. Let $\xi_0 \in \mathcal{H}$ be a cyclic and separating vector for $\mathcal{M}$. We use Δ and J to denote respectively, the modular operator and the modular conjugation associated with the pair $(\mathcal{M}, \xi_0)$. The associated modular automorphism group is denoted by $\sigma_t : \sigma_t(A) = \Delta^{it} A \Delta^{-it}$, $A \in \mathcal{M}$, $t \in \mathbb{R}$. The map $j : \mathcal{M} \to \mathcal{M}'$ is the antilinear $*$-isomorphism defined by $j(A) = JAJ$, $A \in \mathcal{M}$, where $\mathcal{M}'$ is the commutant of $\mathcal{M}$.

The positive cone $\mathcal{P}$ associated with the pair $(\mathcal{M}, \xi_0)$ is the closure of the set $\{Aj(A)\xi_0 : A \in \mathcal{M}\}$. $\mathcal{P}$ can be obtained by the closure of the set $\{\Delta^{1/4} A^* A \xi_0 : A \in \mathcal{M}\}$ and is self-dual in the sense that

$$\{\xi \in \mathcal{H} : \langle \xi, \eta \rangle \geq 0, \ \forall \eta \in \mathcal{P}\} = \mathcal{P}.$$

For the details we refer [6] and Section 2.5 of [4].

The form $(\mathcal{M}, \mathcal{H}, \mathcal{P}, J)$ is the standard form associated with the pair $(\mathcal{M}, \xi_0)$. The Hilbert space $\mathcal{H}$ is the complexification of the real subspace $\mathcal{H}^J := \{\xi \in \mathcal{H} : \langle \xi, \eta \rangle \in \mathbb{R}, \forall \eta \in \mathcal{P}\}$, whose elements are called *J-real*: $\mathcal{H} = \mathcal{H}^J \oplus i\mathcal{H}^J$. Such a positive cone $\mathcal{P}$ gives a rise to a structure of ordered Hilbert space on $\mathcal{H}^J$(denoted by $\leq$) and an anti-unitary involution J on $\mathcal{H}$ by $J(\xi + i\eta) := \xi - i\eta$, $\forall \xi, \eta \in \mathcal{H}^J$. For ξ, $\eta \in \mathcal{H}^J$, $\xi \leq \eta$ means $\eta - \xi \in \mathcal{P}$. Any J-real element $\xi \in \mathcal{H}^J$ can be decomposed uniquely as a difference of two orthogonal, positive elements, called the positive and the negative part of ξ: ξ_+, $\xi_- \in \mathcal{P}$, , $\xi = \xi_+ - \xi_-$, $\langle \xi_+, \xi_- \rangle = 0$. The order interval $\{\eta \in \mathcal{H} : 0 \leq \eta \leq \xi_0\}$, denoted by $[0, \xi_0]$, is a closed convex subset of $\mathcal{H}$, and we denote the nearest point projection onto $[0, \xi_0]$ by $\eta \to \eta_I$.

Let $\mathcal{M}_0 \subset \mathcal{M}$ be the $*$-subalgebra of the σ_t-entire analytic elements [4] and $\mathcal{M}_+$ the subset of positive elements of $\mathcal{M}$. Let ω be a vector state on $\mathcal{M}$ such that $\omega(A) := \langle \xi_0, A\xi_0 \rangle$, $A \in \mathcal{M}$.

Consider the semigroup $\{S_t\}_{t \geq 0}$ of everywhere defined linear maps on $\mathcal{M}$. A semigroup $\{S_t\}_{t \geq 0}$ is said to be *KMS-symmetric* if for all $t \in \mathbb{R}$ and for all $A, B \in \mathcal{M}_0$, one has

$$\omega(S_t(A)\sigma_{-i/2}(B)) = \omega(\sigma_{i/2}(A)S_t(B)). \tag{2.1}$$

A semigroup $\{S_t\}_{t \geq 0}$ is said to be *real* if $S_t(A^*) = (S_t(A))^*$ for all $A \in \mathcal{M}$ and for all $t \geq 0$, *positive preserving* if $S_t(A) \in \mathcal{M}_+$ for all $A \in \mathcal{M}_+$ and for all $t \geq 0$, *sub-Markovian* if $0 \leq S_t(A) \leq \mathbf{1}$ for all $0 \leq A \leq \mathbf{1}$ and for all $t \geq 0$. A semigroup $\{S_t\}_{t \geq 0}$ is said to be *Markovian* if S_t is positive preserving and $S_t(\mathbf{1}) = \mathbf{1}$ for all $t \geq 0$.

Next, we consider a complex valued, closed, positive sesquilinear form on some linear manifold of $\mathcal{H}$: $\mathcal{E}(\cdot, \cdot) : D(\mathcal{E}) \times D(\mathcal{E}) \to \mathbb{C}$ satisfying $\mathcal{E}(\xi, \xi) \geq 0$ for all $\xi \in D(\mathcal{E})$, and also the associated quadratic form: $\mathcal{E}[\cdot] : D(\mathcal{E}) \to \mathbb{C}$, $\mathcal{E}[\xi] := \mathcal{E}(\xi, \xi)$. A quadratic form $(\mathcal{E}, D(\mathcal{E}))$ is said to be *J-real* if $JD(\mathcal{E}) \subset D(\mathcal{E})$ and $\mathcal{E}[J\xi] = \mathcal{E}[\xi]$ for any $\xi \in D(\mathcal{E})$, equivalently, $\mathcal{E}(J\xi, J\eta) = \mathcal{E}(\eta, \xi)$ for all $\xi, \eta \in D(\mathcal{E})$.

A J-real, real-valued, densely defined quadratic form $(\mathcal{E}, D(\mathcal{E}))$ is called *Markovian* (with respect to ξ_0) if

$$\xi \in D(\mathcal{E}) \cap \mathcal{H}^J \ \Rightarrow \ \xi_I \in D(\mathcal{E}), \quad \mathcal{E}[\xi_I] \leq \mathcal{E}[\xi].$$

A closed Markovian form is called a *Dirichlet form*.

For a positive closed form $(\mathcal{E}, D(\mathcal{E}))$, there exists a positive self-adjoint operator $(H, D(H))$ such that

$$\mathcal{E}(\xi,\eta) = \langle \xi, H\eta \rangle, \ \xi, \, \eta \in D(\mathcal{E}),$$

and a strongly continuous, symmetric semigroup $\{T_t\}_{t\geq 0}, T_t = e^{-tH}$. Moreover, when $(\mathcal{E}, D(\mathcal{E}))$ is a Dirichlet form, H is called a Dirichlet operator. For the details, see Section 3.1 of [4].

In [9], the author has used the notion of admissible function to construct Dirichlet forms.

Definition 2.1. An analytic function $f : D \to \mathbb{C}$ on a domain D containing the strip Im $z \in [-1/4, 1/4]$ is said to be *admissible* if the following properties hold:

(a) $f(t) \geq 0$ for all $t \in \mathbb{R}$,
(b) $f(t + i/4) + f(t - i/4) \geq 0$ for all $t \in \mathbb{R}$,
(c) there exist $M > 0$ and $p > 1$ such that the bound

$$|f(t + is)| \leq M(1 + |t|)^{-p}$$

holds uniformly in $s \in [-1/4, 1/4]$.

The function $g(t)$ given by

$$g(t) = \frac{2}{\sqrt{2\pi}} \int (e^{k/4} + e^{-k/4})^{-1} e^{-\frac{1}{2}k^2} e^{-ikt} \, dk \tag{2.2}$$

is admissible. See the proof of Lemma 3.1 of [9].

Using a fixed admissible function f and $x \in \mathcal{M}$ satisfying the condition $\sup_{s\in[-1/4,1/4]} ||\, \sigma_{t+is}(x) \,|| \leq M$ for some $M > 0$ uniformly $t \in \mathbb{R}$, the following (bounded) Dirichlet form $(\mathcal{E}, \mathcal{H})$ was constructed in [3, 9]:

$$\begin{aligned} \mathcal{E}(\eta,\xi) \;\; = \;\; & \int \big\langle \big[\sigma_{t-i/4}(x) - j\,(\sigma_{t-i/4}(x^*))\big]\eta, \\ & \qquad \big[\sigma_{t-i/4}(x) - j\,(\sigma_{t-i/4}(x^*))\big]\xi \big\rangle f(t)dt \\ & + \int \big\langle \big[\sigma_{t-i/4}(x^*) - j\,(\sigma_{t-i/4}(x))\big]\eta, \\ & \qquad \big[\sigma_{t-i/4}(x^*) - j\,(\sigma_{t-i/4}(x))\big]\xi \big\rangle f(t)dt. \end{aligned} \tag{2.3}$$

Putting $x = \frac{1}{\sqrt{2}}(x_1 + ix_2)$, $x_1 = x_1^*, x_2 = x_2^* \in \mathcal{M}$, the above form $(\mathcal{E}, \mathcal{H})$ can be rewritten as

$$\begin{aligned} \mathcal{E}(\eta,\xi) \;\; = \;\; & \int \big\langle \big[\sigma_{t-i/4}(x_1) - j\,(\sigma_{t-i/4}(x_1))\big]\eta, \\ & \qquad \big[\sigma_{t-i/4}(x_1) - j\,(\sigma_{t-i/4}(x_1))\big]\xi \big\rangle f(t)dt \\ & + \int \big\langle \big[\sigma_{t-i/4}(x_2) - j\,(\sigma_{t-i/4}(x_2))\big]\eta, \\ & \qquad \big[\sigma_{t-i/4}(x_2) - j\,(\sigma_{t-i/4}(x_2))\big]\xi \big\rangle f(t)dt \end{aligned} \tag{2.4}$$

for any $\eta, \xi \in \mathcal{H}$. Thus the (bounded) Dirichlet operator H associated with $(\mathcal{E}, \mathcal{H})$ is

$$\begin{aligned} H &= \int [\sigma_{t+i/4}(x_1) - j\,(\sigma_{t+i/4}(x_1))][\sigma_{t-i/4}(x_1) - j\,(\sigma_{t-i/4}(x_1))]f(t)dt \\ &\quad + \int [\sigma_{t+i/4}(x_2) - j\,(\sigma_{t+i/4}(x_2))][\sigma_{t-i/4}(x_2) - j\,(\sigma_{t-i/4}(x_2))]f(t)dt. \end{aligned}$$

Notice that $j(B) = JBJ \in \mathcal{M}'$ and $JB\xi_0 = \Delta^{1/2}B^*\xi_0$ for any $B \in \mathcal{M}$. Using the symmetric embedding map, define the operator G on $\mathcal{M}$ given by

$$\Delta^{1/4}G(A)\xi_0 = H\Delta^{1/4}A\xi_0,\ A \in \mathcal{M},$$

and the semigroup $S_t = e^{-tG}$ on $\mathcal{M}$. Then

$$\begin{aligned} G(A) &= \int G(A,t)f(t)dt \\ &\equiv \int [G_1(A,t) + G_2(A,t)]f(t)dt, \end{aligned} \tag{2.5}$$

where

$$\begin{aligned} G_1(A,t) &= \sigma_{t+i/2}(x_1)\sigma_t(x_1)A + A\sigma_t(x_1)\sigma_{t-i/2}(x_1) \\ &\quad -\sigma_{t+i/2}(x_1)A\sigma_t(x_1) - \sigma_t(x_1)A\sigma_{t-i/2}(x_1) \end{aligned}$$

and

$$\begin{aligned} G_2(A,t) &= \sigma_{t+i/2}(x_2)\sigma_t(x_2)A + A\sigma_t(x_2)\sigma_{t-i/2}(x_2) \\ &\quad -\sigma_{t+i/2}(x_2)A\sigma_t(x_2) - \sigma_t(x_2)A\sigma_{t-i/2}(x_2). \end{aligned}$$

The operator G is a generator of the KMS-symmetric Markovian semigroup $\{S_t\}$ associated to $x \in \mathcal{M}$. See Remark 2.1 of [3] and Theorem 2.12 of [6].

3. Invariant subspaces

In this section we give the sufficient and necessary conditions for x used to construct the semigroup $\{S_t\}$ in Section 2 so that the semigroup acts separately on diagonal and off-diagonal operators with respect to a basis, and study some results.

Let $\mathfrak{h}$ be a complex separable Hilbert space and let $L(\mathfrak{h})$ be the von Neumann algebra of all bounded operators on $\mathfrak{h}$. The faithful, normal, semifinite trace on $L(\mathfrak{h})$ is denoted by Tr. The Hilbert-Schmidt class $L^2(\mathfrak{h})$ is a Hilbert space with the inner product $\langle \xi, \eta \rangle = \mathrm{Tr}(\xi^*\eta)$. Consider the faithful, normal representation π_L of $L(\mathfrak{h})$ given by

$$\pi_L : L(\mathfrak{h}) \longrightarrow \mathcal{L}(L^2(\mathfrak{h})),\ \pi_L(x) = L_x,$$

where L_x is the left multiplication operator, $\xi \mapsto x\xi$. Put $\mathcal{M}(= \mathcal{L}^\infty) = \pi_L(L(\mathfrak{h}))$.

The closed convex cone $L^2_+(\mathfrak{h})$ in $L^2(\mathfrak{h})$ consisting of nonnegative Hilbert-Schmidt operators is a self-dual cone in the sense that

$$L^2_+(\mathfrak{h}) = \{\xi \in L^2(\mathfrak{h}) : \langle \xi, \eta\rangle \geq 0 \ \forall \eta \in L^2_+(\mathfrak{h})\}.$$

The associated antiunitary conjugation J on $L^2(\mathfrak{h})$ is simply the adjoint operator on $L^2(\mathfrak{h})$: $J\xi = \xi^*$. Then $(\mathcal{M}, L^2(\mathfrak{h}), L^2_+(\mathfrak{h}), *)$ is a standard form for $L(\mathfrak{h})$.

The following notation remains highly convenient. For vectors $e, f \in \mathfrak{h}$, let $|e\rangle\langle f|$ denote the operator on $\mathfrak{h}$ given by $|e\rangle\langle f|v = \langle f, v\rangle e$. Thus $|e\rangle\langle f|$ is one rank operator on $\mathfrak{h}$ and so $|e\rangle\langle f| \in L(\mathfrak{h})$.

Now let $\{e_k\}_{k=0}^\infty$ be an orthonormal basis of $\mathfrak{h}$. Let ρ, ω and ξ_0 be a strictly positive density matrix, the corresponding normal state and the corresponding cyclic and separating vector, respectively:

$$\begin{aligned}
&\rho = \sum_{k=0}^\infty r_k |e_k\rangle\langle e_k|, \ r_k > 0 \ \text{ and } \ \sum_{k=0}^\infty r_k = 1,\\
&\omega(L_x) = \mathrm{Tr}(\rho x) = \langle \xi_0, x\xi_0\rangle,\\
&\xi_0 = \rho^{1/2} = \sum_{k=0}^\infty r_k^{1/2} |e_k\rangle\langle e_k|.
\end{aligned}$$

The action of the associated modular operator and the modular group are given by

$$\begin{aligned}
\Delta^{1/2} L_x \xi_0 &= \rho^{1/2} x, \ x \in L(\mathfrak{h}),\\
\sigma_t(L_x) &= L_{\rho^{it} x \rho^{-it}}.
\end{aligned}$$

Since π_L is a monomorphism, we identify $\mathcal{M} = \mathcal{L}^\infty$ with $L(\mathfrak{h})$. We will write the above second term as $\sigma_t(x) = \rho^{it} x \rho^{-it}$.

Consider a fixed element $x \in \mathcal{M}$ such that

$$x = \sum_{l,m\geq 0} x_{lm} |e_m\rangle\langle e_l|, \ x_{lm} \in \mathbb{C}, \tag{3.1}$$

and let

$$\begin{aligned}
x_1 &= \frac{1}{\sqrt{2}}(x + x^*) = \sum_{l,m\geq 0} \frac{x_{lm} + \overline{x_{ml}}}{\sqrt{2}} |e_m\rangle\langle e_l| \equiv \sum_{l,m\geq 0} y_{lm} |e_m\rangle\langle e_l|,\\
x_2 &= \frac{1}{\sqrt{2}\, i}(x - x^*) = \sum_{l,m\geq 0} \frac{x_{lm} - \overline{x_{ml}}}{\sqrt{2}\, i} |e_m\rangle\langle e_l| \equiv \sum_{l,m\geq 0} z_{lm} |e_m\rangle\langle e_l|.
\end{aligned}$$

Then $x_1^* = x_1$, $x_2^* = x_2$ and $x = x_1 + ix_2$.

Assume that

$$\sup_{0\leq s\leq 1/2} \sum_{l,m\geq 0} r_m^{-s} r_l^s |x_{lm}| < \infty. \tag{3.2}$$

In case $\sup_l \mathrm{Card}(\{m | x_{lm} \neq 0\}) < \infty$, the assumption (3.2) is reduced to the condition

$$\sup_{0\le s\le 1/2} \sum_{l\ge 0} r_l^s |x_{lm}| < \infty.$$

It follows from (3.2) that for all $t \in \mathbb{R}$ and $s \in [-1/2, 1/2]$, $\sigma_{t+is}(x)$ is bounded and

$$\begin{aligned}\sigma_{t+is}(x) &= \rho^{it-s} x \rho^{-it+s} \\ &= \sum_{l,m\ge 0} r_m^{it-s} r_l^{-it+s} x_{lm} |e_m\rangle\langle e_l|. \end{aligned} \tag{3.3}$$

Lemma 3.1. *For each $A = |e_k\rangle\langle e_n|$, we have the following relations: for all $t \in \mathbb{R}$,*

$$\begin{aligned}
\sigma_{t+i/2}(x_1)\sigma_t(x_1)A &= \sum_{l,m\ge 0} r_m^{it-1/2} r_l^{1/2} r_k^{-it} y_{lm} y_{kl} |e_m\rangle\langle e_n|, \\
A\sigma_t(x_1)\sigma_{t-i/2}(x_1) &= \sum_{l,m\ge 0} r_n^{it} r_m^{1/2} r_l^{-it-1/2} y_{mn} y_{lm} |e_k\rangle\langle e_l|, \\
\sigma_{t+i/2}(x_1)A\sigma_t(x_1) &= \sum_{l,m\ge 0} r_m^{it-1/2} r_k^{-it+1/2} r_n^{it} r_l^{-it} y_{km} y_{ln} |e_m\rangle\langle e_l|, \\
\sigma_t(x_1)A\sigma_{t-i/2}(x_1) &= \sum_{l,m\ge 0} r_m^{it} r_k^{-it} r_n^{it+1/2} r_l^{-it-1/2} y_{km} y_{ln} |e_m\rangle\langle e_l|
\end{aligned}$$

and

$$\begin{aligned}
\sigma_{t+i/2}(x_2)\sigma_t(x_2)A &= \sum_{l,m\ge 0} r_m^{it-1/2} r_l^{1/2} r_k^{-it} z_{lm} z_{kl} |e_m\rangle\langle e_n|, \\
A\sigma_t(x_2)\sigma_{t-i/2}(x_2) &= \sum_{l,m\ge 0} r_n^{it} r_m^{1/2} r_l^{-it-1/2} z_{mn} z_{lm} |e_k\rangle\langle e_l|, \\
\sigma_{t+i/2}(x_2)A\sigma_t(x_2) &= \sum_{l,m\ge 0} r_m^{it-1/2} r_k^{-it+1/2} r_n^{it} r_l^{-it} z_{km} z_{ln} |e_m\rangle\langle e_l|, \\
\sigma_t(x_2)A\sigma_{t-i/2}(x_2) &= \sum_{l,m\ge 0} r_m^{it} r_k^{-it} r_n^{it+1/2} r_l^{-it-1/2} z_{km} z_{ln} |e_m\rangle\langle e_l|.
\end{aligned}$$

Proof. This is clear from (3.3). □

Applying Lemma 3.1 to (2.5), we have the concrete action of the generator G:

$$G(A) = \int G(A,t) f(t) dt, \quad A = |e_k\rangle\langle e_n|, \tag{3.4}$$

where

$$\begin{aligned}
G(A,t) &= \sum_{l,m\geq 0} r_m^{it} r_k^{-it} \sqrt{\frac{r_l}{r_m}} (y_{lm}y_{kl} + z_{lm}z_{kl})|e_m\rangle\langle e_n| \\
&\quad + \sum_{l,m\geq 0} r_n^{it} r_l^{-it} \sqrt{\frac{r_m}{r_l}} (y_{mn}y_{lm} + z_{mn}z_{lm})|e_k\rangle\langle e_l| \\
&\quad - \sum_{l,m\geq 0} r_m^{it} r_k^{-it} r_n^{it} r_l^{-it} \sqrt{\frac{r_k}{r_m}} (y_{km}y_{ln} + z_{km}z_{ln})|e_m\rangle\langle e_l| \\
&\quad - \sum_{l,m\geq 0} r_m^{it} r_k^{-it} r_n^{it} r_l^{-it} \sqrt{\frac{r_n}{r_l}} (y_{km}y_{ln} + z_{km}z_{ln})|e_m\rangle\langle e_l| \\
&= \sum_{l,m\geq 0} r_m^{it} r_k^{-it} \sqrt{\frac{r_l}{r_m}} (x_{lm}\overline{x_{lk}} + \overline{x_{ml}}x_{kl})|e_m\rangle\langle e_n| \\
&\quad + \sum_{l,m\geq 0} r_n^{it} r_l^{-it} \sqrt{\frac{r_m}{r_l}} (x_{mn}\overline{x_{ml}} + \overline{x_{nm}}x_{lm})|e_k\rangle\langle e_l| \\
&\quad - \sum_{l,m\geq 0} r_m^{it} r_k^{-it} r_n^{it} r_l^{-it} \Big(\sqrt{\frac{r_k}{r_m}} + \sqrt{\frac{r_n}{r_l}}\Big) (x_{km}\overline{x_{nl}} + \overline{x_{mk}}x_{ln})|e_m\rangle\langle e_l|.
\end{aligned}$$

Now we fix the admissible function f given by

$$f(t) = \frac{1}{\sqrt{2\pi}} g = \frac{1}{\pi} \int (e^{k/4} + e^{-k/4})^{-1} e^{-\frac{1}{2}k^2} e^{-ikt}\, dk,$$

where g is defined as in (2.2). Notice that g is the Fourier transform of $h(k) = 2(e^{k/4} + e^{-k/4})^{-1} e^{-\frac{1}{2}k^2}$. Thus

$$\begin{aligned}
\int f(t) e^{ipt}\, dt &= 2(e^{p/4} + e^{-p/4})^{-1} e^{-\frac{1}{2}p^2} \\
&= \operatorname{sech}(\frac{p}{4}) e^{-\frac{1}{2}p^2}
\end{aligned} \tag{3.5}$$

for any $p \in \mathbb{R}$. Applying (3.5) to (3.4), the generator G can be written as, for $A = |e_k\rangle\langle e_n|$,

$$\begin{aligned}
G(A) &= \sum_{l,m\geq 0} \operatorname{sech}(\frac{1}{4} \ln \frac{r_m}{r_k}) e^{-\frac{1}{2}(\ln \frac{r_m}{r_k})^2} \sqrt{\frac{r_l}{r_m}} (x_{lm}\overline{x_{lk}} + \overline{x_{ml}}x_{kl})|e_m\rangle\langle e_n| \\
&\quad + \sum_{l,m\geq 0} \operatorname{sech}(\frac{1}{4} \ln \frac{r_n}{r_l}) e^{-\frac{1}{2}(\ln \frac{r_n}{r_l})^2} \sqrt{\frac{r_m}{r_l}} (x_{mn}\overline{x_{ml}} + \overline{x_{nm}}x_{lm})|e_k\rangle\langle e_l| \\
&\quad - \sum_{l,m\geq 0} \operatorname{sech}(\frac{1}{4} \ln \frac{r_m r_n}{r_k r_l}) e^{-\frac{1}{2}(\ln \frac{r_m r_n}{r_k r_l})^2} \Big(\sqrt{\frac{r_k}{r_m}} + \sqrt{\frac{r_n}{r_l}}\Big) \\
&\qquad \times (x_{km}\overline{x_{nl}} + \overline{x_{mk}}x_{ln})|e_m\rangle\langle e_l|.
\end{aligned} \tag{3.6}$$

Consider two conditions for the sequence $\{x_{lm}\}_{l,m\geq 0}$ of the complex numbers:

$$x_{km}\overline{x_{kl}} + \overline{x_{mk}}x_{lk} = 0, \;\; l \neq m, l \neq k, \, m \neq k, \tag{3.7}$$

$$x_{mm}\overline{x_{mk}} + \overline{x_{mm}}x_{km} = x_{km}\overline{x_{kk}} + \overline{x_{mk}}x_{kk}, \;\; m \neq k \tag{3.8}$$

for all k.

Lemma 3.2. *Let the sequence $\{x_{lm}\}_{l,m\geq 0}$ of the complex numbers satisfy the above condition* (3.7)*. Then for each k there exist at most three nonzero elements $x_{kl_1}, x_{kl_2}, x_{kk}$, $l_1, l_2 \neq k$ of $\{x_{km}\}_{m\geq 0}$ and at most nonzero three elements $x_{l_3k}, x_{l_4k}, x_{kk}$, $l_3, l_4 \neq k$ of $\{x_{mk}\}_{m\geq 0}$.*

Proof. Suppose that there exist three nonzero elements $x_{kl_1}, x_{kl_2}, x_{kl_3}$ such that $l_1, l_2, l_3 (\neq k)$ are mutually different. It follows from (3.7) that we have x_{l_1k}, x_{l_2k}, $x_{l_3k} \neq 0$ and

$$\begin{aligned} x_{kl_1}\overline{x_{kl_2}} + \overline{x_{l_1k}}x_{l_2k} &= 0, \\ x_{kl_1}\overline{x_{kl_3}} + \overline{x_{l_1k}}x_{l_3k} &= 0, \\ x_{kl_2}\overline{x_{kl_3}} + \overline{x_{l_2k}}x_{l_3k} &= 0. \end{aligned}$$

Then $\frac{x_{kl_1}}{\overline{x_{l_1k}}} = -\frac{x_{l_2k}}{\overline{x_{kl_2}}} = -\frac{x_{l_3k}}{\overline{x_{kl_3}}}$, $\frac{\overline{x_{l_2k}}}{x_{kl_2}} = -\frac{\overline{x_{kl_3}}}{x_{l_3k}}$, which implies $\frac{x_{l_3k}}{\overline{x_{kl_3}}} = -\frac{x_{kl_3}}{\overline{x_{l_3k}}}$, that is, $|x_{l_3k}|^2 = -|x_{kl_3}|^2$ and $x_{l_3k} = x_{kl_3} = 0$. This is a contradiction. There does not exist nonzero elements $x_{kl_1}, x_{kl_2}, x_{kl_3}$ such that $l_1, l_2, l_3 (\neq k)$ are mutually different.

The other part will be proved similarly. $\square$

By Lemma 3.2, rearranging the orthonormal basis $\{e_k\}_{k\geq 0}$, the operator x satisfying (3.7) can be expressed as the following matrix form:

$$x = \begin{pmatrix} x_{00} & x_{01} & x_{02} & 0 & 0 & 0 & 0 & \cdots \\ x_{10} & x_{11} & 0 & x_{13} & 0 & 0 & 0 & \cdots \\ x_{20} & 0 & x_{22} & 0 & x_{24} & 0 & 0 & \cdots \\ 0 & x_{31} & 0 & x_{33} & 0 & x_{35} & 0 & \cdots \\ 0 & 0 & x_{42} & 0 & x_{44} & 0 & x_{46} & \cdots \\ 0 & 0 & 0 & x_{53} & 0 & x_{55} & 0 & \cdots \\ 0 & 0 & 0 & 0 & x_{64} & 0 & x_{66} & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots \end{pmatrix}.$$

We call the operator of $\mathcal{M}$ of the form $\sum_{k\geq 0} w_k |e_k\rangle\langle e_k|$, $w_k \in \mathbb{C}$ the diagonal operator and $x \in \mathcal{M}$ satisfying $\mathrm{Tr}(x|e_n\rangle\langle e_n|) = 0$ for all n the off-diagonal operator. Let $\mathcal{M}_d$ be the (diagonal) subalgebra consisting of the diagonal operators and $\mathcal{M}_{od}$ the space of off-diagonal elements. Every operator in $\mathcal{M}$ is expressed as the sum of the diagonal operator and off-diagonal operator.

Theorem 3.3. *Let $x = \sum_{l,m\geq 0} x_{lm}|e_m\rangle\langle e_l|$ satisfy the assumption* (3.2) *and $\{S_t\}_{t\geq 0}$ be the semigroup with generator G in* (3.6) (*and* (3.4))*. x satisfies two* *conditions* (3.7) *and* (3.8) *if and only if* $\mathcal{M}_d$ *and* $\mathcal{M}_{od}$ *are* S_t*-invariant for all* $t \geq 0$*, that is,* $S_t(\mathcal{M}_d) \subset \mathcal{M}_d$ *and* $S_t(\mathcal{M}_{od}) \subset \mathcal{M}_{od}$.

Proof. For $A = |e_k\rangle\langle e_k| \in \mathcal{M}_d$, we get from (3.6) that

$$\begin{aligned} G(A) &= \sum_{l,m\geq 0} \operatorname{sech}(\frac{1}{4}\ln\frac{r_m}{r_k})e^{-\frac{1}{2}(\ln\frac{r_m}{r_k})^2}\sqrt{\frac{r_l}{r_m}}(x_{lm}\overline{x_{lk}} + \overline{x_{ml}}x_{kl})|e_m\rangle\langle e_k| \\ &\quad + \sum_{l,m\geq 0} \operatorname{sech}(\frac{1}{4}\ln\frac{r_k}{r_l})e^{-\frac{1}{2}(\ln\frac{r_k}{r_l})^2}\sqrt{\frac{r_m}{r_l}}(x_{mk}\overline{x_{ml}} + \overline{x_{km}}x_{lm})|e_k\rangle\langle e_l| \\ &\quad - \sum_{l,m\geq 0} \operatorname{sech}(\frac{1}{4}\ln\frac{r_m}{r_l})e^{-\frac{1}{2}(\ln\frac{r_m}{r_l})^2}\Big(\sqrt{\frac{r_k}{r_m}} + \sqrt{\frac{r_k}{r_l}}\Big) \\ &\qquad \times(x_{km}\overline{x_{kl}} + \overline{x_{mk}}x_{lk})|e_m\rangle\langle e_l|. \end{aligned} \tag{3.9}$$

We rewrite in detail as

$$\begin{aligned} & G(A) \\ = & \sum_{l\geq 0}\sqrt{\frac{r_l}{r_k}}(|x_{lk}|^2 + |x_{kl}|^2)|e_k\rangle\langle e_k| \\ & + \sum_{\substack{m\neq k\\ m\geq 0}} \operatorname{sech}(\frac{1}{4}\ln\frac{r_m}{r_k})e^{-\frac{1}{2}(\ln\frac{r_m}{r_k})^2}\sum_{l\geq 0}\sqrt{\frac{r_l}{r_m}}(x_{lm}\overline{x_{lk}} + \overline{x_{ml}}x_{kl})|e_m\rangle\langle e_k| \\ & + \sum_{m\geq 0}\sqrt{\frac{r_m}{r_k}}(|x_{mk}|^2 + |x_{km}|^2)|e_k\rangle\langle e_k| \\ & + \sum_{\substack{l\neq k\\ l\geq 0}} \operatorname{sech}(\frac{1}{4}\ln\frac{r_k}{r_l})e^{-\frac{1}{2}(\ln\frac{r_k}{r_l})^2}\sum_{m\geq 0}\sqrt{\frac{r_m}{r_l}}(x_{mk}\overline{x_{ml}} + \overline{x_{km}}x_{lm})|e_k\rangle\langle e_l| \\ & -2\sum_{l\geq 0}\sqrt{\frac{r_k}{r_l}}(|x_{kl}|^2 + |x_{lk}|^2)|e_l\rangle\langle e_l| \\ & - \sum_{\substack{m\neq k\\ m\geq 0}} \operatorname{sech}(\frac{1}{4}\ln\frac{r_m}{r_k})e^{-\frac{1}{2}(\ln\frac{r_m}{r_k})^2}\Big(\sqrt{\frac{r_k}{r_m}} + 1\Big)(x_{km}\overline{x_{kk}} + \overline{x_{mk}}x_{kk})|e_m\rangle\langle e_k| \\ & - \sum_{\substack{l\neq k\\ l\geq 0}} \operatorname{sech}(\frac{1}{4}\ln\frac{r_k}{r_l})e^{-\frac{1}{2}(\ln\frac{r_k}{r_l})^2}\Big(1 + \sqrt{\frac{r_k}{r_l}}\Big)(x_{kk}\overline{x_{kl}} + \overline{x_{kk}}x_{lk})|e_k\rangle\langle e_l| \\ & - \sum_{\substack{l\neq m\\ l,m\neq k\\ l,m\geq 0}} \operatorname{sech}(\frac{1}{4}\ln\frac{r_m}{r_l})e^{-\frac{1}{2}(\ln\frac{r_m}{r_l})^2}\Big(\sqrt{\frac{r_k}{r_m}} + \sqrt{\frac{r_k}{r_l}}\Big) \\ & \times(x_{km}\overline{x_{kl}} + \overline{x_{mk}}x_{lk})|e_m\rangle\langle e_l|. \end{aligned} \tag{3.10}$$

For $B = |e_k\rangle\langle e_n|,\ k \neq n$ in $\mathcal{M}_{od}$, we also get

$$
\begin{aligned}
& G(B) \\
= \ & \mathrm{sech}(\frac{1}{4}\ln\frac{r_n}{r_k})e^{-\frac{1}{2}(\ln\frac{r_n}{r_k})^2}\sum_{l\geq 0}\sqrt{\frac{r_l}{r_n}}(x_{ln}\overline{x_{lk}}+\overline{x_{nl}}x_{kl})|e_n\rangle\langle e_n| \\
& +\sum_{\substack{m\neq n\\ m\geq 0}}\mathrm{sech}(\frac{1}{4}\ln\frac{r_m}{r_k})e^{-\frac{1}{2}(\ln\frac{r_m}{r_k})^2}\sum_{l\geq 0}\sqrt{\frac{r_l}{r_m}}(x_{lm}\overline{x_{lk}}+\overline{x_{ml}}x_{kl})|e_m\rangle\langle e_n| \\
& +\mathrm{sech}(\frac{1}{4}\ln\frac{r_n}{r_k})e^{-\frac{1}{2}(\ln\frac{r_n}{r_k})^2}\sum_{m\geq 0}\sqrt{\frac{r_m}{r_k}}(x_{mn}\overline{x_{mk}}+\overline{x_{nm}}x_{km})|e_k\rangle\langle e_k| \\
& +\sum_{\substack{l\neq k\\ l\geq 0}}\mathrm{sech}(\frac{1}{4}\ln\frac{r_n}{r_k})e^{-\frac{1}{2}(\ln\frac{r_n}{r_k})^2}\sum_{m\geq 0}\sqrt{\frac{r_m}{r_l}}(x_{mn}\overline{x_{ml}}+\overline{x_{nm}}x_{lm})|e_k\rangle\langle e_l| \\
& -\mathrm{sech}(\frac{1}{4}\ln\frac{r_n}{r_k})e^{-\frac{1}{2}(\ln\frac{r_n}{r_k})^2}\sum_{l\geq 0}\Big(\sqrt{\frac{r_k}{r_l}}+\sqrt{\frac{r_n}{r_l}}\Big)(x_{kl}\overline{x_{nl}}+\overline{x_{lk}}x_{ln})|e_l\rangle\langle e_l| \\
& -\sum_{\substack{l\neq m\\ m\geq 0}}\mathrm{sech}(\frac{1}{4}\ln\frac{r_m r_n}{r_k r_l})e^{-\frac{1}{2}(\ln\frac{r_m r_n}{r_k r_l})^2}\Big(\sqrt{\frac{r_k}{r_m}}+\sqrt{\frac{r_n}{r_l}}\Big) \\
& \times(x_{km}\overline{x_{nl}}+\overline{x_{mk}}x_{ln})|e_m\rangle\langle e_l|. \qquad (3.11)
\end{aligned}
$$

By (3.10) and (3.11), $G(A) \in \mathcal{M}_d$ and $G(B) \in \mathcal{M}_{od}$ if and only if the following relations hold:

$$
\begin{aligned}
x_{km}\overline{x_{kl}}+\overline{x_{mk}}x_{lk} &= 0,\ \ l\neq m,\, l\neq k,\, m\neq k, \\
\sum_{l\geq 0}\sqrt{\frac{r_l}{r_m}}(x_{lm}\overline{x_{lk}}+\overline{x_{ml}}x_{kl}) &= \Big(\sqrt{\frac{r_k}{r_m}}+1\Big)(x_{km}\overline{x_{kk}}+\overline{x_{mk}}x_{kk}),\ \ m\neq k, \\
\sum_{m\geq 0}\sqrt{\frac{r_m}{r_l}}(x_{mk}\overline{x_{ml}}+\overline{x_{km}}x_{lm}) &= \Big(1+\sqrt{\frac{r_k}{r_l}}\Big)(x_{kk}\overline{x_{kl}}+\overline{x_{kk}}x_{lk}),\ \ l\neq k
\end{aligned}
$$

for all k. The above relations are reduced to (3.7) and (3.8).

In this case we have

$$
\begin{aligned}
G(A) &= 2\sum_{l\geq 0}\sqrt{\frac{r_l}{r_k}}(|x_{lk}|^2+|x_{kl}|^2)|e_k\rangle\langle e_k| \\
&\quad -2\sum_{l\geq 0}\sqrt{\frac{r_k}{r_l}}(|x_{kl}|^2+|x_{lk}|^2)|e_l\rangle\langle e_l|,
\end{aligned} \qquad (3.12)
$$

and

$$
\begin{aligned}
& G(B) \\
= & \sum_{\substack{m\neq n\\ m\geq 0}} \operatorname{sech}(\frac{1}{4}\ln\frac{r_m}{r_k})e^{-\frac{1}{2}(\ln\frac{r_m}{r_k})^2}\sum_{l\geq 0}\sqrt{\frac{r_l}{r_m}}(x_{lm}\overline{x_{lk}}+\overline{x_{ml}}x_{kl})|e_m\rangle\langle e_n| \\
& +\sum_{\substack{l\neq k\\ l\geq 0}} \operatorname{sech}(\frac{1}{4}\ln\frac{r_n}{r_k})e^{-\frac{1}{2}(\ln\frac{r_n}{r_k})^2}\sum_{m\geq 0}\sqrt{\frac{r_m}{r_l}}(x_{mn}\overline{x_{ml}}+\overline{x_{nm}}x_{lm})|e_k\rangle\langle e_l| \\
& -\sum_{\substack{l\neq m\\ m\geq 0}} \operatorname{sech}(\frac{1}{4}\ln\frac{r_m r_n}{r_k r_l})e^{-\frac{1}{2}(\ln\frac{r_m r_n}{r_k r_l})^2}\Big(\sqrt{\frac{r_k}{r_m}}+\sqrt{\frac{r_n}{r_l}}\Big) \\
& \quad\times(x_{km}\overline{x_{nl}}+\overline{x_{mk}}x_{ln})|e_m\rangle\langle e_l|,
\end{aligned}
$$

which implies $S_t(\mathcal{M}_d)\subset\mathcal{M}_d$, $S_t(\mathcal{M}_{od})\subset\mathcal{M}_{od}$. The proof is completed. □

Remark 3.4. Since $\sigma_t(A)=A$ for all $A\in\mathcal{M}_d$ and $t\in\mathbb{R}$ there exists a projection (σ_t-compatible conditional expectation) $P_{\mathcal{M}_d}$ of $\mathcal{M}$ onto $\mathcal{M}_d$ (Proposition 2.6.6 of [11]). The subalgebra $\mathcal{M}_d$ and the space $\mathcal{M}_{od}$ are invariant for the semigroup $\{S_t\}_{t\geq 0}$ in Theorem 3.3. So the restriction $S_t^d := S_t|_{\mathcal{M}_d}$ of S_t on $\mathcal{M}_d$ is also a semigroup and

$$S_t^d(P_{\mathcal{M}_d}(A)) = P_{\mathcal{M}_d}(S_t(A))$$

for all $A\in\mathcal{M}$.

Notice that the semigroup $\{S_t\}_{\geq 0}$ is a KMS-symmetric semigroup on $\mathcal{M}$ (see (2.1)) and $S_t(\mathbf{1})=\mathbf{1}$ for all $t\geq 0$, and so $\{S_t\}_{t\geq 0}$ is ρ-invariant in the sense that

$$\operatorname{Tr}(\rho S_t(A)) = \operatorname{Tr}(\rho A),\ \ A\in\mathcal{M}.$$

Theorem 3.5. *Let $\{S_t\}_{t\geq 0}$ be the semigroup with generator G in* (3.6) *associate to $x\in\mathcal{M}_d$ as in* (3.1).

(a) $S_t(A)=A$ for all $A\in\mathcal{M}_d$ and $t\geq 0$.

(b) A density matrix $\tilde\rho=\sum_{k\geq 0}\tilde{r_k}|e_k\rangle\langle e_k|$ is an invariant state for $\{S_t\}_{\geq 0}$, that is, Tr$(\tilde\rho S_t(A))$ = Tr$(\tilde\rho A)$ for all $A\in\mathcal{M}$ and all $t\geq 0$.

Proof. (a) By (3.9), we have $G(A)=0$, which is equivalent to $S_t(A)=A$ for all $A\in\mathcal{M}_d$.

(b) Let $A\in\mathcal{M}$. Then $A\in\mathcal{M}$ can be written as the sum $A=A_d+A_{od}$ with $A_d\in\mathcal{M}_d$ and $A_{od}\in\mathcal{M}_{od}$. Since $x\in\mathcal{M}_d$, we get from (a) that $S_t(A_d)=A_d$ for all $t\geq 0$. By $S_t(A_{od})\in\mathcal{M}_{od}$ for all $t\geq 0$, we have $\operatorname{Tr}(\tilde\rho S_t(A_{od}))=0$ and so

$$\operatorname{Tr}(\tilde\rho S_t(A)) = \operatorname{Tr}(\tilde\rho S_t(A_d)) = \operatorname{Tr}(\tilde\rho A_d) = \operatorname{Tr}(\tilde\rho A)$$

for all $t\geq 0$. The proof is completed. □

References

[1] L. Accardi, F. Fagnola, and S. Hachicha, *Generic q-Markov semigroups and speed of convergence of q-algorithms*, Inf. Dim. Anal. Quantum Probab. Related Topics **9** (2006), 567–594.

[2] S. Albeverio and R. Høegh-Krohn, *Dirichlet forms and Markovian semigroups on C^*-algebras*, Comm. Math. Phys. **56** (1977), 173–187.

[3] C. Bahn, C. K. Ko, and Y. M. Park, *Dirichlet forms and symmetric Markovian semigroups on CCR algebras with quasi-free states*, J. Math. Phys. **44** (2003), 723–753.

[4] O. Bratteli and D. W. Robinson, *Operator algebras and quantum statistical mechanics*, Springer-Verlag, New York-Heidelberg-Berlin, vol I (1979), vol. II (1981).

[5] R. Carbone, F. Fagnola, and S. Hachicha, *Generic quantum Markov semigroups: the Gaussian gauge invariant case*, priprint.

[6] F. Cipriani, *Dirichlet forms and Markovian semigroups on standard forms of von Neumann algebras*, J. Funct. Anal. **147** (1997), 259–300.

[7] F. Cipriani, F. Fagnola, and J. M. Lindsay, *Spectral Analysis and Feller Properties for Quantum Ornstein-Uhlenbeck Semigroups*, Comm. Math. Phys. **210** (2000), 85–105.

[8] F. Fagnola and R. Quezada, *Two-photon and emission process*, Inf. Dim. Anal. Quantum Probab. Related Topics **8** (2005), 573–591.

[9] Y. M. Park, *Construction of Dirichlet forms on standard forms of von Neumann algebras*, Inf. Dim. Anal. Quantum Probab. Related Topics **3** (2000), 1–14.

[10] K. R. Parthasarathy, *An Introduction to Quantum Stochastic Calculus*, Birkhäuser, Basel, 1992.

[11] V. S. Sunder, *An Invitation to von Neumann Algebras*, Springer-Verlag, Newyork Berlin Heidelberg London Paris Tokyo, 1986.

Veni Choi
Division of General Studies
Ajou University
Suwon 443-749, Korea
E-mail address: veni@ajou.ac.kr

Chul Ki Ko
University College
Yonsei University
Seoul 120-749, Korea
E-mail address: kochulki@yonsei.ac.kr