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A Genetic Algorithm with a New Repair Process for Solving Multi-stage, Multi-machine, Multi-product Scheduling Problems

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Abstract. Companies that produce capital goods need to schedule the production of products that have complex product structures with components that require many operations on different machines. A feasible schedule must satisfy operation and assembly precedence constraints. It is also important to avoid deadlock situations. In this paper a Genetic Algorithm (GA) has been developed that includes a new repair process that rectifies infeasible schedules that are produced during the evolution process. The algorithm was designed to minimise the combination of earliness and tardiness penalties and took into account finite capacity constraints. Three different sized problems were obtained from a collaborating capital goods company. A design of experimental approach was used to systematically identify that the best genetic operators and GA parameters for each size of problem.

Keywords: Scheduling, Repair Process, Product Structure, Genetic Algorithm, Metaheuristics

1. INTRODUCTION

Sequencing determines the order of tasks based upon operation and assembly precedence relationships. Scheduling is defined as "the allocation of resources over time to perform a collection of tasks" (Baker, 1974). A schedule specifies sequence and timing, normally expressed as a set of start and due times (Hicks and Pongcharoen, 2006). Scheduling is a combinatorial optimisation problem that is classified as an NP hard problem (King and Spackis, 1980), which means that the amount of computation required to find solutions increases exponentially with problem size.

Various assumptions have been made in order to simplify, formulate and solve scheduling problems. The most common assumptions can be summarised as follows: a successor operation is performed immediately after its predecessor has finished, providing that the machine is available; each machine can handle only one operation at a time; each operation can only be performed on one machine at a time; there is no interruption of operations; there is no rework; setup and transfer times are of zero or uniform duration; and tasks are independent.

Production scheduling in the capital goods industry is difficult for several reasons. Firstly, demand is highly variable and uncertain. The products (e.g. steam turbine generators, power station boilers and transformer) are complex and are produced from components that require a large number of operations on machines with high capital and operating cost (which means that high utilisation is important). There are many operation and assembly dependency relationships. There are also multiple finite capacity resource constraints and the performance objectives may vary for different product families. Finally, the bespoke nature of production leads to large variations in product mix.

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Classical job shop and flow shop scheduling problems generally consist of a set of independent tasks. This is known as single stage scheduling (He and Hui, 2007) which means that there are no precedence constraints arising from assembly requirements (Blazewicz *et al.*, 1996a). Scheduling is important because companies seek to minimise lead-times and simultaneously achieve high resource utilisation.

There is a comprehensive literature related to classical flow and job shop scheduling (see for example Garey *et al.*, 1976; Graham *et al.*, 1979; Reisman *et al.*, 1994; Ruiz and Maroto, 2005). However, there is very little research that has considered multi-stage, multi-product scheduling problems with finite capacity multi-resource constraints. Reeja and Rajendran (2000) highlighted the lack of production scheduling research that has taken account of assembly relationships and constraints. Fry *et al.* (1989) recognized that there is a strong relationship between product structure and sequencing rule performance.

The objectives of this paper are to: i) describe a Genetic Algorithm based scheduling tool (GAST) for scheduling multiple products with deep and complex product structures; ii) explain a new repair process embedded within the GAST that rectifies infeasible schedules that may be generated during evolution process; iii) demonstrate the use of an effective experimental design for investigating the influence of problem size on the GA parameter settings using three representative problems that were obtained from a collaborating capital goods company.

The next section of this paper presents the literature review related to Genetic Algorithms. Section 3 explains the Genetic Algorithm that was developed to schedule the manufacture of complex products. Section 4 presents the experimental design and provides an analysis of the results. These are followed by the conclusions, which appear in section 5.

2. LITERATURE REVIEW

There are two categories of optimisation algorithm: conventional and approximation optimisation algorithms (Nagar *et al.*, 1995; Blazewicz *et al.*, 1996a). Conventional optimisation algorithms are usually based upon mathematical models. Examples include Integer Linear Programming, Branch and Bound and Dynamic Programming. Approximation optimisation algorithms are based upon constructive approaches (e.g. dispatching rules, etc.) and/or stochastic search techniques (e.g. Simulated Annealing, Taboo Search, Genetic Algorithm, Particle Swarm Optimization, Ant Colony Optimisation, Shuffled Frog Leaping and Artificial Immune Systems).

Genetic Algorithms (GA) have several advantages. GA deal with a coding of the problem instead of decision variables (Syarif *et al.*, 2002). They require limited domain knowledge (to represent the problem in terms of genes and chromosomes and also an objective function for evaluating the fitness of chromosomes) and use stochastic transition rules to guide the search (Goldberg, 1989). GA perform multiple directional search using a set of candidate solutions, whereas most conventional methods conduct single directional search (Gen and Cheng, 1997).

There are many research articles related to GA and their applications in the fields of production and operations management. Aytug *et al.* (2003) reviewed more than 110 GA articles published between 1996~2002. They found that GA parameters and operators had mostly been selected in an ad hoc fashion, rather than using a systematic design of experiments approach. A survey conducted by Chaudhry and Luo (2005) stated that 67.98% of 178 GA related research articles had used GA to solve scheduling or facility layout problems.

Much of the scheduling literature relates to classical job shop or flow shop problems and ignores assembly operations (Reeja and Rajendran, 2000). Multi-stage, multi-machine, multi-product scheduling (MMMS) is the allocation of resources over time to perform a collection of operations required by components, which are subsequently sub-assembled and assembled into products in accordance with the product structure. The number of levels of product structure indicates the number of stages of assembly precedence relationships. The leaf nodes within the product structure represent components. Since some components may require a number of operations to be performed on various machines, the sequence of operations for each component must consider the operation precedence constraint inherent its manufacturing process.

Figure 1 shows an example of a typical product structure obtained from a collaborating company engaged in the capital goods industry. It can be seen that a final

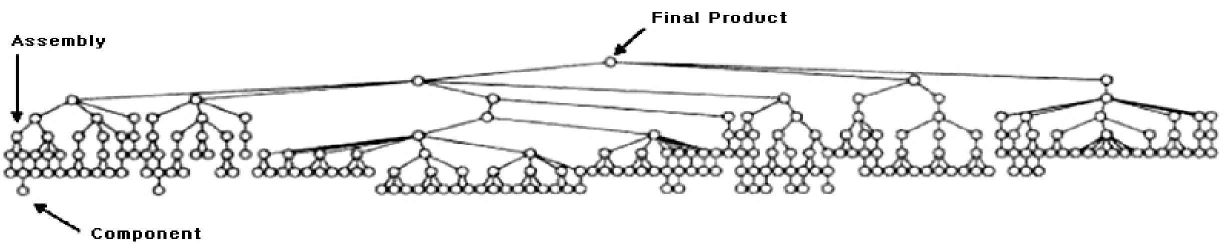


Figure 1. A typical product structure from a collaborating company (Hicks and Pongcharoen, 2006).

product (P) required assemblies, subassemblies, parts and components (C), in each of which a sequence of operations (O) to be performed on multiple machines (M) is specified. In order to formulate the multi-stage, multi-machine, multi-product scheduling (MMMS) model, the following notation is introduced for using in the model.

Notation:

i	operation i^{th} ($i = 1, \cdots, O$)
j	part or component j^{th} ($j = 1, \cdots, C$)
k	final product k^{th} ($k = 1, \cdots, P$)
m	machine m^{th} ($m = 1, \cdots, M$)
R_m	ready time of machine m^{th} (date)
C_k	completion time of product k^{th} (date)
D_k	due date of product k^{th} (date)
C_{jk}	completion time of component j^{th} in product k^{th} (date)
D_{jk}	due date of component j^{th} in product k^{th} (date)
E_k	earliness duration of product k^{th} (days)
E_{jk}	earliness duration of component j^{th} in product k^{th} (days)
T_k	tardiness duration of product k^{th} (days)
SU_{ijkm}	setup time of operation i^{th} for component j^{th} in product k^{th} on machine m^{th} (minute)
ST_{ijkm}	start time of operation i^{th} for component j^{th} in product k^{th} on machine m^{th} (minute)
PT_{ijkm}	processing time of operation i^{th} for component j^{th} in product k^{th} on machine m^{th} (minute)
FT_{ijkm}	finishing time of operation i^{th} for component j^{th} in product k^{th} on machine m^{th} (minute)
TT_{ijkm}	transfer time of operation i^{th} for component j^{th} in product k^{th} on machine m^{th} (minute)
$X_{ijkabcm}$	1 if operation i^{th} for component j^{th} in product k^{th} precedes operation a^{th} for component b^{th} in product c^{th} on machine m^{th}; and 0 otherwise (minute)
Pe	earliness penalty (currency unit per day)
Pt	tardiness penalty (currency unit per day)
$S(x)$	set of child items of item x
Sh	working hour per shift (minutes per shift)

The scheduling objective is based on the Just in Time philosophy that aims to minimise the combination of earliness penalties for components and final products and tardiness penalties of final products. The mathematical model for multi-stage multi-machine multi-product scheduling (MMMS) problem is as follows:

Penalty cost

$$= \sum_{j=1}^{C}\sum_{k=1}^{P} Pe(E_{jk}) + \sum_{k=1}^{P} Pe(E_k) + \sum_{k=1}^{P} Pt(T_k) \tag{1}$$

Subject to:

$$ST_{ijkm} \geq R_m \qquad \forall i, j, k, m \tag{2}$$

$$FT_{ijkm} = ST_{ijkm} + SU_{ijkm} + PT_{ijkm} + TT_{ijkm} \qquad \forall i, j, k, m \tag{3}$$

$$C_{jk} \geq FT_{ijkm} \qquad \forall i, j, k, m \tag{4}$$

$$E_{jk} = (D_{jk} - C_{jk})/Sh \qquad \forall j, k \tag{5}$$

$$E_k = (D_k - C_k)/Sh \qquad \forall k \tag{6}$$

$$T_k = (C_k - D_k)/Sh \qquad \forall k \tag{7}$$

$$ST_{ihkm} - ST_{ijkm} \geq SU_{ijkm} + PT_{ijkm} + TT_{ijkm} \qquad \forall i, k, m, j \in S(h) \tag{8}$$

$$ST_{gjkm} - ST_{ijkm} \geq SU_{ijkm} + PT_{ijkm} + TT_{ijkm} \qquad \forall j, k, m, g = i + 1 \tag{9}$$

$$X_{ijkabcm} + X_{abcijkm} = 1 \qquad \forall a, b, c, i, j, k, m \tag{10}$$

$$X_{ijkabcm} \in \{0, 1\} \qquad \forall a, b, c, i, j, k, m \tag{11}$$

$$E_{jk}, E_k, T_k \geq 0 \qquad \forall j, k \tag{12}$$

$$ST_{ijkm}, R_m \geq 0 \qquad \forall i, j, k, m \tag{13}$$

$$FT_{ijkm}, ST_{ijkm}, SU_{ijkm}, PT_{ijkm}, TT_{ijkm} \geq 0 \qquad \forall i, j, k, m \tag{14}$$

This mathematical model was derived to describe the MMMS problem considered in this research. The objective function (1) contains three parts: i) an earliness penalty for components; ii) an earliness penalty for final products; and iii) a tardiness penalty for final products. Constraint (2) ensures that the start time of all operations is not earlier than the time the machines are ready. Constraint (3) makes sure that the finishing time for each operation is determined by the start, setup, machining and transfer times. Constraint (4) ensures that the completion time for each component is not before the finishing time. The number of days earliness and tardiness for components and final products are defined by constraints (5), (6) and (7). Part precedence constraints relating to the product structure are defined by constraint (8). For example, if part j is child item of part h, then the start time of the parent part (ST_{ihkm}) minus the start time of the child item (ST_{ijkm}) should be greater or equal to the sum of setup, processing and transfer times for the child item. Constraint (9) makes sure that the operation precedence constraints within a component are satisfied. For example, if a successor operation (g) is performed after its predecessor operation (i) is finished, the start time of the successor operation (ST_{gjkm}) minus the start time of the predecessor operation (ST_{ijkm}) should be greater or equal to the sum of setup, processing and transfer times of the predecessor operation. Constraint (10) ensures that only one operation can be performed on a machine using the decision variables defined by constraints in (11). Constraints (12)-(14) guarantee non-negative values for those defined variables.

There is only limited research that relates to this multi -stage multi-machine multi-product scheduling (MMMS) problem. Pongcharoen *et al.* (2004) described a Genetic Algorithm tool that was developed for solving the MMMS problem and reported that the schedules obtained from GA outperformed the schedules obtained from a collaborating company. Pongcharoen *et al.* (2002) investigated the significance of the proposed repair process and found the best values for GA parameters using regression analysis. However, the computational experiments conducted in these papers ignored the genetic operations. The repair process used consisted of four steps: i) operation precedence adjustment; ii) part precedence adjustment; iii) timing assignment; and iv) deadlock adjustment. The part precedence adjustment was designed to satisfy product structure constraints i.e. an assembly requires all its subassemblies and components to be complete before it can be produced. A deadlock situation may arise when there is a conflict between operation and part precedence constraints for different items that both require a common resource (Pongcharoen *et al*., 2002).

Previous research relating to the multi-stage multi-machine multi-product scheduling (MMMS) problem has not provided an exact mathematical formulation of the problem as outlined above. Chen and Ji (2007) applied a mixed integer programming method to solve a simple MMMS problem. Due to the limitation of their proposed method, the numerical experiments were based on a small-size problem that consisted of 2 final products with a requirement for 21 operations to manufacture 21 items (excluding final products). Some of the components were duplicated within the product structure. Each item only required one operation. Operation precedence constraints, setup and transfer times were ignored in their formulation.

3. GENETIC ALGORITHM FOR MMMS

The Genetic Algorithm mechanism starts by encoding the problem to produce a list of genes. The genes are represented by either numeric (binary or real), or alphanumeric characters. Blazewicz *et al*. (1996b) suggested that the binary chromosome representation is often unsuitable for combinatorial optimization problems because it is very difficult to represent solutions. Therefore numeric or alphanumeric coding is often used. The next step is to randomly combine genes to produce a population of chromosomes, each of which represents a possible solution. Genetic operations (crossover and mutation) are next performed on chromosomes, which are randomly selected from the population as parents to produce offspring. A fitness function (objective function) is used to measure the chromosomes' fitness value, from which the probability of their survival is determined. The most famous process is the 'Roulette Wheel' (Goldberg, 1989) which may also be used for chromosome selection. The GA process is repeated until a termination condition is satisfied.

In this paper, the general Genetic Algorithm described by Goldberg (1989) was modified to include a new repair process for scheduling the manufacture of complex products in the multi-stage multi-machine environment. A schematic of the Genetic Algorithm based scheduling tool (GAST) is shown in Figure 2. It includes the following steps: i) chromosome presentation and initialisation; ii) a repair process; iii) fitness evaluation; iv) roulette wheel selection; v) a check of the termination conditions; and vi) genetic operations (crossover and mutation). The GAST program includes a graphic user interface and graphical outputs (e.g. Gantt charts). The program consists of nearly 10,000 lines of TCL programming language code (Ousterhout, 1994).

3.1 Chromosome Representation and Initialisation

In this research the operations on each part were encoded into genes represented by alphanumeric strings. They had two parts: a part code (P) and an operation (O) number (see Figure 3). All the genes were randomly sequenced to generate a chromosome. This was repeated to generate a population of the specified size. It should be emphasized that the chromosome representation considered in this work was not separated into sub-chromosomes, which had been the approach adopted by previous research by the authors (Pongcharoen *et al*., 2002 and 2004). The undivided chromosome used was beneficial to the repair process that is described in the next sub-section.

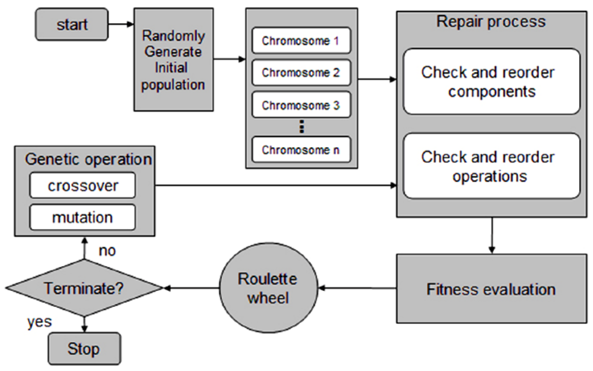


Figure 2. Structure of the GA based Scheduling Tool (GAST).

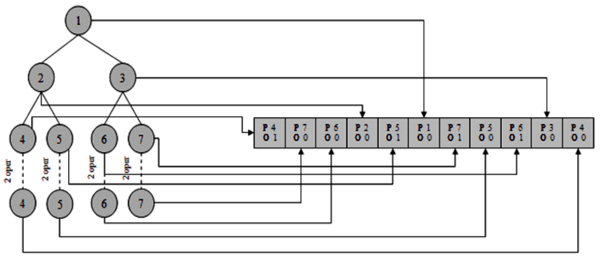


Figure 3. Chromosome representation.

3.2 Repair Process

The processes of chromosome initialisation and genetic operations within the evolution procedure within the GA may produce infeasible chromosomes (schedules) that represent impractical solutions that contravene constraints. There are three ways to deal with infeasible solutions: i) discard them; ii) apply a high penalty in the fitness function so that they are unlikely to survive; or iii) repair them (Blum and Roli, 2003). Discarding infeasible solutions or applying a high penalty is only an option when a large proportion of the population is feasible. Otherwise, the GA process could be terminated if all chromosomes in the population are infeasible. Several researchers have used a repair process for rectifying infeasible chromosomes in timetabling research (e.g. Caldeira and Agostinho, 1997; Colorni *et al*., 1998) and scheduling research (Pongcharoen *et al*., 2004). The repair processes are problem specific due to differences between problem domains, chromosome representation, structures of the solution space and the nature of constraints. The undivided chromosome representation used in this work allowed a new compact repair process to be developed that has only two stages: i) part precedence adjustment; and ii) operation precedence adjustment. These processes are described in the following sub-sections.

3.2.1 Part Precedence Adjustment

In previous research part precedence adjustment was based upon re-sequencing based upon the level of a part within the product structure. In this research part precedence adjustment was based upon the assembly lines defined in its product structure. For example, if there is a product structure (e.g. the product structure in Figure 3), part number 1, 2 and 3 require one operation whilst the remaining parts (part no. 4, 5, 6 and 7) require two operations. The process of checking and reordering parts was based on four assembly lines specified in the product structure. Figure 4 shows an example of checking and reordering parts in the first assembly line. The process began by selecting genes involved in the first assembly line for precedence checking regarding to the assembly line. If the selected genes had been in the wrong order (infeasible), then the selected genes are reordered based on its assembly requirement and finally replaced back to the positions they are selected but in a new feasible sequence. This process was repeated until all assembly lines in the product structure were considered.

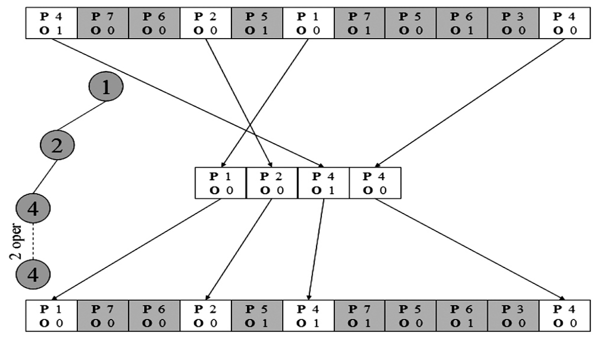


Figure 4. Example of the new part precedence adjustment.

The part precedence adjustment described allows the repair process to always avoid the deadlock situations that arose in previous research. This means that there was no conflict arising from operation and part precedence constraints. The deadlock adjustment was therefore excluded from the repair process proposed as it was unnecessary. The new repair process is therefore easier and requires fewer steps than those used in previous research. It is therefore more computationally efficient.

3.2.2 Operation Precedence Adjustment

The process of operation precedence adjustment was applied to each part that required two or more operations. For example, the product structure shown in Figure 3 has four parts (part no. 4, 5, 6 and 7) that required two operations for each part. Figure 5 shows an example of checking and reordering of the two operations required for part number 4. The genes corresponding to the operations on each part are identified and the order of operations is checked to ensure that is satisfies operation precedence constraints. If the selected genes are in the wrong order, then the selected genes are reordered in a new sequence. The process is repeated until the operation precedence constraints for all parts are satisfied.

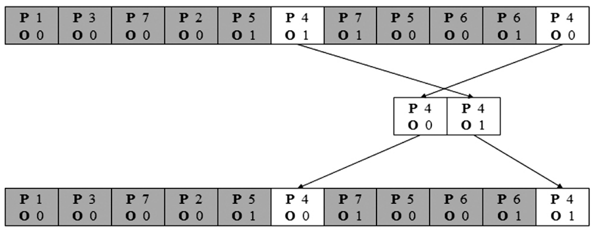


Figure 5. Example of operation precedence adjustment.

3.3 Fitness Function

The next stage was to measure the fitness of the chromosomes. The quality of schedules was evaluated by using the fitness (objective) function given in Equation (1), which aggregated the penalty cost arising from earliness and tardiness.

3.4 Roulette Wheel Selection

This stage produced a new generation with the same population size as the initial population. The roulette wheel (Goldberg, 1989) approach was used for chromosome selection. It used a random number generator over the range 0-1 (Wichmann and Hill, 1982). The probability of survival and number of replicates of a chromosome in the next generation was determined by its fitness. The GA process was repeated until the termination criterion was satisfied.

3.5 Genetic Operations: Crossover and Mutation

There are two types of genetic operator: crossover and mutation. A crossover operator generally combines the characteristics of two parents to produce an offspring, whilst mutation usually produces random change in one chromosome. Many crossover and mutation operators have been developed and reviewed in the literature (see for example Pongcharoen *et al*., 2001, which considered a comprehensive list). In this research, the nine crossover and nine mutation operators shown in Table 1 were considered.

A Genetic Algorithm requires parameters and operators to be specified. There is no particular configuration that can guarantee to find the best solution for different problem sizes or domains. The configuration of GA structure should therefore be systematically investigated through using a design of experiments approach (Montgomery, 2001), which is described in the next section.

Table 1. Crossover and mutation operators.

Crossover operators (COP)	Mutation operators (MOP)
Enhanced edge recombination (EERX)	Enhanced 2-operation random swap (E2OR)
Order crossover (OX)	Inverse mutation (IM)
Partially mapped crossover (PMX)	Shift operation mutation (SOM)
Cycling crossover (CX)	Three operations adjacent swap (3OAS)
Position based crossover (PBX)	Three operations random swap (3ORS)
Maximal preservation crossover (MPX)	Two operations adjacent swap (2OAS)
One point crossover (1PX)	Two operations random swap (2ORS)
Two point centre crossover (2PCX)	Centre inverse mutation (CIM)
Mixed-crossover (MXOV)	Mixed-mutation (MMUT)

4. EXPERIMENTAL DESIGN AND ANALYSIS

In this work, three sizes of industrial scheduling problem adopted from Pongcharoen *et al*. (2004) were considered (see Table 2). Data, including production schedules, product structure relationships, process plans and resource loading information were obtained from a collaborating company that was in the make/engineer to order capital good industry. The first (small) problem, involved two different products (245 and 451), which had a combined requirement of twenty-five machining operations on eight resources with nine assembly operations. The second and third problems were termed medium and large, respectively. The large problem involved 118 machining and 17 assembly operations on 17 resources. There were interactions and contention for resources.

The aim of the computational experiments was to investigate the performance of GA and to identify the best configuration of parameters and operators for the three problem sizes. The GA parameters and operators were considered as factors, each of which had three levels (see Table 3).

The first factor was the combination of the population size and the number of generations (P*G), determined the total number of chromosomes (candidate solutions) created and therefore had a strong influence on the execution time. A large number of generations with a large population size increased the amount of search and increased the probability of finding an optimal solution. Due to the limitation of execution time and resources required for each computational run, the combination of P*G considered in this work was fixed at 2,500 chromosomes. The next two factors were the probabilities of crossover (%C) and mutation (%M). The range considered for these factors were adopted from previous research (Todd, 1997; Pongcharoen *et al*., 2002). Finally, nine crossover (COP) and nine mutation operations (MOP) shown in Table 1 were considered as last two factors.

With these experimental factors and levels, a full factorial experimental design would require 2,187 (3x3x 3x9x9) computational runs for a single replication. In order to reduce this, a novel experimental design was

Table 2. Industrial scheduling problem.

Problem number	Part number	Characteristics of scheduling problems				
		Number of Products	Number of Components	Machining/Assembly Operation	Number of Resources	Levels of product structure
1	245 and 451	2	6	25/9	8	4
2	229 and 451	2	8	57/10	7	4
3	4 and 228	2	12	118/17	17	4

Table 3. Experimental factors and its levels.

Factors	Levels	Values		
		Low (-1)	Medium (0)	High (+1)
Propulation * Generation (P * G)	3	25 * 100	50 * 50	100 * 25
Probability of Crossover (%C)	3	0.1	0.5	0.9
Probability of Mutation (%M)	3	0.05	0.1	0.15
Crossover operators (COP)	9		See Table 1	
Mutation operation (MOP)	9		See Table 1	

adopted to reduce the number of runs required. The proposed design embedded a one-third (3^{3-1}) fractional factorial experimental design within the Latin Square experimental design. The factorial design (see Table 4) involved nine combination of treatments or settings denoted by the alphabet letters A, B, C, ···, I. Each combination was then embedded into the Latin Square design (see Table 5), which was based on the concept of a blocking experiment (Montgomery, 2001). It can be seen that each alphabet letter (treatment) was applied once and only once in each row (mutation) and column (crossover). Using the proposed experimental design, the total numbers of runs were reduced from 2,187 to 81 for each replication. This design dramatically reduced the computational effort by 96%.

Experimental results were obtained from five replicates that used different random seed numbers. The results were then analysed using a general linear form of the analysis of variance and main effect plots were produced. Table 6 shows the *F* and *p* values for the main factors for each problem size. It can be seen that all the main factors were statistically significant with a 95% confidence interval as they had *p* values that were less than or equal to 0.05. However, P*G was statistically insignificant for medium problem. This means that the performance of GA was dependent upon the configuration of the GA parameters and operators used. It should be noted that the average execution time were 20, 50 and 130 seconds for the small, medium and large problem, respectively (using a PC with AMD 1.3 GHz CPU with 128 MB RAM).

Table 4. The one-third (3^{3-1}) fractional factorial design.

Combine	P*G	%C	%M	U = 0
A	25/100	0.1	0.05	000
B	25/100	0.5	0.15	012
C	50/50	0.1	0.1	101
D	100/25	0.1	0.15	202
E	25/100	0.9	0.1	021
F	50/50	0.5	0.05	110
G	50/50	0.9	0.15	122
H	100/25	0.5	0.1	211
I	100/25	0.9	0.05	220

In order to identify the appropriate settings of the significant parameters, main effect plots for the statistically significant parameters are presented in Figure 6. It can be seen that, for the small problem, the best results

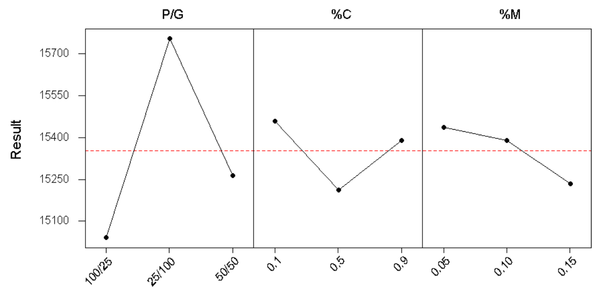
Table 5. The Latin Square design.

Design	CX	EERX	MPX	IPX	OX	PBX	PMX	2PCX	MOXV
2OAS	A	I	H	G	F	E	D	C	B
3OAS	B	A	I	H	G	F	E	D	C
2ORS	C	B	A	I	H	G	F	E	D
3ORS	D	C	B	A	I	H	G	F	E
IM	E	D	C	B	A	I	H	G	F
SOM	F	E	D	C	B	A	I	H	G
CIM	G	F	E	D	C	B	S	I	H
E2ORS	H	G	F	E	D	C	B	A	I
MMUT	I	H	G	F	E	D	C	B	A

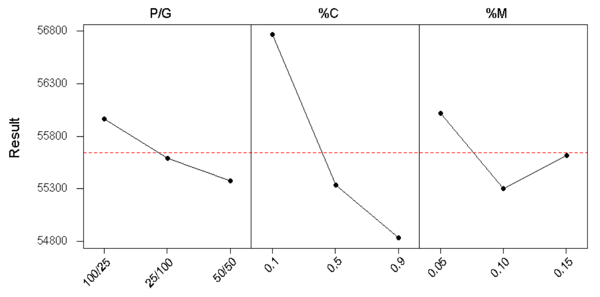
Table 6. The analysis of variance on three problem sizes.

		Problem sizes					
		Small		Medium		Large	
Source	DF	F	P	F	P	F	P
P * G	2	36.46	0.000	1.99	0.138	13.29	0.000
%C	2	4.46	0.012	22.31	0.000	21.56	0.000
%M	2	3.09	0.047	2.87	0.058	7.64	0.001
COP	8	8.17	0.000	35.21	0.000	40.14	0.000
MOP	8	4.71	0.000	2.67	0.007	2.81	I0.005
Seed	4	4.82	0.001	16.8	0.000	19.27	0.000
Error	378						
Total	404						

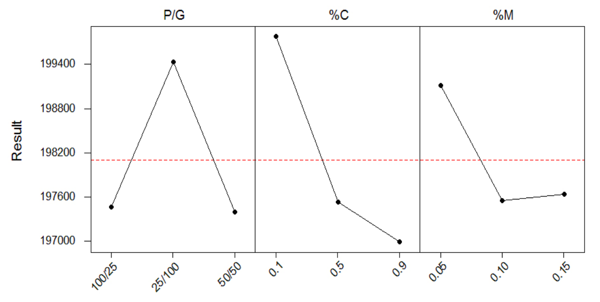
were obtained from the setting of P*G at 100*25 with the probabilities of crossover and mutation set at 0.5 and 0.15 respectively. For the medium problem, the best configuration was 50*50, 0.9 and 0.1, respectively. Whilst the best setting for the large problem was found to be at 50*50, 0.9 and 0.1, respectively. These results indicated that the best GA configurations are problem specific and are influenced by the structure of the problem domain, the size of the solution space and the nature of the constraints.



(a) Small problem



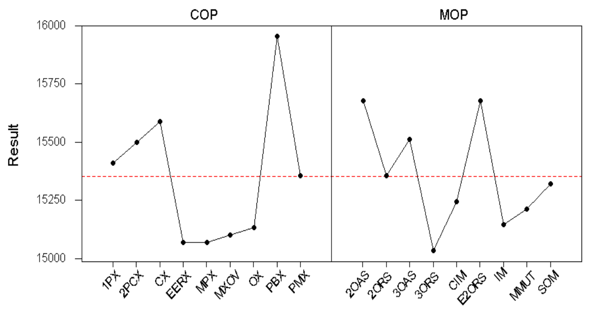
(b) Medium problem



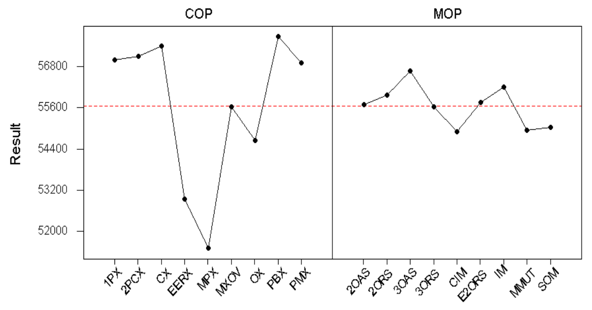
(c) Large problem

Figure 6. Main Effect Plot of the GA parameters.

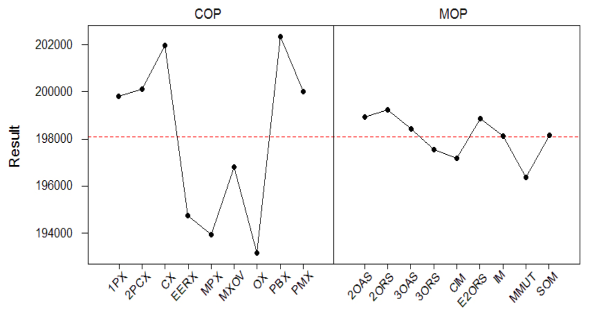
Main effect plots for both statistically significant operators are presented in Figure 7. This suggested that the best crossover operators for small, medium and large size problems were MPX, MPX and OX respectively. For mutation, the appropriate operators were 3ORS, CIM and MMUT. It can be concluded that the efficiency of GA operators were differentiated according to the size of the scheduling problem.



(a) Small problem



(b) Medium problem



(c) Large problem

Figure 7. Main Effect Plot of the GA operators.

Even though the efficiency of the GA operators was different for the various problem sizes considered, it was worth investigating which operator generally performed best. Table 7 shows the mean and standard deviation of the penalty costs associated with the schedules obtained for the different problems. When considering the best and the second best performance of both crossover and mutation operators for each problem sizes, the maximal preservation crossover (MPX) and the mixed mutation (MMUT) operators were found to help the GA process to achieve the lowest mean penalty costs.

Table 7. The mean and standard deviation of COP and MOP.

	Small		Medium		Large	
	Mean	SD	Mean	SD	Mean	SD
Crossover						
1PX	15411(6)	937	57000(6)	2284	199811(5)	4571
2PCX	15500(7)	1034	57100(7)	1718	200122(7)	4111
CX	15589(8)	1083	57400(8)	1938	201956(8)	3834
EERX	15067(2)	252	52933(2)	3715	194744(3)	4279
MPX	**15067(1)**	229	**51511(1)**	5230	193922(2)	6449
MXOV	15100(3)	294	55622(4)	1995	196789(4)	3677
OX	15133(4)	360	54656(3)	2840	**193167(1)**	4430
PBX	15956(9)	1283	57667(9)	1822	202356(9)	3837
PMX	15356(5)	889	56911(5)	1923	200000(6)	3999
Mutation						
2OAS	15678(9)	1262	55689(5)	2859	198944(8)	5055
2ORS	15356(6)	816	55978(7)	2650	199244(9)	4615
3OAS	15511(7)	1084	56667(9)	2527	198433(6)	5735
3ORS	**15033(1)**	126	55633(4)	2647	197533(3)	5338
CIM	15244(4)	529	**54900(1)**	5075	197189(2)	6876
E2ORS	15678(8)	1207	55756(6)	4068	198878(7)	5651
IM	15144(2)	229	56200(8)	2130	198133(4)	4215
MMUT	15211(3)	420	54944(2)	3841	**196367(1)**	6202
SOM	15322(5)	886	55033(3)	4243	198144(5)	5112

5. CONCLUSIONS

This paper described the Genetic Algorithm based Scheduling Tool (GAST) for solving the multi-stage multi-machine multi-product scheduling (MMMS) problem. A mathematical model of MMMS problem was proposed. The model considered setup and transfer times and the operation precedence constraints within each component, which had been ignored by previous research. The proposed algorithm included a new repair process that was embedded within the proposed GA for rectifying infeasible schedules that were generated in the evolution process. A new repair process was used that always avoided deadlock, which was a problem in previous research. The algorithm minimises the combination of earliness and tardiness penalties with finite resource capacity. The GAST was applied to solve three representative problems obtained from a collaborating capital goods company.

The computational experiments were aimed to investigate the appropriate settings of Genetic Algorithm parameters and mechanisms for various problem sizes using an advanced experimental design. The proposed design dramatically decreased the computational time and computing resource required. The analysis of experimental results obtained from the GAST indicated that all the Genetic Algorithm parameters and mechanisms influenced the results obtained for all three problem sizes. However, the efficiency of crossover and mutation operators varied depending on the problem size. It suggested that the appropriate setting of GA parameters and mechanisms were dependent on the size and the structure of the solution space, which was directly related to the problem size.

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