Peak-to-Average Power Ratio Reduction of OFDM Signals Using Evolutionary Techniques

George D. Pantos, Panagiotis D. Karamalis, and Philip Constantinou

Abstract: **In this paper, the application of genetic algorithms (GAs) for orthogonal frequency division multiplexing (OFDM) signal peak-to-average power ratio (PAPR) reduction is investigated. A GA is applied in order to enhance the performance of some known techniques for OFDM PAPR reduction and the potential benefits are analyzed. Using the proposed techniques, the system designer can take advantage of the GA versatility, robustness, and adaptability to specific system requirements, in order to achieve a convenient trade-off between effectiveness and computational burden.**

Index Terms: **Genetic algorithms, orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR).**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising modulation technique that can support high data rate transmission in harsh propagation environments. Thus, it is adopted in several wireless standards (e.g., [1]–[4]) and it is considered as a candidate modulation technique for 4th generation communication systems. However, one of the major implementation drawbacks of OFDM, at the transmitter side, is the high peak-to-average power ratio (PAPR) value it demonstrates. High values of PAPR may lead an amplifier to its non-linear region of operation, introducing nonlinear distortion, spectral regrowth and intermodulation among OFDM sub-carriers. Anticipation of non-linear effects on signals with high PAPR value either require amplifiers with large dynamic range of linear operation, which are very expensive, or impose the use of a large back-off, inducing a severe degradation on system power efficiency.

When designing an OFDM transmission system, special precautions should be taken to prevent high PAPR values. Several techniques, have been proposed in order to control the PAPR of the OFDM signal [5]. A simple approach is deliberate clipping of OFDM signal before amplification [6]. This is a very easy to implement method, albeit it may cause in-band distortion and out-of-band noise. An alternative technique is to use block coding schemes to reduce PAPR [7]. However, it is very difficult to design proper coding schemes for large number of sub-carriers. In addition, the redundant bits of the coding scheme reduce the information transmission rate. Recently, multiple signal representation (MSR) techniques have been proposed, namely partial transmit sequences (PTS) [8], [9], selective mapping (SLM) [10] and interleaving [11]. Using MSR techniques, a remarkable reduction of PAPR is achieved, however side information should be provided to the receiver in order to detect the transmitted OFDM symbols. An efficient method to reduce the PAPR of OFDM signals is tone reservation (TR). This method exploits the so called *virtual sub-carriers* of OFDM, i.e., the unused IFFT inputs, to introduce properly selected peak cancelling symbols (complex numbers) in order to reduce the PAPR of the transmitted OFDM symbol [12], [13]. The sub-carriers used to reduce the PAPR value of OFDM symbol are called peak reduction carriers (PRC). The problem of optimum selection of PRCs' values is a convex optimization problem [14], [15] that leads to increased computational complexity and low convergence rates. Lawrey [16] proposes the selection of the PRC values using an exhaustive search technique. This approach is reasonable for OFDM systems employing a small number of sub-carriers, e.g., 16, however it is inapplicable when the number of sub-carriers is 64 or more. An alternative approach is to use a genetic algorithm (GA) to select the PRC values, provided that some pre-defined criteria are fulfilled [17], [18]. The aim of this paper is to demonstrate some sub-optimum solutions to the OFDM PAPR reduction problem, based on the TR concept and by exploiting the intrinsic advantages of GAs. The rest of the paper is organized as follows: Starting with some definitions and describing the problem formulation in Section II, we go through the description of GA application in the proposed PAPR reduction technique, in Section III. Simulation results are presented in Section IV and, finally, Section V concludes the paper.

II. PROBLEM FORMULATION

A. Tone Reservation Method

Let's consider a continuous time OFDM signal

$$x(t) = \sum_{n=0}^{N-1} X_n \cdot e^{j2\pi n \cdot \Delta f \cdot t},\ 0 \le t < T_s \qquad (1)$$

where $j = \sqrt{-1}$ and Δf is the frequency separation between any two adjacent sub-channels. The OFDM symbol duration T_s is equal to Δf^{-1} to ensure the orthogonality among the sub-channels. The OFDM coefficients X_n are typically taken from a fixed modulation constellation (e.g., QPSK). The PAPR is [19]

$$\mathrm{PAPR} = \frac{\max\limits_{0 \le t < T_s} |x(t)|^2}{E\left\{|x(t)|^2\right\}} \qquad (2)$$

where the denominator denotes the maximum instantaneous power of the OFDM signal.

Usually, the PAPR of $x(t)$ can be estimated from its LN samples during the symbol period, where L is called the *oversampling factor*. These samples may be easily obtained from the

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LN-point inverse discrete Fourier transform (IDFT) of $\{X_n\}$ with $(L-1)N$ zero-padding. The sampled version of the OFDM signal, at rate $\frac{LN}{T_s}$, is given by

$$x\left[{}^{k}\!/\!_{L}\right] = x\left(\frac{kT_s}{LN}\right) = \sum_{n=0}^{N-1} X_n \cdot e^{j2\pi nk/N},\ k = 0, 1, \cdots, LN-1. \tag{3}$$

The PAPR estimation considering the rate $\frac{LN}{T_s}$ sampled version of the OFDM signal is [14]

$$\text{PAPR}_L = \max_{k\in[0,NL)} \frac{\left|x\left[{}^{k}\!/\!_{L}\right]\right|^2}{E\left\{\left|x\left[{}^{k}\!/\!_{L}\right]\right|^2\right\}}. \tag{4}$$

When $L = 1$ these samples are called *Nyquist-rate samples.* The PAPR estimate for the Nyquist-rate samples is

$$\text{PAPR}_{Nyq} = \max_{k\in[0,N)} \frac{|x[k]|^2}{E\left\{|x[k]|^2\right\}}. \tag{5}$$

Obviously, PAPR $\geq$ PAPR$_L$ $\geq$ PAPR$_{Nyq}$.

Most PAPR reduction methods can only be implemented on the discrete-time signals. However, the Nyquist-rate samples, given by (3), for $L = 1$, do not necessarily coincide with the peaks of the continuous-time signal. In [11], it is shown that an oversampling factor of four, $L = 4$, is enough to estimate the continuous PAPR given in (2). More about the relationship of the maxima discrete and continuous OFDM signal can be found in [20], [21].

In a real OFDM system FFT/IFFT chips are employed and the complex data do not occupy all the inputs of an N-IFFT, since $N = 2^q$ for an integer q. Let $\mathbf{X}_D = [X_0, X_1, \cdots, X_{D-1}]^T, D < N$ be the original data (complex symbols) to be transmitted using an OFDM system. The remaining $N - D$ inputs of the N-IFFT are set to zero. The concept of the tone reservation method is to form the IFFT input vector by appending P more symbols to $\mathbf{X}_D$, e.g.,

$$\mathbf{X} = [\underbrace{X_0, \cdots, X_{D-1}}_{\mathbf{X}_D}, \underbrace{X_D, \cdots, X_{D+P-1}}_{\mathbf{X}_{DP}}, \underbrace{0, \cdots, 0}_{N-D-P}]^T, \quad D + P \leq N. \tag{6}$$

The P redundant symbols are the PRCs and they do not affect the original OFDM symbol due to sub-carrier orthogonality. At the receiver part, the PRCs are dropped after the FFT, as seen in Fig. 1. Appending a small number of PRCs usually does not require increased IFFT size. However, the number of the used PRCs should not exceed a small portion of the number of the real data sub-carriers, in order to maintain power efficiency. For example, considering an OFDM system with 385 data sub-carriers, a 512 IFFT/FFT is employed. Thus, the PRCs may occupy some of the remaining 127 IFFT inputs. Using 20 PRCs increases the transmitted power by 5.2%. If we use 80 PRCs, the power increment raises to 21.0%. Since PRCs do not carry data, this increment induces a severe degradation of system's power efficiency.

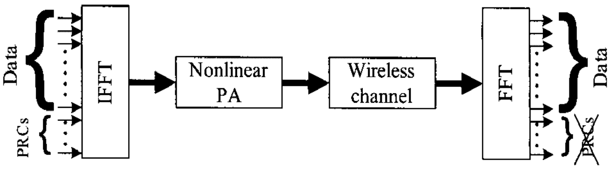

Fig. 1. OFDM transmission system using PRCs.

The problem is to identify the vector $\mathbf{X}_{DP}$ *so that the IFFT output* $\mathbf{x} = [x_0, \cdots, x_{N-1}]^T$ *achieves an acceptable PAPR value.* A straightforward approach [16] is to perform an exhaustive search through all possible solutions, considering the M-QAM constellation points as possible PRCs. This approach is reasonable when considering QPSK modulation with a small number of PRCs, e.g., in [16] the method is applied in an OFDM system with 16 sub-carriers. For higher order modulation techniques and/or more PRCs, the search space is huge and an exhaustive search is not applicable. A systematic approach is to consider the problem as a special form of a convex optimization problem [14]–[15]. A simpler gradient algorithm [14] provides an approximate solution with reduced complexity. In this paper, we use a GA in order to determine the amplitude and phase of the PRCs prior to transmission of the OFDM symbol.

B. Proposed Techniques

The output of the N-IFFT is

$$\mathbf{x} = \mathbf{Q}\mathbf{X} \tag{7}$$

where $\mathbf{Q}$ is the N-IFFT matrix, namely:

$$\mathbf{Q} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & w & \cdots & w^{N-1} \\ 1 & w^2 & \cdots & w^{2\cdot(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & w^{N-1} & \cdots & w^{(N-1)\cdot(N-1)} \end{bmatrix} \tag{8}$$

with $w = e^{j\frac{2\pi}{N}}$. Substituting $\mathbf{X} = [\ \mathbf{X}_D \quad \mathbf{X}_{DP} \quad \mathbf{0}\ ]^T$, we may rewrite (7) as:

$$\mathbf{x} = \mathbf{Q}[\ \mathbf{X}_D \quad \mathbf{X}_{DP} \quad \mathbf{0}\ ]^T = \mathbf{Q}_D\mathbf{X}_D^T + \mathbf{Q}_{DP}\mathbf{X}_{DP}^T \tag{9}$$

where $\mathbf{Q}_D$ is the sub-matrix of $\mathbf{Q}$ that contains its first D columns, and $\mathbf{Q}_{DP}$ is the sub-matrix of $\mathbf{Q}$ that contains columns D to $D + P - 1$. The complex data vector $\mathbf{X}_D$ and the matrix $\mathbf{Q}$ are known. Let

$$\mathbf{Q}_D\mathbf{X}_D^T = \mathbf{a} = \begin{bmatrix} a_0 & a_1 & \cdots & a_{N-1} \end{bmatrix}^T. \tag{10}$$

We can rewrite (9) in a more compact form as

$$\mathbf{x} = \mathbf{a} + \mathbf{q} \tag{11}$$

where

$$\mathbf{q} = \mathbf{Q}_{DP}\mathbf{X}_{DP}^T = \begin{bmatrix} q_0 & q_1 & \cdots & q_{N-1} \end{bmatrix}^T. \tag{12}$$

For a given vector $\mathbf{a}$, the problem is to identify the vector $\mathbf{q}$ that minimizes $\|\mathbf{a}+\mathbf{q}\|_\infty^2$, where $\|\mathbf{v}\|_\infty$ denotes the ∞−norm of vector $\mathbf{v}$, i.e., the maximum of the absolute values of its components.

We consider the following alternative approaches, shown in Fig. 2: "Method A" is the case that the possible positions of PRCs are fixed for all OFDM symbols, hence no side information is required in the receiver. The GA searches for PRCs' values in order to achieve PAPR value below the predefined threshold. It has been revealed [14], [17] that the same PAPR reduction is achieved when PRCs are appended after the data symbols or when they are uniformly distributed among data, as long as the PRCs' positions are fixed for all OFDM symbols. An alternative approach is "method B" where we do not use fixed positioned PRCs, instead we use $P < N - D$ PRCs and the GA searches for both values and positions of PRCs. This approach adds flexibility to GA and it is expected to further reduce the achieved PAPR value with no need for side information. Simulations have shown [5] that contiguous PRCs provide worse results in comparison with randomly distributed PRCs. This fact motivates "method C," that is the technique that PRCs may occupy any of the IFFT inputs, i.e., we use the sub-carriers $\{i_0, i_1, \cdots, i_{D-1}\}$ to transmit data, whereas the subcarriers $\{j_0, j_1, \cdots, j_{P-1}\}$ may be used as PRCs. In that case

$$\mathbf{x} = \mathbf{0}_{N\times 1} + \tilde{\mathbf{Q}}_D\mathbf{X}_D^T + \tilde{\mathbf{Q}}_{DP}\mathbf{X}_{DP}^T \tag{13}$$

where $\tilde{\mathbf{Q}}_D$ is the sub-matrix of $\mathbf{Q}$ that contains the columns $\{i_0, i_1, \cdots, i_{D-1}\}$, and $\tilde{\mathbf{Q}}_{DP}$ is the sub-matrix of $\mathbf{Q}$ that contains the columns $\{j_0, j_1, \cdots, j_{P-1}\}$. In this case, we should first select the P proper PRC positions in the range 0 to $N-1$ and then use D of the remaining $N-P$ inputs for data transmission. This is an efficient alternative that boosts the tone reservation method performance, however side information is required to the receiver in order to identify the real data in the OFDM symbol. Furthermore, the complexity is increased as well. In [14], a random set of potential PRCs' positions is evaluated and the set that achieves the lowest PAPR value is selected. In our technique, the GA adds one more parameter to the evolutionary procedure: PRCs' positions.

For all the proposed techniques the PAPR of the initial OFDM symbol is compared to a threshold PAPR value, i.e., the maximum acceptable PAPR value. If the initial PAPR value of the OFDM symbol exceeds that threshold, then the GA is employed in order to provide a modified OFDM symbol, with acceptable PAPR, using PRCs. The first step of the GA is the random selection of a set of possible solutions (initial population). If one of those solutions achieves the target PAPR value, then the problem reduces to a random selection problem and the procedure stops. If the target value is not met, then the GA is executed. The number of data subcarriers, the number of PRCs, the set of possible PRCs' positions, as well as the acceptable PAPR threshold should be defined along with the GA parameters, prior to the execution of the algorithm.

The complexity of the proposed technique is a random variable

$$C_{\max} = InitPop \cdot Gen \cdot NL\log_2(NL)$$

where $InitPop$ is the size of the Initial Population, Gen is the number of generations performed to achieve PAPR threshold, N is the number of subcarriers and L is the oversampling factor. It should be noted that if one of the randomly selected solutions provided by the initial population of the GA fulfills the PAPR value threshold, then the GA is not executed. In that case, the problem reduces to a random selection problem and its complexity is

$$C_{\text{Rand.Sel.}} = InitPop \cdot NL\log_2(NL).$$

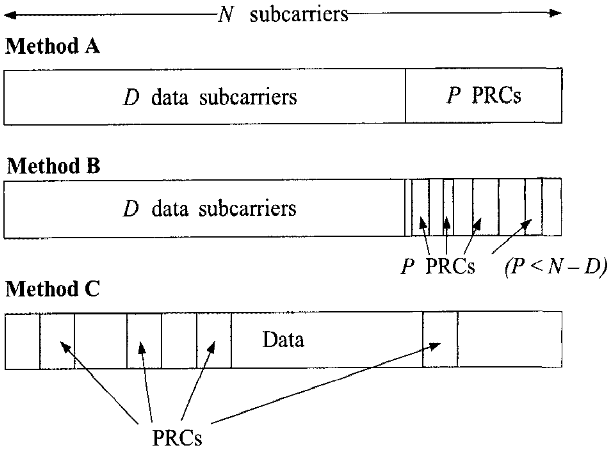

Fig. 2. Alternatives for PRCs positions.

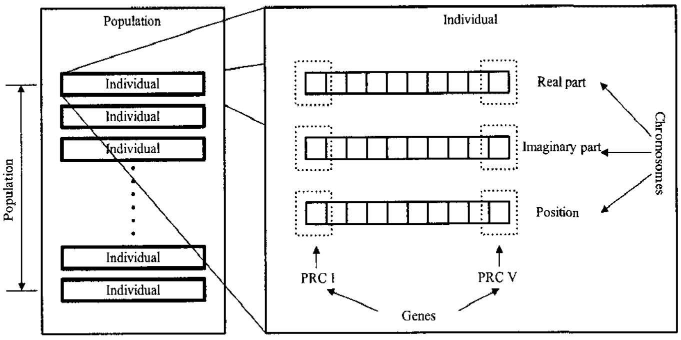

Fig. 3. Genetic algorithm terms.

III. APPLICATION OF THE GENETIC ALGORITHM

The fact that the PAPR reduction method leads to a minimization problem with extremely large search space motivates the investigation of potential use of a GA which can efficiently explore large search spaces. The concept is simple: The GA seeks for a set of possible solutions (values and positions of PRCs) that would minimize the PAPR.

A. Terminology

It is useful to refer to the basic GA terminology used throughout the paper (more about GAs can be found in [22], [23]). As revealed in Fig. 3, a *population* is the array of individuals under examination by the GA. An *individual* is a representation of a complete solution to the problem, containing information about the positions and values of the PRCs. It consists of three *chromosomes*: One for the real part, one for the imaginary part, and one for the position of the PRC.

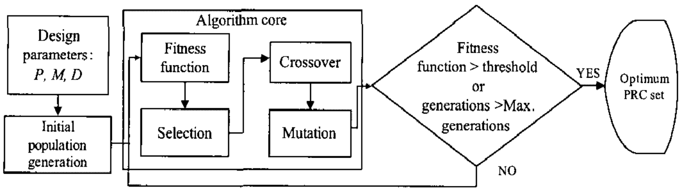

Fig. 4. Genetic algorithm flow chart.

B. Fitness Function

Individuals are characterized using the PAPR that corresponds to the PRCs they designate, as defined in (4). The goal of the GA is to find the set of PRCs that minimizes the above expression. The inverse of the PAPR value therefore designates the fitness of each individual as a solution to the problem. In GA terms, it is called the *fitness function* $F(\text{Individual})$. A normalized fitness function, bounded in $[0, 1]$ or $[0\%, 100\%]$, can be derived if the designer has set a specific PAPR value $\text{PAPR}_{\text{goal}}$ as a goal.

$$F_{\text{norm}}(\text{Individual}) = \frac{\text{PAPR}_{\text{goal}}}{\text{PAPR}(\text{Individual})}. \tag{14}$$

C. Algorithm

The algorithm proposed is a modified simple genetic algorithm (SGA) with three core operators and integer representation, as described in [24]. A flow chart of the applied algorithm is shown in Fig. 4.

The first step is to generate an initial population of solutions, i.e., individuals. In absence of any hint, values are chosen randomly. The next step is to evaluate each individual's fitness and rank the population in declining fitness order. This procedure enables the GA to resolve the total generation fitness and pass the information on to the core of the algorithm, which consists of the three central genetic operators: Selection, crossover, and mutation [22].

The selection (or reproduction) operator is an efficient way to choose the individuals that will give birth to the new generation.

In the present implementation of the GA, all individuals have the same probability to be selected until the entire population becomes highly correlated and the selection strategy changes to roulette-wheel selection [22], [23] for faster convergence. The second operator, crossover, is the method of exchanging genetic information between two individuals, the *parents*, in order to form another pair of individuals, the *offsprings*, sharing their attributes. This operator aims at recombining the qualities of two individuals and producing a higher-quality pair for the next generation. The crossover strategy used here is uniform crossover [23], [25]: The operator randomly chooses gene positions and exchanges the values between the two parents (Fig. 5). Another aspect of generation updating is the population refresh percentage, namely the number of offsprings as a fraction of the entire population. For static cases, where the population size doesn't change, the less fit individuals are replaced by the new breed, while a number of the old generation fittest individuals are preserved. This strategy, called *generation overlapping*, ensures that the population state can only be improved. As long as only the crossover operator alters the generations, the population tends to become uniform, converging to a fitness maximum that is not certain to be the global maximum. In order to diversify the population and force the algorithm to search in previously unexplored areas of the search space, there is a need to introduce *mutation*, an operator that randomly alters the population state. This is achieved by choosing two genes within the same chromosome and exchanging their values (Fig. 6).

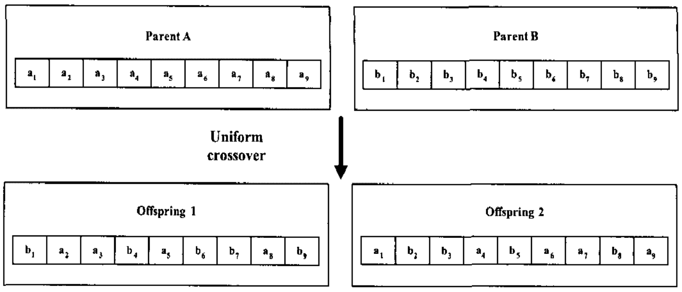

Fig. 5. The uniform crossover operator.

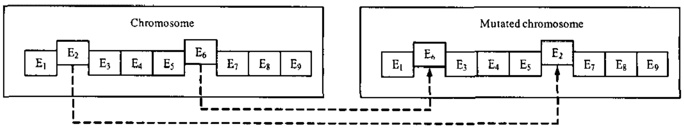

Fig. 6. The mutation operator.

The algorithm execution halts when the number of generations exceeds a predefined maximum, or the fitness function exceeds the anticipated threshold or the population state remains stable for a number of generations. These thresholds are chosen in a way to avoid premature termination of the algorithm or unproductive continuation after reaching a satisfactory solution.

IV. RESULTS

Let us now proceed to the application of GA-assisted tone reservation method in OFDM PAPR reduction. In order to evaluate the proposed techniques, we consider an OFDM system with 100 subcarriers carrying QPSK data, and 20 PRCs, hence a 128−IFFT should be used. Furthermore, we consider 4× oversampling. A simple GA, presented in Fig. 5, has been used. Throughout the simulation an initial population of 40 individuals has been considered. Mutation probability has been set to 10%, whereas 5% of the population form an elite population. We perform Monte-Carlo simulation, with 10^6 runs, with PAPR value threshold set to 8.5 dB for method A and method B and 6.5 dB for method C, and compare the PAPR statistics with the ones of the original OFDM signal. The results are shown in Fig. 7. Considering 0.01% clipping probability as reference, the corresponding PAPR value for method A is 9.3 dB, i.e., 2.4 dB improvement compared to the original OFDM signal. The corresponding PAPR values for methods B and C are 8.6 dB and 7.2 dB, respectively. Simulations showed that the problem reduces to a random selection problem with probability about 45%.

In order to evaluate the performance advantage of the GA versus random selection, we compare the achieved PAPR statistics with the ones achieved by random selection. We perform the

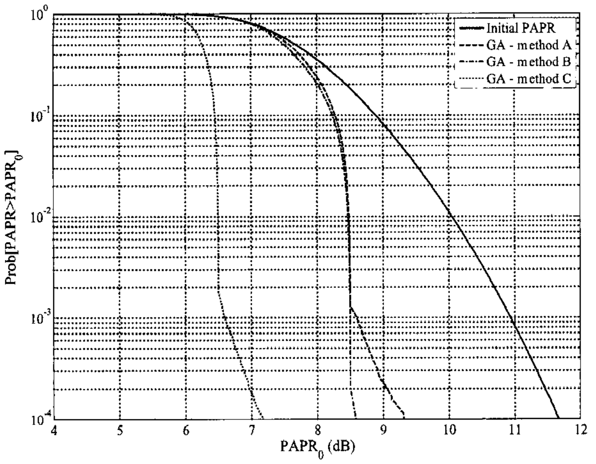

Fig. 7. CCDF of PAPR value of an OFDM signal with 100 data subcarriers, modified by the proposed techniques.

Table 1. Comparison of GA with random selection technique (clipping probability = 10^{-4}).

Method	Initial PAPR value	PAPR value with random selection	PAPR value with GA
A	11.6 dB	10.55 dB	9.7
B	11.6 dB	10.60 dB	9.7
C	11.6 dB	9.35 dB	7.8

Table 2. Tone reservation techniques applied to 256-OFDM system.

Method	PAPR reduction (clipping probability = 10^{-5})
GA method A	1.70 dB
GA method B	2.30 dB
GA method C	3.50 dB
Method [27]	3.03 dB
Method [14]	2.50 dB

following simulation scenario: For each trial, a random OFDM symbol is generated. In case that the PAPR value of the symbol exceeds the predefined threshold value, a genetic algorithm with population $P = 30$ is applied, in order to reduce the PAPR value, using GA-assisted tone-reservation technique. If the number of the generations produced by the GA is G, then for the same OFDM symbol we try PG randomly selected solutions and we keep the best one, in terms of PAPR reduction. Hence, for each iteration step we get a GA and a random selection solution, both of the same complexity. The procedure is executed 10^6 times for each method (A, B, and C) and the results are shown in Table 1. Simulations have shown that GA approach outperforms the Random Selection technique, by 0.85 to 1.55 dB, when considering the 10^{-4} level of the CCDF. This is the gain obtained using the evolutionary procedure.

Table 2 summarizes the results of application of the proposed GA-assisted tone reservation methods, compared to iterative techniques proposed in [14], [27]. An OFDM system with 256 subcarriers has been used, with 244 of them carrying QPSK data. In proposed method B, 8 PRCs have been used in the 12 available positions.

V. CONCLUSIONS

In this paper, a GA was applied in conjunction with the tone reservation method, in order to reduce PAPR of OFDM signals. Various alternative scenarios were described and simulated. The results of the simulations proved the effectiveness of the described methods and their applicability to various system requirements. Moreover, with the proposed techniques it is up to the system's designer to tradeoff between complexity and required effectiveness.

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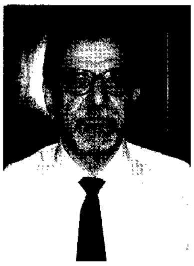

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