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Desirability Function Modeling for Dual Response Surface Approach to Robust Design

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Abstract. Many quality engineering practitioners continue to have a considerable interest in implementing the concept of response surface methodology to real situations. Recently, dual response surface approach is extensively studied and recognized as a powerful tool for robust design. However, existing methods do not consider the information provided by customers and design engineers. In this regard, this article proposes a flexible optimization model that incorporates that information via desirability function modeling. The optimization scheme and its modeling flexibility are demonstrated through an illustrative example by comparing the proposed model with existing ones.

Keywords: Robust Design, Desirability, Response Surface Methodology

1. INTRODUCTION

Robust design (RD) is a cost-effective methodology for determining the best settings of the control factors that make product performance insensitive to the influence of noise factors. Taguchi first introduced the concept of RD by using orthogonal arrays and signal-to-noise ratios. Even if the inclusion of noise factors for design optimization has been considered as an innovative concept by researchers, there is a general consensus that three major problems are embodied in the Taguchi's approach. First, signal-to-noise ratios are heuristic tools to minimize the quality loss. Secondly, orthogonal arrays are not convincing, particularly when there are high interactions among control and noise factors. Finally, his approach lacks a sequential formal investigation (see, e.g., Box 1985). A review article by Myers *et al*. (1989) revealed that practitioners continue to have a considerable interest in applying the concept of response surface methodology to many quality engineering problems. RSM can be used to estimate the relationship between the response (y) and control variables (x_i 's). For example, consider a system involving a response y which depends on k control variables $(x_1, x_2, \cdots, x_k)$. Assuming a second-order polynomial model, the fitted response function can be expressed as

$$\hat{y}(\boldsymbol{x}) = \hat{\beta}_{\boldsymbol{0}} + \sum_{i=1}^{k} \hat{\beta}_i x_i + \sum_{i=1}^{k} \sum_{j=i}^{k} \hat{\beta}_{ij} x_i x_j \qquad (1)$$

Thus, the design engineers can find the optimal setting for the control variables that optimizes the response. It was pointed out that such a model works well when the variance of the response is relatively small and stable, but when the variance is unstable, classical response surface method could be misleading (Kim and Lin 1998).

Since first introduced by Myers and Carter (1973), a

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dual response surface approach, which models both the process mean and standard deviation as separate responses, has received a lot of attention in the context of RD (see, e.g., Vining and Myers 1990, Castillo and Mongomery 1993, Lin and Tu 1995, and Kim and Lin 1998). Let $\hat{\mu}(\boldsymbol{x})$ and $\hat{\sigma}(\boldsymbol{x})$ represent the fitted response functions for the mean and the standard deviation of the quality characteristic, respectively. Assuming a second-order polynomial model for the response functions, we get

$$\hat{\mu}(\boldsymbol{x}) = \hat{\beta}_0 + \sum_{i=1}^{k} \hat{\beta}_i x_i + \sum_{i=1}^{k}\sum_{j=i}^{k} \hat{\beta}_{ij} x_i x_j$$
and
$$\hat{\sigma}(\boldsymbol{x}) = \hat{\gamma}_0 + \sum_{i=1}^{k} \hat{\gamma}_i x_i + \sum_{i=1}^{k}\sum_{j=i}^{k} \hat{\gamma}_{ij} x_i x_j \qquad (2)$$

Now, these response functions can be used in optimization models to find optimal setting for control variables.

Castillo and Mongomery (1993) suggested an optimization model to find an optimal setting of control factors, which minimizes the standard deviation while keeping bias at zero. However, the solution may not minimize the expected quality loss due to the unrealistic constraint of keeping the process mean at the target value. Later, Lin and Tu (1995) suggested that minimizing the expected loss might provide better solution than that of models based on zero-bias logic, even if the process mean might be off the target. All the above approaches did not consider the information fed by customers and design engineers, if available. For example, if a customer allows a certain level of bias in process mean and concerns about taking the variance down, the optimization model by Vining and Myers (1990) and Castillo and Montgomery (1993) might be inappropriate. To the contrary, if a design engineer did not care about the variance up to a reasonable level and emphasized getting process mean as close to the target value as possible, the approach taken by Lin and Tu (1995) might be inadequate. In the example discussed in Lin and Tu (1995), the squared bias has rare effect on the expected quality loss. Consequently, minimization of the expected quality loss is mostly determined by the variance while sacrificing the performance in bias. Furthermore, such a quite good performance in variance may hardly be obtained in some practical situations. Those kinds of information need to be considered in designing processes.

In this regard, Kim and Lin (1998) proposed an optimization model that balances the grade of membership functions (or equivalently, the degree of desirability functions) for process mean and standard deviation. However, balanced desirability could not maximize the level of composite desirability. Thus, the optimal solution may not be the most desirable one from the viewpoint of a customer or a design engineer. The main purpose of this paper is to propose an optimization model that incorporates the information on mean and variance, provided by customers and design engineers, to construct the desirability functions, and then maximizes the composite desirability of mean and standard deviation. Thus, the proposed model is flexible in the sense that preferences of customers and design engineers can be reflected. Modeling flexibility of our approach is demonstrated in an example and compared with existing methods.

2. DESIRABILITY FUNCTION

Since first introduced by Harrington (1965), the desirability functions have been used extensively to simultaneously optimize several responses by balancing performances of the responses (Derringer and Suich 1980, Derringer 1994). Balancing performances inevitably involves the trade-off among responses. When balancing, the most ambiguous and important point is how to trade-off responses having different magnitudes. Desirability functions provide the basis for responses with different magnitudes to be measured in the same scale.

Any levels of a response can be mapped onto the desirability function, which ranges from zero to one. A zero-level of desirability implies that performances of the corresponding response may not be acceptable, while a desirability of one can be considered as the most satisfactory level of the response's performance. The desirability function can be obtained by transforming a performance level of the response to a desirability value. Suppose a response y, which depends on k control variables $(x_1, x_2, \cdots, x_k)$, is estimated by a second-order polynomial model as equation (1). Assuming that a specific target value is the most satisfactory level of the response y and performances below a lower limit or above an upper limit are unacceptable (i.e., nominal-the-best). Then, a desirability function can be constructed as

$$d_y = \begin{cases} \left(\dfrac{\hat{y}(\boldsymbol{x}) - y_{\min}}{\tau - y_{\min}}\right)^s, & \text{if } y_{\min} < \hat{y}(\boldsymbol{x}) < \tau \\ \left(\dfrac{y_{\max} - \hat{y}(\boldsymbol{x})}{y_{\max} - \tau}\right)^t, & \text{if } \tau < \hat{y}(\boldsymbol{x}) < y_{\max} \\ 0, & \text{otherwise,} \end{cases} \qquad (3)$$

where $y_{\max}$ and $y_{\min}$ represent the upper and lower limits, respectively, and τ is the target value. For the case of "smaller-the-better" responses, the desirability functions can be modeled in a similar way.

$$d_y = \begin{cases} 1, & \text{if } \hat{y}(\boldsymbol{x}) \le y_{\min} \\ \left(\dfrac{y_{\max} - \hat{y}(\boldsymbol{x})}{y_{\max} - y_{\min}}\right)^r, & \text{if } y_{\min} < \hat{y}(\boldsymbol{x}) < y_{\max} \\ 0, & \text{if } \hat{y}(\boldsymbol{x}) \ge y_{\max}. \end{cases} \qquad (4)$$

In equations (3) and (4), r, s, and t are user-specified weights to allow the engineer to accommodate nonlinear desirability functions. (Derringer and Suich 1980) If the values of r, s, and t are set to one, the desirability functions are linear. Applying the desirability functions to multiple responses system, we need to find a composite desirability which is defined as the geometric mean of the individual desirabilities (Derringer and Suich 1980, Derringer 1994, Castillo *et al*. 1996). For an n responses system, the overall performance of the system is determined by the composite desirability D, which can be expressed as

$$D = \left[d_1 d_2 \cdots d_n\right]^{\frac{1}{n}} \tag{5}$$

where d_i's represent desirability functions of the responses and defined by equations (3) and (4) according to the characteristics of corresponding responses. The rationale behind using the geometric mean is that if the desirability of any response is zero at some operating condition then the product usually turns out to be unacceptable regardless of the performances of other responses. As the dual response surface approach to robust design involves two responses, mean and standard deviation, the concept of composite desirability may be applied to find the optimal setting of control factors.

3. PROPOSED OPTIMIZATION STRATEGY

The objective of RD is to find an optimal setting of control factors which minimizes bias and variance of process simultaneously. However, the optimal solution minimizing bias is usually different from one minimizing variance, and vice versa. Thus, we need to compromise between individual solutions. Vining and Myers (1990) and Castillo and Montgomery (1993) provided compromise solutions by assigning preemptive priority to minimizing bias. Lin and Tu (1995) yielded to compromise solutions by giving equal weights to squared bias and variance. Proposed optimization model is to provide the most desirable compromise solution that maximizes the composite desirability of mean and standard deviation. The nominal-the-best case for process mean will be focused to illustrate the proposed model, and it is obvious that standard deviation is a smaller-the-better case.

Assume that second-order polynomial models are appropriate to approximate response functions of process mean and standard deviation as in equation (2). The lower and upper limits of bias and standard deviation need to be set to define individual desirabilities. It is quite reasonable assuming that an engineer can specify the upper and lower limits to avoid unusual derailments of the process. As in equations (3) and (4), the desirability functions for process mean and standard deviation are given by

$$d_\mu = \begin{cases} \left(\dfrac{\hat{\mu}(\boldsymbol{x}) - \mu_{\min}}{\tau - \mu_{\min}}\right)^s, & \text{if } \mu_{\min} < \hat{\mu}(\boldsymbol{x}) < \tau \\ \left(\dfrac{\mu_{\max} - \hat{\mu}(\boldsymbol{x})}{\mu_{\max} - \tau}\right)^t, & \text{if } \tau < \hat{\mu}(\boldsymbol{x}) < \mu_{\max} \\ 0, & \text{otherwise,} \end{cases} \tag{6}$$

$$d_\sigma = \begin{cases} 1, & \text{if } \hat{\sigma}(\boldsymbol{x}) \le \sigma_{\min} \\ \left(\dfrac{\sigma_{\max} - \hat{\sigma}(\boldsymbol{x})}{\sigma_{\max} - \sigma_{\min}}\right)^r, & \text{if } \sigma_{\min} < \hat{\sigma}(\boldsymbol{x}) < \sigma_{\max} \\ 0, & \text{if } \hat{\sigma}(\boldsymbol{x}) \ge \sigma_{\max}, \end{cases} \tag{7}$$

where $\mu_{\min}$ and $\mu_{\max}$ represent the lower and upper limits of process mean, respectively, while $\sigma_{\min}$ and $\sigma_{\max}$ are those of standard deviation. The value of $\sigma_{\min}$ is usually set to zero, however, may take positive values if the engineer does not care about the variability up to a certain level in some cases (Kim and Lin 1998). The linear desirability functions (i.e., $r = s = t = 1$) are focused in this paper. Nonlinear desirability functions are also studied later to accommodate the user-specified weights.

RD requires a simultaneous optimization of mean and standard deviation. Since a solution that optimizes both mean and standard deviation hardly exists, we need to make a compromise between them. The proposed optimization model, which provides a compromise solution maximizing the composite desirability D, can be written as

$$\begin{aligned} \text{Maximize} \quad & D = \left(d_\mu \cdot d_\sigma\right)^{1/2} \\ \text{subject to} \quad & \mu_{\min} \le \hat{\mu}(\boldsymbol{x}) \le \mu_{\max} \\ & \sigma_{\min} \le \hat{\sigma}(\boldsymbol{x}) \le \sigma_{\max} \\ & \boldsymbol{x} \in [\boldsymbol{x_L}, \boldsymbol{x_U}], \end{aligned}$$

where $\boldsymbol{x_L}$ and $\boldsymbol{x_U}$ are lower and upper limits for $\boldsymbol{x}$, respectively.

The most significant advantage of our model over the existing ones comes from its modeling flexibility. Customers' or design engineers' preferences can be included in the proposed model either by adjusting the limits of mean and bias or by assigning appropriate user-specified weights. For example, the desirability function can be asymmetric as the design engineer's preference changes at different rates. If the process mean lower than target value need to be avoided, the lower limit may be set closer to target value than the upper limit. User-specified weights are another significant means to incorporate the engineer's preference into the model. A larger weight may be assigned if it is highly desirable for the standard deviation to be as small as possible. On the other hand, a smaller value of r may be specified if the engineer does not care about the standard deviation within the limits.

4. AN ILLUSTRATIVE EXAMPLE

To demonstrate the usefulness of the proposed model, the following example is taken from Box and Draper (1987), which was also used in Vining and Myers (1990), Lin and Tu (1995), and Kim and Lin (1998). A 3^3 factorial design with three replicates at each design point is used to investigate the effects of three factors, speed (x_1), pressure (x_2) and distance (x_3), upon the application of coloring inks onto packaging labels as shown in Table 1. The average and standard deviation of three replicates at each design point are also presented in the table. The quality characteristic of interest is the quality of printing machine whose target value is 500. The quadratic fitted response functions for process mean and standard deviation were given by

$$\begin{aligned}\hat{\mu}(\boldsymbol{x}) = {} & 327.6 + 177.0x_1 + 109.4x_2 + 131.5x_3 \\ & + 32.0x_1^2 - 22.4x_2^2 - 29.1x_3^2 \\ & + 66.0x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3\end{aligned}$$

and

$$\begin{aligned}\hat{\sigma}(\boldsymbol{x}) = {} & 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 \\ & + 4.2x_1^2 - 1.3x_2^2 + 16.8x_3^2 + 7.7x_1x_2 \\ & + 5.1x_1x_3 + 14.1x_2x_3,\end{aligned}$$

respectively (Vining and Myers (1990)). The constraints for the control factors were $-1 \le x_i \le 1$ for i = 1, 2, 3. Suppose it is necessary for bias from the target not to exceed 10 (that is, $\mu_{\min} = 490$ and $\mu_{\max} = 510$). Furthermore, if it is technologically infeasible achieving variance below 1500 and a design engineer specified 2100 as the maximum allowable level of variance, then $\sigma_{\min}$ and $\sigma_{\max}$ are set $\sqrt{1500}$ and $\sqrt{2100}$, respectively. Assuming linear desirability functions for process mean and standard deviation, then the optimization model can be written as

$$\text{Maximize} \quad D = \left(d_\mu \cdot d_\sigma\right)^{\frac{1}{2}} \Big|_{r=s=t=1}$$

$$\begin{aligned}\text{subject to} \quad & 490 \le \hat{\mu}(\boldsymbol{x}) \le 510 \\ & \sqrt{1500} \le \hat{\sigma}(\boldsymbol{x}) \le \sqrt{2100} \\ & -1 \le x_i \le 1,\ i = 1, 2, 3.\end{aligned}$$

The optimal setting is found to be (1.000, 0.102, -0.257) with composite desirability of 0.331 (desirabilities of process mean and variance are 0.804 and 0.136, respectively). The result is summarized and compared with those of Vining and Myers (1990), Castilo and Mongomery (1993), Lin and Tu (1995), and Kim and Lin (1998) in Table 2. Note that the desirability of Vining and Myers (1990) is zero since the standard deviation falls above the upper limit even though the mean exactly falls at the target value. If the upper limit of standard deviation had been set at a value greater than $\sqrt{2679.70}$, the desirability would have taken a positive value. The results of Castillo and Mongomery (1993) can be obtained by tightening the limits of process mean, i.e., by setting $\mu_{\min} \cong \mu_{\max} \cong \tau$. The optimization model taken by Kim and Lin (1998) always yields to the same level of desirabilities for process mean and standard deviation. However, the balanced desirabilities between process mean and standard deviation could not provide the most desirable solution. The proposed model provides the most desirable

Table 1. Data for example.

u	x_1	x_2	x_3	y_{u1}	y_{u2}	y_{u3}	$\bar{y}_u$	s_u
1	-1	-1	-1	34	10	28	24.0	12.5
2	0	-1	-1	115	116	130	120.3	8.4
3	1	-1	-1	192	186	263	213.7	42.8
4	-1	0	-1	82	88	88	86.0	3.7
5	0	0	-1	44	178	188	136.7	80.4
6	1	0	-1	322	350	350	340.7	16.2
7	-1	1	-1	141	110	86	112.3	27.6
8	0	1	-1	259	251	259	256.3	4.6
9	1	1	-1	290	280	245	271.7	23.6
10	-1	-1	0	81	81	81	81.0	0.0
11	0	-1	0	90	122	93	101.7	17.7
12	1	-1	0	319	376	376	357.0	32.9
13	-1	0	0	180	180	154	171.3	15.0
14	0	0	0	372	372	372	372.0	0.0
15	1	0	0	541	568	396	501.7	92.5
16	-1	1	0	288	192	312	264.0	63.5
17	0	1	0	432	336	513	427.0	88.6
18	1	1	0	713	725	754	730.7	21.1
19	-1	-1	1	364	99	199	220.7	133.8
20	0	-1	1	232	221	266	239.7	23.5
21	1	-1	1	408	415	443	422.0	18.5
22	-1	0	1	182	233	182	199.0	29.4
23	0	0	1	507	515	434	485.3	44.6
24	1	0	1	846	535	640	673.7	158.2
25	-1	1	1	236	126	168	176.7	55.5
26	0	1	1	660	440	403	501.0	138.9
27	1	1	1	878	991	1161	1010.0	142.5

solution with a little higher expected loss.

A designer's preference may be reflected either by setting limits or by assigning weights. By varying the limits of process mean and standard deviation, the desirabilities of proposed model are compared with those of existing models. As expected, the proposed model shows better performances in terms of composite desirability across various settings of limits. Table 3 shows the desirabilities of various models by varying $\sigma_{\max}$ from $\sqrt{1950}$ to $\sqrt{2150}$, while keeping $\mu_{\min}$, $\mu_{\max}$, and $\sigma_{\min}$ at 490,

Table 2. Comparison of results($\mu_{\min} = 490$, $\mu_{\max} = 510$, $\sigma_{\min} = \sqrt{1500}$, and $\sigma_{\max} = \sqrt{2100}$).

Model	Optimal Setting	Mean (desirability)	Variance (desirability)	Expected Quality Loss	Composite Desirability
VM	(0.614, 0.228, 0.100)	500.00 (1.000)	2679.70 (0.000)	2679.70	0.000
CM	(1.000, 0.118, -0.259)	500.00 (1.000)	2033.74 (0.103)	2033.74	0.320
LT	(1.000, 0.073, -0.251)	494.44 (0.444)	1974.02 (0.192)	2005.08	0.300
KL	(1.000, 0.055, -0.248)	492.32 (0.232)	1951.79 (0.232)	2010.77	0.232
PM	(1.000, 0.102, -0.257)	498.04 (0.804)	2012.69 (0.136)	2016.53	0.331

Note: VM (Vining and Myers 1990), CM (Castillo and Mongomery 1993), LT (Lin and Tu 1995), KL (Kim and Lin 1998), and PM (Proposed Model).

Table 3. Comparison of results by varying $\sigma_{\max}$ ($\mu_{\min} = 490$, $\mu_{\max} = 510$, $\sigma_{\min} = \sqrt{1500}$).

Model	Summary	$\sigma_{\max}$		
		$\sqrt{1950}$ (= 44.16)	$\sqrt{2050}$ (= 45.28)	$\sqrt{2150}$ (= 46.37)
CM	Optimal Setting	(1.000, 0.118, -0.259)	(1.000, 0.118, -0.259)	(1.000, 0.118, -0.259)
	Mean (Desirability)	500.00 (1.000)	500.00 (1.000)	500.00 (1.000)
	Std. Dev. (Desirability)	45.10 (0.000)	45.10 (0.027)	45.10 (0.166)
	Composite Desirability	0.000	0.166	0.408
LT	Optimal Setting	(1.000, 0.073, -0.251)	(1.000, 0.073, -0.251)	(1.000, 0.073, -0.251)
	Mean (Desirability)	494.44 (0.444)	494.44 (0.444)	494.44 (0.444)
	Std. Dev. (Desirability)	44.43 (0.000)	44.43 (0.129)	44.43 (0.254)
	Composite Desirability	0.000	0.240	0.334
KL	Optimal Setting	(1.000, 0.039, -0.246)	(1.000, 0.050, -0.248)	(1.000, 0.059, -0.249)
	Mean (Desirability)	490.39 (0.039)	491.78 (0.178)	492.79 (0.279)
	Std. Dev. (Desirability)	43.95 (0.039)	44.11 (0.178)	44.24 (0.279)
	Composite Desirability	0.039	0.178	0.279
PM	Optimal Setting	(1.000, 0.045, -0.247)	(1.000, 0.083, -0.254)	(1.000, 0.119, -0.260)
	Mean (Desirability)	491.08 (0.108)	495.75 (0.575)	500.00 (1.000)
	Std. Dev. (Desirability)	44.03 (0.024)	44.59 (0.105)	45.10 (0.166)
	Composite Desirability	0.050	0.246	0.408

510, and $\sqrt{1500}$, respectively. The optimization model by Lin and Tu (1995) shows better performances for small values of $\sigma_{\max}$. On the other hand, the model by Castillo and Montgomery (1993) turns out to be more desirable as $\sigma_{\max}$ increases. Note that our proposed model keeps track of better performances across various values of $\sigma_{\max}$. When $\sigma_{\max} = \sqrt{1950}$, composite desirabilities of Castillo and Montgomery (1993) and Lin and Tu (1995) are zero since standard deviations exceed the upper limit. On the other hand, when $\sigma_{\max} = \sqrt{2150}$, the optimal solution is equivalent to that of Castillo and Montgomery (1993) since less emphasis would be placed on standard deviation.

Table 4 illustrates the desirabilities by varying $\mu_{\min}$ from 480 to 495, while keeping $\mu_{\max}$, $\sigma_{\min}$, and $\sigma_{\max}$ at 510, $\sqrt{1500}$, and $\sqrt{2100}$, respectively. Our model still chases better performances among existing models. Note that the desirability of proposed model approaches to that of Castillo and Montgoemry (1993) as the lower limit gets closer to the target value. This is because more emphasis would be placed on taking process mean to the target value. Actually, the optimal solution of proposed model is equivalent to their solution when the lower limit is set to 495. Table 5 compares the results of various models for some values of $\mu_{\min}$ The composite desir-

Table 4. Comparison of results by varying $\mu_{\min}$ ($\mu_{\max} = 510$, $\sigma_{\min} = \sqrt{1500}$, and $\sigma_{\max} = \sqrt{2100}$).

Model	Summary	$\mu_{\min}$		
		485	490	495
CM	Optimal Setting	(1.000, 0.118, -0.259)	(1.000, 0.118, -0.259)	(1.000, 0.118, -0.259)
	Mean (Desirability)	500.00 (1.000)	500.00 (1.000)	500.00 (1.000)
	Std. Dev. (Desirability)	45.10 (0.103)	45.10 (0.103)	45.10 (0.103)
	Composite Desirability	0.320	0.320	0.320
LT	Optimal Setting	(1.000, 0.073, -0.251)	(1.000, 0.073, -0.251)	(1.000, 0.073, -0.251)
	Mean (Desirability)	494.44 (0.646)	494.44 (0.444)	494.44 (0.000)
	Std. Dev. (Desirability)	44.43 (0.192)	44.43 (0.192)	44.43 (0.254)
	Composite Desirability	0.352	0.300	0.000
KL	Optimal Setting	(1.000, 0.029, -0.244)	(1.000, 0.055, -0.248)	(1.000, 0.084, -0.254)
	Mean (Desirability)	489.25 (0.284)	492.32 (0.232)	495.86 (0.172)
	Std. Dev. (Desirability)	44.56 (0.284)	44.18 (0.232)	44.60 (0.172)
	Composite Desirability	0.284	0.232	0.172
PM	Optimal Setting	(1.000, 0.081, -0.253)	(1.000, 0.083, -0.254)	(1.000, 0.119, -0.260)
	Mean (Desirability)	495.55 (0.703)	498.04 (0.804)	500.00 (1.000)
	Std. Dev. (Desirability)	44.56 (0.178)	44.86 (0.136)	45.10 (0.103)
	Composite Desirability	0.354	0.330	0.320

Table 5. Effects of Weight *r* ($\mu_{\min} = 490$, $\mu_{\max} = 510$, $\sigma_{\min} = \sqrt{1500}$, and $\sigma_{\max} = \sqrt{2100}$).

Weight *r*	Optimal Setting	Mean	Variance	Expected Quality Loss
0.1	(1.000, 0.119, -0.260)	500	2033.80	2033.80
0.7	(1.000, 0.114, -0.259)	499.46	2027.93	2028.23
1.0	(1.000, 0.102, -0.257)	498.04	2012.69	2016.53
2.5	(1.000, 0.074, -0.252)	494.60	1975.92	2005.09
5.0	(1.000, 0,058, -0.249)	492.69	1955.65	2009.15

ability of Lin and Tu (1995) is zero since the process mean is smaller than the lower limit when $\mu_{\min} = 495$.

Finally, the effects of weights are also studied by varying the value of the weight of standard deviation (r) as in Table 4. As the weight increases, the standard deviation gets smaller while sacrificing the performance in process mean. When $r = 0.1$, i.e., a lot smaller weight is assigned to standard deviation, the process mean is set exactly at the target while sacrificing the performance in standard deviation. Note that the solution obtained is equivalent to that of Castillo and Montgomery (1993) since much more emphases are placed on process mean. On the other hand, for the case of $r = 5.0$, i.e., much larger weight is assigned to standard deviation, the process mean is quite off the target while the variance reduces more or less compared with the cases having smaller weights. The weights of process mean can also be examined in a similar way. As the weights gets larger, process mean is expected to approach to target value while standard deviation increases.

5. CONCLUDING REMARKS

In this paper, we proposed a versatile optimization model for robust design. The proposed model is to maximize the composite desirability of process mean and variance, and thus provides the most desirable compromise solution. It can accommodate a variety of design engineers' and customers' preferences either by adjusting the limits on process mean and standard deviation or by assigning different weight to desirabilities of mean and standard deviation. For example, a process mean that is very close to the target value can be achieved either by assigning large weight to process mean or by tightening the limits of process mean. Once the preferences for the process are identified, the proposed model provides better (or at least equal) solutions compared with existing models.

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