IEMS Vol. 7, No. 3, pp. 257-265, December 2008.

Optimal Lot-sizing and Pricing with Markdown for a Newsvendor Problem

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Selected paper from APIEMS 2006

Abstract. This paper deals with the joint decisions on pricing and ordering for a monopolistic retailer who sells perishable goods with a fixed lifetime or demand period. The newsvendor-typed problem is formulated as a two-period inventory system where the first period represents the inventory of fresh or new-arrival items and the second period represents the inventory of items that are older but still usable. Demand may be for either fresh items or for somewhat older items that exhibit physical decay or deterioration. The retailer is allowed to adjust the selling price of the deteriorated items in the second period, which stimulates demand and reduces excess season-end or stale inventory. This paper develops a stochastic dynamic programming model that solves the problem of pre-season decisions on ordering-pricing and a within-season decision on markdown pricing. We also develop a fixed-price model as a benchmark against the dual-price dynamic model. To illustrate the effect of the dual-price policy on expected profit, we conduct a comparative study between the two models. Extension to a generalized multi-period model is also discussed.

Keywords: Inventory, Perishable, Pricing-Ordering, Dynamic Pricing

1. INTRODUCTION

The This paper considers a single product with a fixed length of lifetime in a two-period inventory system. The first period represents the inventory of fresh or new arrival items and the second period represents the inventory of items that are older but still usable. New items not sold within a certain period will be markdown in the second period. The older and deteriorated items not sold within a certain period will be salvaged. The new items are replenished from an outside vendor with infinite capacity, stochastic demands occur independently in both periods, and the unfilled demands are lost.

The two-period structure is similar to the price protection model proposed by Lee *et al*. (2000) and the blood management model given in Goh *et al*. (1993). Lee *et al*., focused on channel coordination issues in a bilateral monopoly, dealing with the optimal trade terms such as seller's price and rebate and buyer's order quantity. Goh *et al*. dealt with the issuing policies of new and old items for blood management. In this paper, our interest is in the pricing-ordering decisions for a monopolistic retailer who determines the replenishment quantity, sets the initial price for the new items, and makes a follow-up markdown for the old items so that the expected profit is maximized over the fixed shelf life or demand period.

The fixed lifetime perishability problem falls into the general framework of the classic newsboy model. Goyal and Giri (2001), Nahmias (1982), and Raafat (1991) provided extensive review of the problem. However, most of

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the references therein exploit the problem by deriving proper ordering policy under the implicit assumption of first-in-first-out issuing policy. In many real systems such as food and fashion retailing, customers determine the issuing policy. If the valuation of fresh items is higher, stock consumptions in the display shelf will be last-in-first-out. In this paper, the valuations of perishable items with different ages are characterized by different distributions of reservation prices (see Lazear 1986 for details). Reservation price is defined as the maximum price that customers are willing to pay for the product, i.e., customers buy the product only if their reservation price is higher than or equal to the product's price. Hence, it tends to have a continuous distribution. In the fashion and food industries, the reservation price distribution shifts to the left over time, i.e., customers in general are willing to pay higher price for the new items and lower price for the aged items. Lazear formulated the obsolescence-prone pricing behavior by a simple two-period dynamic model. Pashigian (1988) and Pashigian and Bowen (1991) investigated the model empirically. The specific version of newsboy problem assumes that the reservation price is random, and management knows its probability distribution. The retailer therefore can adjust price in response to the shift of reservation price distribution during its lifetime.

Due to its effectiveness for short-term control over on-hand stock, price change is increasingly prevalent in practice. The average weekly frequency of price changes at the largest U.S. supermarket chains ranges from 3223 to 4316, yielding an average of 15.66 percent of their products each week (Levy *et al*., 1997). The supermarket chains tend to achieve more price flexibility by changing prices more often, i.e., multiple times each week. Pashigian (1988) and Pashigian and Bowen (1991) showed that fashion goods increase in the percentage markup and the frequency of sales since 1970. In the less fashion products like automobiles, there still exist within-season price declines (Pashigian *et al*., 1995). In the airline industry, fare changes are routine and commonly used as a demand-responsive mechanism (see McGill and van Ryzin 1999, Weatherford and Bodily 1992, and Belobaba 1987). For instance, American Airlines made up to 50,000 daily fare changes (Smith *et al*., 1992). In the context of electronic commerce, on-line retailers adjust their prices more readily than conventional retailers in response to structure changes in supply or demand (Brynjolfsson and Smith 2000).

The reasons of price changes are numerous such as to reflect cost changes or demand fluctuations, to response to competitor's price changes, or to comply temporary promotions. This paper focuses on permanent markdown or clearance sales over product lifetime. The price adjustment in the second period is an attempt to compensate for a shifting demand function due to the physical decay or deterioration of the aged items. The advantage of using the proposed strategy, or so-called dual-price or two-fare pricing policy in airlines practice, is that by adjusting the price over product life time it is possible to synthesize a wide range of deteriorating losses and to induce customers to change their buying behaviors.

This paper is organized as follows. Section 2 provides a survey of related research. The underlying problem is formulated as a two-period dynamic programming model in section 3. For comparison purpose, a fixed-price model, i.e., without markdown in the second period, is also introduced in the section. Section 4 illustrates the mathematical behavior of the models and conducts a comparative study between dual-price and fixed-price policies. Extension to a generalized multi-period model and concluding remarks are provided in section 5.

2. LITERATURE REVIEW

Many research works on pricing-ordering/production have been developed in the past decades. Eliashberg and Steinberg (1993) and Yano and Gilbert (2004) provided a comprehensive survey of early works on this area. For the deterministic demand, Rajan *et al*. (1992) developed a continuous time pricing and ordering policy for perishable goods. Abad (1996) extended their work by allowing backlogged. Under demand uncertainty circumstance, Smith and Achabal (1998) developed optimal pricing and inventory policies that take into account of price, reduced assortment, and seasonal effects on sales rates. Gilbert (2000) and Deng and Yano (2006) considered the problem of joint decisions on pricing and production schedules for multi-period settings. Bernstein and DeCroix (2004) considered joint decisions on pricing and capacity, instead of pricing and production/inventory decisions, in a multi-tier assembly system. Federgruen and Heching (1999) exploited the pricing-ordering problem for the cases of multi-period as well as infinite sales horizon. A replenishment order may be placed at the beginning of some or all of the periods. In this paper, we consider the pricing-ordering decisions for the newsboy-typed problem, i.e., a fixed selling horizon with only one replenishment order at the beginning of the period.

Gallego and van Ryzin (1994) formulated the underlying problem using intensity control mechanism and obtained structural monotonicity results for the optimal dynamic price that is a function of the stock level and the length of the selling horizon. Their work motivates many researchers to develop more general models from a variety of perspectives. Gallego and van Ryzin (1997) generalized the model that allows time-varying demand and multi-product with a network structure. Bitran and Mondschein (1997) developed both continuous time and periodical pricing models for compound Poisson process demand that is a function of the price through the distribution of reservation prices. Chun (2003) studied the variant of the periodic model proposed by Bitran and Mondschein that considers a negative binomial distributed demand function. Zhao and Zheng (2000) exploited the problem by allowing non-homogeneous demand where both the intensity of the customer arrival process and the reservation price distribution may change over time. A recent joint price/inventory newsvendor model was proposed by Raz and Porteus (2006), who used a standard approach to approximating a given distribution with a finite number of representative fractiles. A comprehensive review in this stream of research work can be found in Petruzzi and Dada (1999).

Another stream of works addresses the problem of deciding the optimal timing of price changes within a given menu of allowable prices over a fixed horizon. Feng and Gallego (1995) were among the initiatives that determined the optimal timing and direction of a single price change (markdown or markup). Feng and Xiao (1999, 2000a, 2000b) and Feng and Gallego (2000) extended their research by considering more realistic situations. In this paper, the switching time of prices is exogenous. We consider both cases with deterministic and random demands in the numerical examples provided.

In the context of unknown demand distribution, some representative works include Burnetas and Smith (2000) and van Ryzin and McGill (2000). Both of them developed adaptive algorithms to approximate optimal solutions. Advantages of the proposed algorithms are the simplicity of implementation and without knowing the demand distribution. However, one of their drawbacks is the requirement of long trial period to converge. For the products with short life cycle or seasonal effects on demand, the algorithms may be inappropriate in practice.

Our work is closest in spirit to Bitran and Mondschein (1997), who focused on determining the optimal pricing policies given a fixed amount of perishable inventory. Our work addresses the preseason problem of setting replenishment quantity as well as list price, and the within-season problem of optimal markdown pricing. Both papers assume a stochastic arrival of customers coupled with different valuations of the product. As time elapses, however, they changed the intensity rather than the distribution of reservation prices to mimic the behavior of declining-prone market. We take the approach of shifting the distribution to the left over time that retains the characteristics of consumer behaviors.

3. THE MODEL

We assume that the arrival of potential customers in the store is a Poisson process with arrival rate λ_k and their reservation price distribution is $F_k(p)$ at period k, $k = 1, 2$. Hence the demand is a non-homogeneous compound Poisson process with intensity $\lambda_k(1-F_k(p))$ where $1-F_k(p)$ represents the probability that a customer's reservation price is higher than or equal to product's price p. Figure 1 illustrates the Weibull distributed reservation prices that shift to the left over time in the two-period system. The price-sensitive and decline-prone demand induces the joint decisions on pricing-ordering for the new items and a follow-up markdown for the aged items.

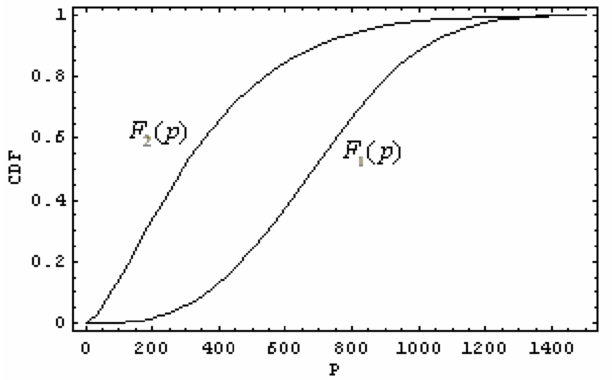

Figure 1. Weibull distributed reservation prices.

In this section, we present both the dual-price (with one-time markdown) and the fixed-price (without markdown) models for the underlying system and prove their necessary and sufficient conditions. Before presenting the models, we introduce additional notation: p_k Selling price at the market per unit at period k (decision variables); $f_k(p)$ Probability density function for the reservation price at period k; $\xi_k(p)$ Random variable denoting the price-sensitive demand at period k; w Purchasing cost per unit from the supplier, and γ Time-discounting factor per period

In addition, goodwill cost for demand unmet by the retailer is assumed to be negligible and the salvage value of any unsold items at the end of the second period is zero. Since for any positive salvage value s we can always define a new price $p \leftarrow (p-s)$, and a new function of reservation price $F_k(p) \leftarrow F_k(p+s)$ that transforms the problem into the zero-salvage-value case.

3.1 Model 1: The Dual-Price Policy

The retailer replenishes Q items at the beginning of period 1 and the units are delivered ready for sale in the first period at a full price. As time elapses, the aged items are transferred to the markdown period for sale at a discounted price. To solve the decision problem for the two-period system, we work backward starting with period 2. At the end of period 1, the leftover stock is q, and the expected profit in the second period is given by

$$\begin{aligned}
&\pi_2(q,\ p) \\
&= \sum_{y=0}^{q} py \Pr(\xi_2(p)=y) + \sum_{y=q+1}^{\infty} pq \Pr(\xi_2(p)=y) \\
&= pq - \sum_{y=0}^{q} p(q-y)\Pr(\xi_2(p)=y) \qquad (1)
\end{aligned}$$

Moving back to the first period, the expected profit to the system is given by:

$$
\begin{aligned}
&\pi_1(Q,\ p) \\
&= -wQ + \sum_{x=0}^{Q}[px + \gamma\pi_2(Q-x,\ p)]\Pr(\xi_1(p)=x) \\
&+ \sum_{x=Q+1}^{\infty} pQ\Pr(\xi_1(p)=x) \\
&= (p-w)Q \\
&-\sum_{x=0}^{Q}[p(Q-x)+\gamma\pi_2(Q-x,\ p)]\Pr(\xi_1(p)=x) \quad (2)
\end{aligned}
$$

In general, it is very difficult to prove its optimality without further assumption on the reservation price distribution. To remain focus, reservation price is assumed to be two-parameter Weibull distributed: $F_k(p) = 1 - e^{-(p/\beta)^{\alpha}}$ with shape parameter $\alpha > 1$ and scale parameter $\beta > 0$. Choosing Weibull is due to its convenient analytical properties. Besides, it is a generalized form of exponential functions that can represent a large variety of behaviors for the reservation prices. For example, a form of the Weibull with $\alpha = 3.25$ is almost identical to the unit normal distribution. As $\alpha \to \infty$, it becomes degenerate at β (Law and Kelton 1991). With the assumption of Weibull, we state the following propositions with proofs.

Proposition 1: *Given an initial stock* q, *there exists an upper bound* $p_{\max}$ *such that the expected profit function* $\pi_2(q,\ p)$ *is concave in* p *for* $p \le p_{\max}$.

Proof: The first-order condition for the profit function can be determined through marginal analysis, which takes into account marginal profit and marginal loss. A necessary condition for price to be the optimal is that the seller has no incentive to modify this price, i.e., its expected marginal profit equals to its expected marginal loss:

$$\lambda_2(1-F_2(p))dp = p\lambda_2 f_2(p)dp \tag{3}$$

The left-hand side of equation (3) represents the marginal revenue obtained by increasing the price by a small amount dp comes from being able to sell the goods at a higher price. Yet this premium comes at a cost since a fraction of customers who were willing to buy the good at p are no longer willing to buy it at $p+dp$. Since price increment is small enough the expected number of losing customers is no more than one. Therefore the marginal loss is the unit sale p multiplying with the expected number of losing customers $\lambda_2 f_2(p)dp$. Equation (3) can be simplified into the following:

$$1 - F_2(p) - pf_2(p) = 0 \tag{4}$$

A sufficient condition for a price satisfying (4) to be optimal is

$$-2f_2(p) - pf_2'(p) \le 0 \tag{5}$$

Substituting $f_2(p)$ with Weibull density function, manipulating algebraic operations, and rearranging terms, we have

$$e^{-(p/\beta)^{\alpha}}\alpha\beta^{-\alpha}p^{\alpha-1}(\alpha p^{\alpha}\beta^{-\alpha} - \alpha - 1) \le 0 \tag{6}$$

Since $e^{-(p/\beta)^{\alpha}}\alpha\beta^{-\alpha}p^{\alpha-1} > 0$, it is equivalent to show

$$\alpha p^{\alpha}\beta^{-\alpha} - \alpha - 1 \le 0\,. \tag{7}$$

Letting $p_{\max} = ((1+\alpha)/\alpha)^{1/\alpha}\beta$, equation (7) holds for $p \le p_{\max}$. Thus the profit function is concave in p and has a unique solution that corresponds to the optimal price. □

Proposition 2: *Given an initial stock Q, the expected profit function* $\pi_1(Q, p)$ *has a unique optimal solution.*

Proof: In the multi-period case, the marginal profit and the marginal loss are $\lambda_1(1-F_1(p))dp$ and $(p - \gamma(\pi_2(x+1,\ p) - \pi_2(x,\ p)))\lambda_1 f_1(p)dp$, respectively. The lost sale due to increasing price in period 1 is partially offset by the possibility of selling the additional stock in the second period. Let $q = x+1$, and rearranging terms, the first order condition becomes:

$$p - (1-F_1(p))/f_1(p) = \gamma(\pi_2(q,\ p) - \pi_2(q-1,\ p)) \tag{8}$$

The LHS represents the loss associated by not selling one unit of product in period 1 and the RHS representing the marginal profit by selling the addition stock in period 2. Let

$$M(p) = p - (1-F_1(p))/f_1(p) \tag{9}$$

The equation has a unique solution that corresponds to the optimal solution if $M(p)$ is a monotonic function in p, i.e., $M(p)$ is an increasing function of p. Substituting $f_1(p)$ in equation (9) with Weibull density function, manipulating algebraic operations, and rearranging terms, yields

$$M(p) = p(1-\alpha^{-1}(\beta/p)^{\alpha}) \tag{10}$$

Equation (10) obviously increases in p and therefore completes the proof. It is worth noting that Bitran and Mondschein (1993) provided a similar proof for a continuous-time dynamic model. □

Proposition 3: *The expected profit function* $\pi_1(Q, p)$ *is concave in Q.*

Proof: Substituting π_2 with equation (1) and letting p_1 and p_2 represent the prices in both periods, we have

$$\begin{aligned}&\pi_1(Q, p_1, p_2) = \\ &[(p_1 - w) - (p_1 - \gamma p_2)\Pr(\xi_1(p_1) \le Q) \\ &-\gamma p_2 \Pr(\xi_1(p_1) + \xi_2(p_2) \le Q)]Q \\ &+\sum_{x=0}^{Q}(p_1 - \gamma p_2)x\Pr(\xi_1(p_1) = x) \\ &+\sum_{x=0}^{Q}\sum_{y=0}^{Q-x}\gamma p_2 \Pr(\xi_1(p_1) = x, \xi_2(p_2) = y)\end{aligned} \tag{11}$$

The first order condition of the model is given by

$$\begin{aligned}&(p_1 - w) - (p_1 - \gamma p_2)\Pr(\xi_1(p_1) \le Q) \\ &\quad -\gamma p_2 \Pr(\xi_1(p_1) + \xi_2(p_2) \le Q) = 0\end{aligned} \tag{12}$$

It is easy to verify the second order condition is satisfied. □

The concavity property of the profit functions is critical for the development of the solution procedure. In the beginning of period 2, we solve equation (1) to obtain optimal markdown price p_2^* and the expected profit π_2 that are functions of the leftover units q, $0 \le q \le Q$. Moving back to the first period, the optimal initial price p_1^* and the expected profit π_1 can be obtained by solving equation (2) for a given Q. Since π_1 is concave in Q, we can find the optimal ordering quantity Q^* that satisfies the following conditions:

$$\pi_1(Q^*, p_1^*) \ge \pi_1(Q^* - 1, p_1^*)$$

and

$$\pi_1(Q^*, p_1^*) \ge \pi_1(Q^* + 1, p_1^*).$$

3.2 Model 2: The Fixed-price Policy

The fixed-price model is obtained by substituting π_2 in equation (2) with equation (1), manipulating algebraic operations, and rearranging and canceling terms:

$$\begin{aligned}&\pi_f(Q, p) \\ &= [(p - w) - (1-\gamma)p\Pr(\xi_1(p) \le Q) \\ &-\gamma p \Pr(\xi_1(p) + \xi_2(p) \le Q)]Q \\ &+\sum_{x=0}^{Q}(1-\gamma)px\Pr(\xi_1(p) = x) \\ &+\sum_{x=0}^{Q}\sum_{y=0}^{Q-x}\gamma p \Pr(\xi_1(p) = x, \xi_2(p) = y)\end{aligned} \tag{13}$$

Proposition 4: *The expected profit function* $\pi_f(Q, p)$ *is concave in* p *and Q, and therefore has a unique solution* (Q_f^*, p_f^*) *that maximizes the expected profit* π_f.

Proof: The proof of the proposition is similar to the dual-price model and hence is omitted.

Proposition 5: *The fixed-price policy is sub-optimal in the two-period system, i.e.,* $\pi_f(Q_f^*, p_f^*) \le \pi_1(Q^*, p_1^*)$.

Proof: The proof can be accomplished through contradictory arguments. If (Q_f^*, p_f^*) is the optimal policy for the two-period system, we have no incentive to modify neither Q_f^* nor p_f^*, and no markdown for the deteriorated items which contradict to the assumptions on shifting reservation price distributions.

4. NUMERICAL EXAMPLE

To illustrate the mathematical behavior of proposed models and the effects of price adjustments on the expected profit, we conduct some numerical experiments. As a point of comparison, we begin by establishing a base case with the following data: the shape and scale parameters of reservation price distributions: ($\alpha_1 = 3$, $\beta_1 = 773$) for the new items and ($\alpha_2 = 1.4$, $\beta_2 = 379$) for the older items (see Figure 1), customer arrival rates $\lambda_1 = \lambda_2 = 20$, time-discounting factor $\gamma = 0.9$, and unit purchase cost $w = 400$. The settings generate the following means and standard deviations of the reservation prices: ($\mu_1 = 690$, $\sigma_1 = 250$) and ($\mu_2 = 345$, $\sigma_2 = 250$) for the new and old items, respectively. To facilitate further comparative study, we compute the average weighted mean and standard deviation of the reservation price: $\bar{\mu} = (\lambda_1\mu_1 + \lambda_2\mu_2)/(\lambda_1 + \lambda_2) = 517.5$ and $\bar{\sigma} = ((\lambda_1\sigma_1^2 + \lambda_2\sigma_2^2)/(\lambda_1 + \lambda_2))^{1/2} = 250$. Finally, the deterioration rate of the product is defined as $\Delta\mu = (\mu_1 - \mu_2)/\bar{\mu} = 0.655$.

In the base case, we first studied the analytical properties of the profit functions such as concavity in p and their necessary and sufficient conditions where we assume the order quantity $Q = 11$ and the leftover quantity $q = 11$ in thetwo-period system.WE used Mathematica 4.1 to solve the problems and the results are graphically shown in Figures 2 and 3. In this example, the optimal pricing policy is to set an initial price by 720 and a follow-up adjustment to 374 that generate a total profit of 2647 (refer to Table 1). It is worth mentioning that the pricing policy is inventory dependent and $p_{\max} = 476$ in period 2 that provides an upper bound for the search algorithm.

The effectiveness of price adjustment on the expected profit is shown in Figure 4 (and is detailed in Table 1), from which the dual-price policy outperforms the fixed-price by 8.3 percent approximately in profit increment. In the U.S., the net profit margin for the multi-store supermarket chains is 1-3 percent of revenue (Montgomery 1994). The reported average profit margin of the airline industry in the 10-year period from 1978 to 1988 is 1.6 percent (Feldman 1990). An interview of the first author with the manager of local grocery chain stores also revealed that the average gross profit before-tax is around 2.9 percent of the sale. Given the thin margins of most retailers, 8.3 percent increment is significant that can make the difference between a profitable and unprofitable business. In addition, several remarks can be drawn from Figure 4: the optimal prices (p_1^*, p_f^*) decrease in the number of inventory, the expected profit functions (π_1, π_f) are concave in inventory that coincides with Proposition 3, and the prices and profits generated from the dual-priced policy are higher than that of the fixed policy especially in the case of large inventory.

In what follows we conducted a series of comparative studies by changing key factors such as the standard deviation $\overline{\alpha}$, deterioration rate $\Delta\mu$, time-discounting factor γ, and the distribution of demand intensity λ_k.

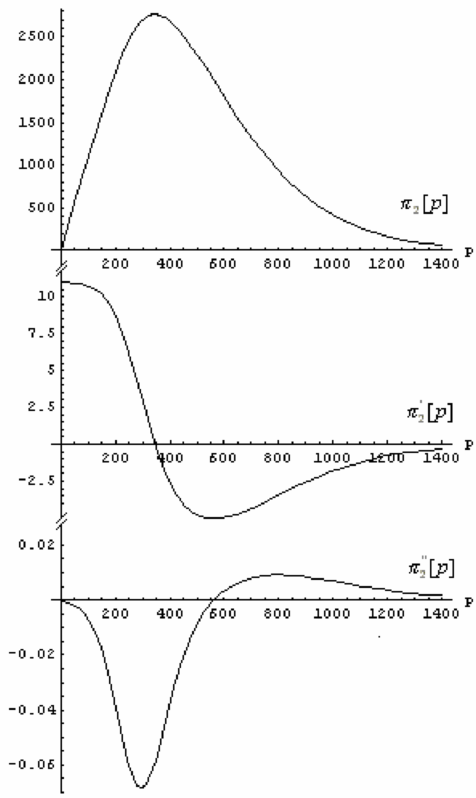

Figure 2. Profit function in period 2.

In order to isolate the effect of uncertainty in the reservation prices, we keep a constant mean $\overline{\mu} = 517.5$ and vary the parameters of Weibull distributions to obtain different values of $\overline{\alpha}$ or $\Delta\mu$. Table 1 shows the optimal initial ordering-pricing policy, expected profit, and percentage of change in profit for the base case and eight variants of it. Note that $(\pi_1^i - \pi_1^0)/\pi_1^0$ represents the percentage of change in profit between case i and base case for the dual-price policy, $(\pi_f^i - \pi_f^0)/\pi_f^0$ is for the fixed policy, and $(\pi_1 - \pi_f)/\pi_f$ evaluates the effect of price adjustments on profit increment.

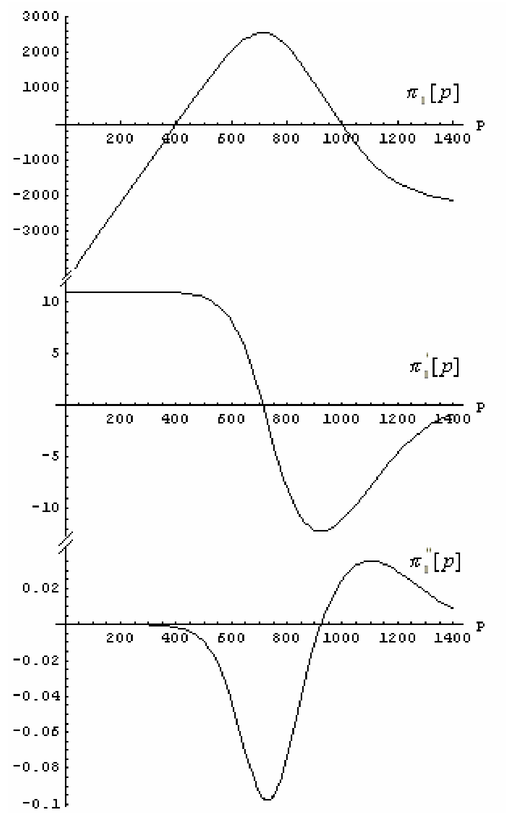

Figure 3. Profit function in period 1.

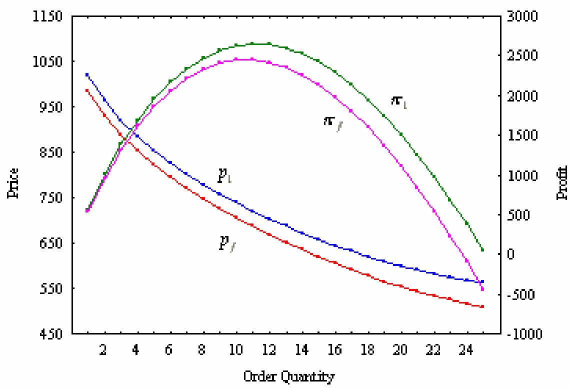

Figure 4. The price and profit as functions of the initial inventory.

Table 1. Numerical results.

	Case	Dual-price				Fixed-price				$\frac{(\pi_1-\pi_f)}{\pi_f}$
		Q^*	p_1^*	π_1	$\frac{\pi_1^i-\pi_1^0}{\pi_1^0}$	Q_f^*	p_f^*	π_f	$\frac{\pi_f^i-\pi_f^0}{\pi_f^0}$	
0	Base case	11	720	2647	-	11	687	2444	-	8.30%
1	$\overline{\sigma}\cdot K$	10	759	2668	0.79%	10	729	2523	3.20%	5.77
2	$\overline{\sigma}/K$	12	692	2676	1.10	11	672	2402	-1.70	11.42
3	$\Delta\mu\cdot K$	11	732	2784	5.18	11	700	2576	5.40	8.05
4	$\Delta\mu/K$	11	707	2512	-5.10	11	674	2321	-5.01	8.23
5	$\gamma=1$	12	711	2760	4.30	11	689	2510	-2.80	9.97
6	$\gamma=0.8$	11	712	2543	-3.90	10	703	2385	-2.40	6.66
7	$\lambda_1\sim DT(0,\lambda,2\lambda)$	11	735	2408	-9.02	10	703	2069	-15.34	16.41
8	$\lambda_1\sim DU(0,2\lambda)$	11	747	2197	-17.00	9	721	1778	-27.25	23.59

The first important finding from the table is that the dual-price policy tends to order more stock and set higher initial price than the fixed. Further, dual-price policy outperforms the fixed-price in all cases that empirically supports Proposition 5 under the design of experiments. The above are largely due to the following reasons: higher price setting in the initial period of selling horizon can generate the premium from the customers who are early adaptors and markdowns on the older items can stimulate demand and reduce the loss of excess season-end inventory.

The eight variants are studied in further details. Cases 1-2 examine the effect of changing standard deviation of the reservation price by a magnitude factor K, K = 1.2, i.e., $\sigma\leftarrow\sigma\cdot K$ and $\sigma\leftarrow\sigma/K$. In case 1, increasing α causes less order quantities ($Q,^*$ Q_f^*) and higher prices (p_1^*, p_f^*), and generates more profits (π_1, π_f), while decreasing α in case 2 comes out an inconsistent results such as a higher profit π_1 that can be interpreted by the discrete effect on order quantity. Cases 3-4 use an analog approach in changing $\Delta\mu$, and generate similar results as in cases 1-2. The effect of time-discounting factor is examined in cases 5-6. It is worth mentioning that both γ = 1.0 and γ = 0.8 cause the decrement in profit π_f. It also can be interpreted by the discrete effect on order quantity.

In the last two cases, we take the random intensity into account. In the experiment, we consider both discrete triangularly distributed intensity $\lambda_1\sim DTriang(0,\lambda,2\lambda)$ and discrete uniformly distributed intensity $\lambda_1\sim DU(0,2\lambda)$ where λ = 20, and let $\lambda_2=2\lambda-\lambda_1$. The new settings allow us to observe the random effect of intensity yet retaining the mean unchanged. In the circumstance of higher variance (i.e., case 8), the retailer will price higher yet generate less profit. In addition, the dual price policy outperforms the fixed significantly, i.e., the effect of clearance sales on profit increment is more crucial in higher uncertainty demand. These cases, while numerically simplified, illustrate the nature of the models.

5. DISCUSSION AND EXTENSION

In this paper, we have formulated two models that determine optimal order quantity and price for a monopolistic retailer who sells perishable goods over a finite horizon. The first model allows a within-season price adjustment to drive sale of slow moving items; the second adopts a stick price police over the demand period or shelf life. These models provide structure and quantitative insights into the interplay between inventory and pricing decisions, such as higher level of inventory leading to lower price. We have also observed that higher uncertainty in the demand leads to higher price and less expected profit. Besides, the comparative study has provided some managerial and economic implications. The dual-price policy significantly outperforms the fixed price in profit generated, especially in the case of higher inventory or higher demand uncertainty. In a general circumstance, the dual-price tends to order more stock and price higher. It is due in large part to the effective markdowns that increases sale of deteriorated items and reduces the risk of excess inventory at the end of selling period.

The model we have used in this paper has some limitations. One of the most critical limitations is the assumption of independent demand, i.e., it does not take into account the effect of substitution, complement, or correlation between products. Considering demand correlation in the model is a new dimension for the underlying problem that requires a significant revision in model formulation and advanced solution technique. We hope future work will follow. Moreover, the assumption on two-typed demand that is either for new items or for old items may restrict its applicability. Fortunately, our model can be easily extended to an n-period model that will solve the problem with n-typed demands.

Let q_k be the quantity left for sales at the beginning of period k with restrictions of $q_{k+1} \le q_k \le Q$, $k > 1$, and let π_k be the profit generated from the k selling periods. We have the following recursive dynamic model for the general multi-period problem:

$$\begin{aligned}
&\pi_k(q_k, p) \\
&= -wq_k + \sum_{x=0}^{q_k}[px + \gamma\pi_{k+1}(q_k - x, p)]\Pr(\xi_k(p) = x) \\
&+ \sum_{x=q_k+1}^{\infty} pq_k \Pr(\xi_k(p) = x) \\
&= (p-w)q_k - \sum_{x=0}^{Q}[p(q_k - x) \\
&+\gamma\pi_{k+1}(q_k - x, p)]\Pr(\xi_k(p) = x) \qquad (14)
\end{aligned}$$

In practice, the n-period or continuous-time model is unrealistic because of the coordination and management costs associated with the dynamic pricing strategy and the confusing information that customers receive about the product's value. Levy *et al*. (1997) provided evidences of menu costs in grocery chains. On average, the cost of one-time price change is \$0.52 per item, \$105,887 per store, and consumes equivalently 35.2 percent of net margins.

In our models, the demand is a function of price through the distribution of reservation price that is Weibull distributed. The assumption of Weibull is for brevity; our models are indeed quite general that can use other probability functions such as uniform distribution. However, estimating densities for reservation price distributions is challenging that requires systematic data collection and analysis with the aid of sophisticated information technology.

Several interesting extensions under consideration include backlogging and mid-life or end-of-life returns. When customers are willing to wait to obtain fresh stock, the seller may use backlogging as a strategy to control deteriorating costs, especially in the case of selling highly perishable goods. In contrast to backlogging, a return policy allows retailers giving back excess stock to vendors, which provides flexibility in inventory and pricing strategy. As a concluding remark, we are aware of an emerging research issue inspired by Friend and Walker (2001) that direct retailing management toward an integrated and streamlined approach, i.e., optimizing the entire merchandizing chain: from buying to stocking to pricing.

ACKNOWLEDGMENT

This research was partially supported by the National Science Council (Taiwan) under Grant NSC91-2416-H-008-001.

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