

Estimation of Smoothing Constant of Minimum Variance and its Application to Industrial Data

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Abstract. Focusing on the exponential smoothing method equivalent to (1, 1) order ARMA model equation, a new method of estimating smoothing constant using exponential smoothing method is proposed. This study goes beyond the usual method of arbitrarily selecting a smoothing constant. First, an estimation of the ARMA model parameter was made and then, the smoothing constants. The empirical example shows that the theoretical solution satisfies minimum variance of forecasting error. The new method was also applied to the stock market price of electrical machinery industry (6 major companies in Japan) and forecasting was accomplished. Comparing the results of the two methods, the new method appears to be better than the ARIMA model. The result of the new method is apparently good in 4 company data and is nearly the same in 2 company data. The example provided shows that the new method is much simpler to handle than ARIMA model. Therefore, the proposed method would be better in these general cases. The effectiveness of this method should be examined in various cases.

Keywords: ARIMA Model, Minimum Variance, Exponential Smoothing Method, Forecasting.

1. INTRODUCTION

Many methods for time series analysis have been presented such as autoregressive model (AR model), autoregressive moving average model (ARMA model) and exponential smoothing method (ESM) (Jenkins, 1994; Kobayashi, 1993; Brown, 1963). In particular, ESM is recognized as a practical simple method presenting various improvements such as adding compensating item for time lag, coping with the time series with trend (Winters, 1984), utilizing Kalman Filter (Maeda, 1984), Bayes Forecasting (West and Harrison, 1989), adaptive ESM (Kern, 1982), exponentially weighted Moving Averages with irregular updating periods (Johnston, 1993), and plural method of forecasts averaging (Makridakis and Winkler, 1983). For example, Maeda (1984) calculates smoothing constant in relation to S/N ratio under the assumption that the observation noise was added to the system. To account for this, the author calculates for the supposed noise because of the difficulty in grasping observation noise. The study shows that it does

not pursue optimum solution from the very data themselves which should be derived by the estimations made. Ishii *et al.*, (1991) pointed out that the optimal smoothing constant is the solution for infinite order equation, but no analytical solution was provided. This study proposes a new method of estimation of smoothing constant in ESM (Takeyasu, 2002). Since the equation of ESM is equivalent to (1, 1) order ARMA model equation, a new method of estimation is presented.

Generally, smoothing constant is arbitrarily selected. However, this study derives a theoretical solution in a simple manner. It makes an estimation of ARMA model parameter and then, estimates smoothing constants. The new method is further examined through an application to the stock market price of electrical machinery industry (6 major companies in Japan). Variance of forecasting error of new method is compared with those of ARIMA model.

The paper is divided into five sections. Section 2 describes the ESM using ARIMA model and the estimation method of smoothing constant using system identification of ARIMA model. Section 3 provides details on forecast-

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ing method. Section 4 examines forecasting accuracy. Section 5 discusses the results. Section 6 gives a summary of the study.

2. DESCRIPTION OF EXPONENTIAL SMOOTHING METHOD USING ARIMA MODEL

2.1 Exponential Smoothing Method and ARIMA model

In the exponential smoothing method, forecasting at time $t+1$ is stated by the following equations:

$$\hat{x}_{t+1} = \hat{x}_t + \alpha(x_t - \hat{x}_t) \quad (1)$$

$$= \alpha x_t + (1 - \alpha)\hat{x}_t \quad (2)$$

Where

\hat{x}_{t+1} : forecasting at $t+1$

x_t : realized value at t

α : smoothing constant ($0 < \alpha < 1$)

Equation (2) is re-stated as

$$\hat{x}_{t+1} = \alpha(1 - \alpha)^t x_{t-1} \quad (3)$$

The study considers the following (1, 1) order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1} \quad (4)$$

Generally, (p, q) order ARMA model is stated as [1, 3]:

$$x_t + \sum_{i=1}^p a_i x_{t-i} = e_t + \sum_{j=1}^q b_j e_{t-j} \quad (5)$$

Where

$\{x_t\}$: Sample process of Stationary Ergodic Gaussian Process $x(t)$ $t = 1, 2, \dots, N, \dots$

$\{e_t\}$: Gaussian White Noise with 0 mean σ_e^2 variance

MA process in (5) is supposed to satisfy convertibility condition. Utilizing the relation that:

$$E[e_t | e_{t-1}, e_{t-2}, \dots] = 0,$$

we get the following equation from (4)

$$\hat{x}_t = x_{t-1} - \beta e_{t-1} \quad (6)$$

Operating this scheme on $t+1$, we get

$$\begin{aligned} \hat{x}_{t+1} &= \hat{x}_t + (1 - \beta)e_t \\ &= \hat{x}_t + (1 - \beta)(x_t - \hat{x}_t) \end{aligned} \quad (7)$$

If $1 - \beta = \alpha$, equation (7) is the same with (1), that is, equation of ESM is equivalent to (1, 1) order ARMA model, or is said to be (0, 1, 1) order ARIMA model because 1st order AR parameter is -1 (Jenkins, 1994; Tokumaru *et al.*, 1982).

2.2 Estimation of Smoothing Constant Utilizing System Identification of ARIMA model

Comparing with (4) and (5), we obtain

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases}$$

From (1) and (7),

$$\alpha = 1 - \beta$$

Therefore, (8) can be obtained.

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases} \quad (8)$$

Now, the smoothing constant can be derived after identifying the parameter of MA part of ARMA model. In many cases, the MA part of ARMA model becomes non-linear equations, as described below.

Let (5) be

$$\tilde{x}_t = x_t + \sum_{i=1}^p a_i x_{t-i} \quad (9)$$

$$\tilde{x}_t = e_t + \sum_{j=1}^q b_j e_{t-j} \quad (10)$$

The autocorrelation function of \tilde{x}_t is expressed as \tilde{r}_k . From (9) and (10), the following non-linear equations are obtained which are well known.

$$\tilde{r}_k = \left\{ \begin{array}{l} \tilde{r}_k = \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j} \quad (k \leq q) \\ 0 \quad (k \geq q+1) \\ \tilde{r}_0 = \sigma_e^2 \sum_{j=0}^q b_j^2 \end{array} \right\} \quad (11)$$

For these equations, recursive algorithm was developed. In this paper, parameter to be estimated is only b_1 , which can be calculated as follows.

From (4), (5), (8) and (11), we get

$$\left. \begin{array}{l} q = 1 \\ a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \\ \tilde{r}_0 = (1 + b_1^2) \sigma_e^2 \\ \tilde{r}_1 = b_1 \sigma_e^2 \end{array} \right\} \quad (12)$$

If we set

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0} \quad (13)$$

it follows that

$$\rho_1 = \frac{b_1}{1 + b_1^2} \quad (14)$$

b_1 is calculated as follows:

$$b_1 = \frac{1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1} \quad (15)$$

In order to have real roots, ρ_1 must satisfy

$$|\rho_1| \leq \frac{1}{2} \quad (16)$$

From invertibility condition, b_1 must satisfy

$$|b_1| < 1 \quad (17)$$

From (15), using the next relation,

$$(1 - b_1)^2 \geq 0$$

$$(1 + b_1)^2 \geq 0$$

(17) always satisfies the condition.

Since $\alpha = b_1 + 1$, b_1 is within the range of $-1 < b_1 < 0$, we get

$$\left. \begin{array}{l} b_1 = \frac{1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \\ \alpha = \frac{1 + 2\rho_1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \end{array} \right\} \quad (18)$$

which satisfies the said condition. This presents a simple theoretical solution.

Emphasizing that the equation of ESM is equivalent to (1, 1) order ARMA model equation, the smoothing constant can be estimated after estimating ARMA model parameter.

It can be estimated only by calculating 0th and 1st order autocorrelation function.

3. FORECASTING THE STOCK MARKET PRICE OF ELECTRICAL MACHINERY INDUSTRY

3.1 Forecasting

By using 6 electrical machinery industry data for three and a half years, from January 2001 to October 2004 (Fuji Electric Holdings, Oki Electric Industry, TDK, Pioneer, Kyocera, Canon), we forecast the stock market price and examine forecasting accuracy through a comparison with the original data.

The stock market price of electrical machinery industries are shown in Figure 1 to Figure 6.

3.2 Forecasting by ARIMA Model

The TSP (Time Series Processor) was used in forecasting by ARIMA model.

Log L (Log of Likelihood Function) from the original data are calculated for each order (0, 0, 1) (0, 0, 2) (1, 0, 0) (1, 0, 1) (1, 0, 2) (2, 0, 0) (2, 0, 1) (2, 0, 2) (0, 1, 1) (0, 1, 2) (1, 1, 0) (1, 1, 1) (1, 1, 2) (2, 1, 0) (2, 1, 1) (2, 1, 2) ARIMA model to obtain AIC. AIC is calculated using (19).

$$AIC = n * \text{Log } L + 2 * (p + d + q) \quad (19)$$

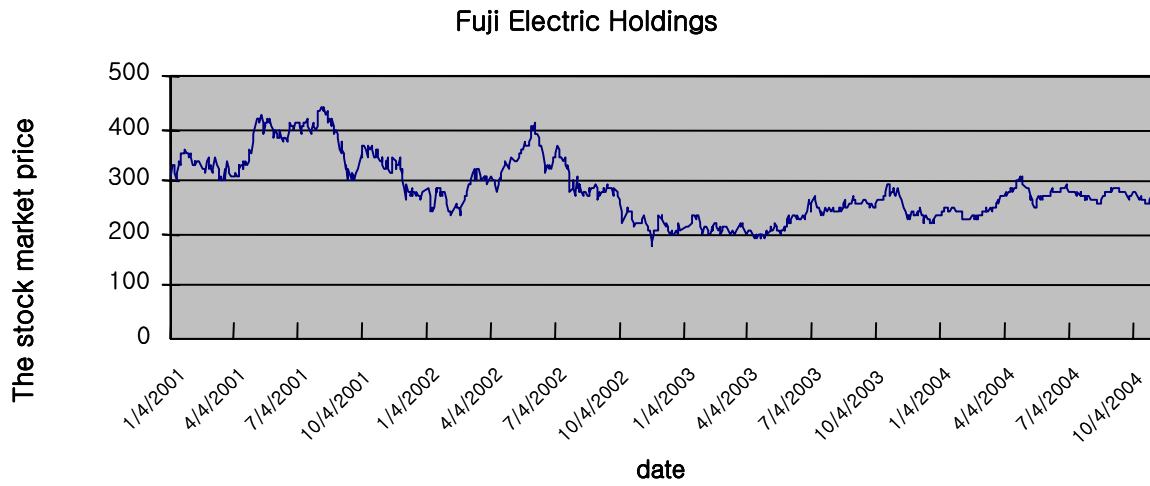


Figure 1. Stock market price of Fuji Electric Holdings from January 2001-October 2004

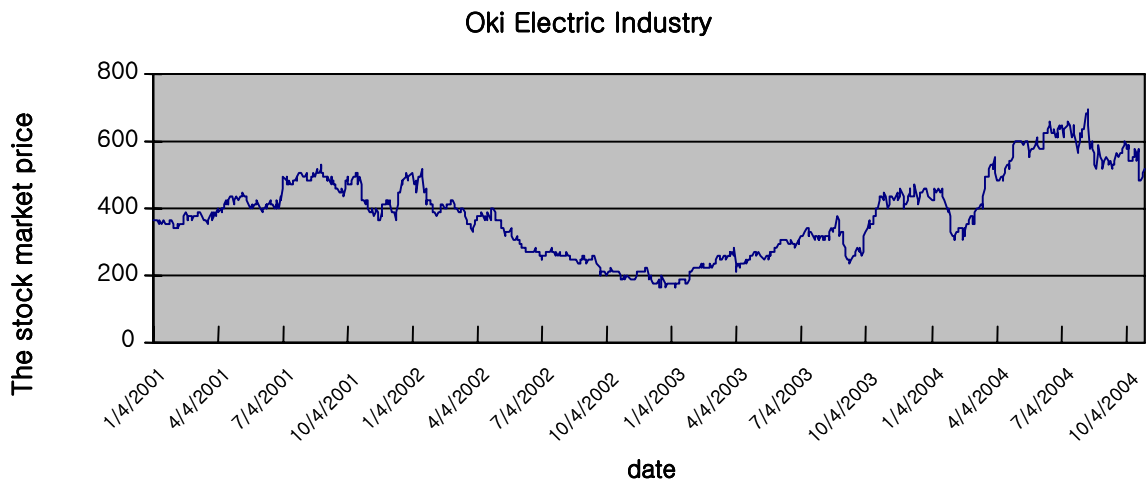


Figure 2. Stock market price of Oki Electric Industry from January 2001 to October 2004

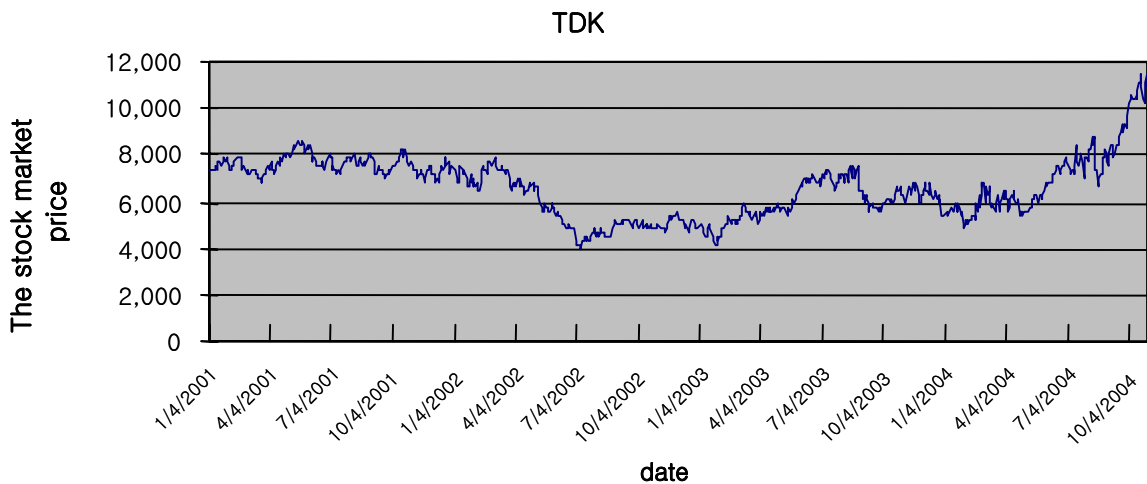


Figure 3. Stock market price of TDK from January 2001 to October 2004

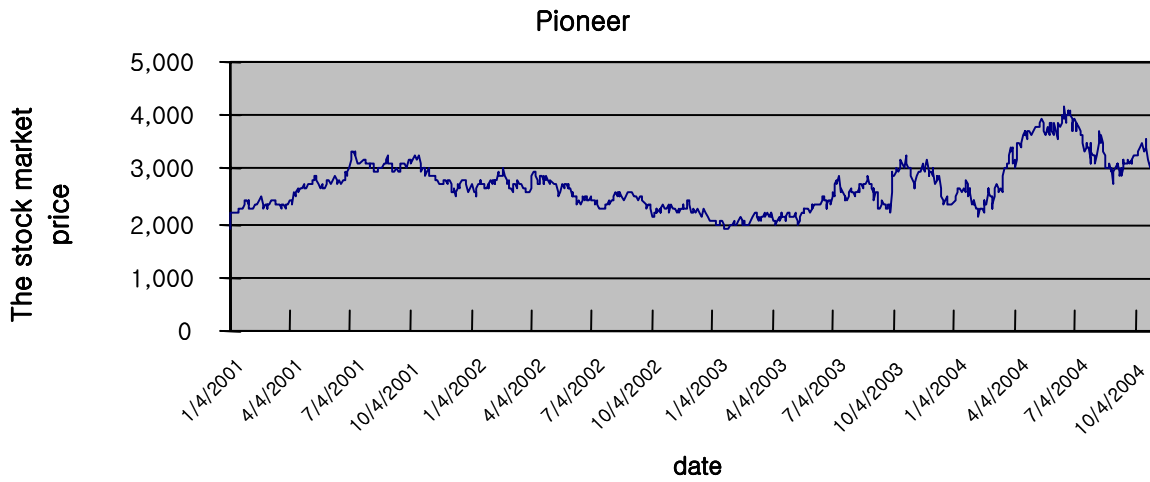


Figure 4. Stock market price of Pioneer from January 2001 to October 2004

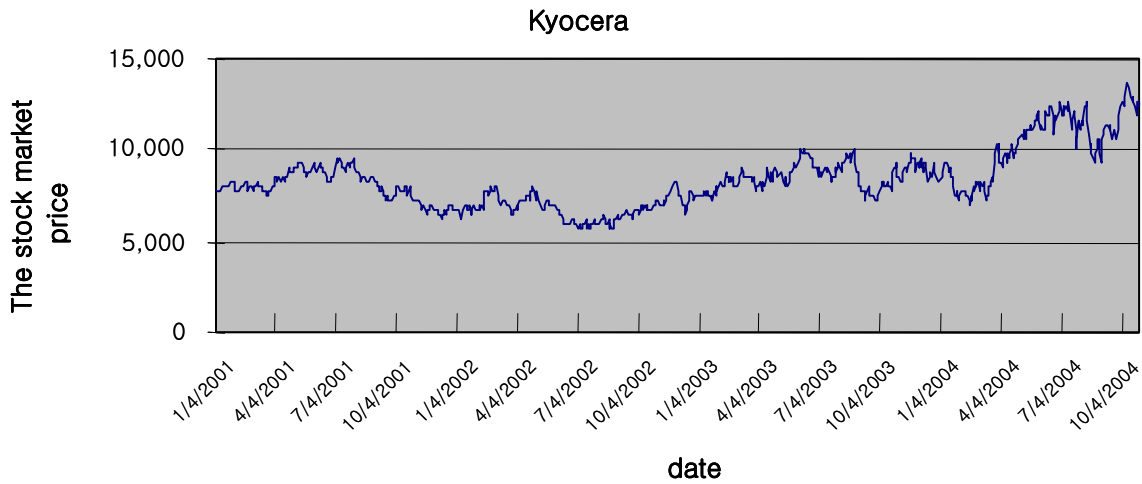


Figure 5. Stock market price of Kyocera from January 2001 to October 2004

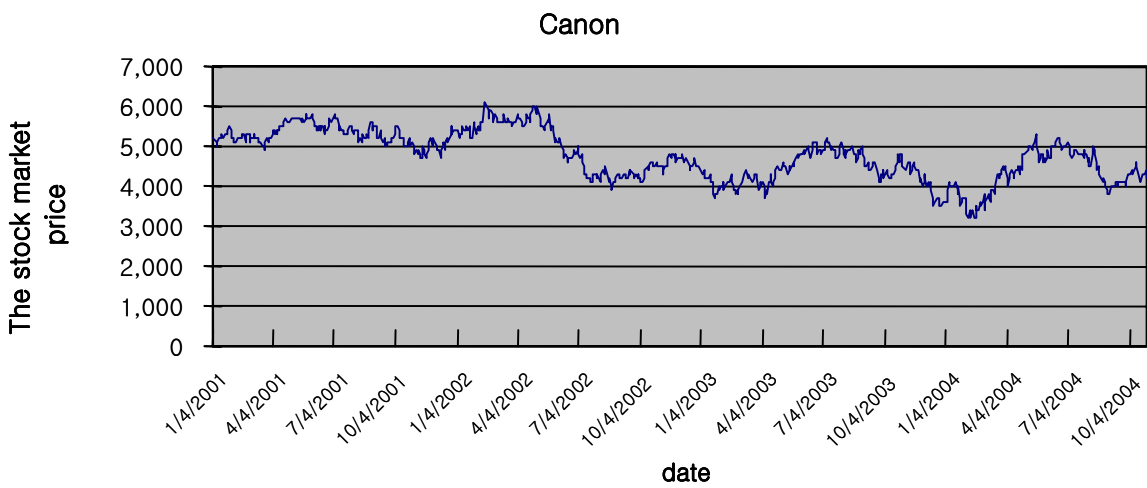


Figure 6. Stock market price of Canon from January 2001 to October 2004

Table 1 shows AIC value in (0, 0, 1) order ARIMA model.

Table 1. AIC value in (0, 0, 1) order ARIMA model

	Log L	AIC
Fuji Electric Holdings	-6044.24	-5693672.08
Oki Electric Industry	-5272.9	-4967069.8
TDK	-7527.88	-7091260.96
Pioneer	-6581.59	-6199855.78
Kyocera	-7725.36	-7277287.12
Canon	-6822.95	-6427216.9

AIC value is at minimum by ARIMA (0, 0, 1) model at all companies, suggesting that it is the best fitting model. Next, forecast was made by ARIMA model which is at minimum in AIC. Then, this method is compared with newly proposed method concerning forecasting accuracy.

4. COMPARISON CONCERNING FORECASTING ACCURACY

The original data and forecasting value of six companies were normalized to allow comparison. Normalization was performed using (20).

$$\tilde{x}_i = \frac{x_i - \bar{x}}{\sigma} \tag{20}$$

The average and variance were also calculated for the original data. A comparison between the two methods concerning forecasting accuracy using variance was made.

The average forecasting error was calculated using (21).

$$\bar{\varepsilon} = \frac{1}{N} \sum_{i=1}^N \varepsilon_i \tag{21}$$

Where

$$\varepsilon_i = \hat{x}_i - x_i \tag{22}$$

Variance was calculated using (23).

$$\sigma_{\varepsilon}^2 = \frac{1}{N-1} \sum_{i=1}^N (\varepsilon_i - \bar{\varepsilon})^2 \tag{23}$$

The variance of forecasting error, given by Table 2, is smaller by exponential smoothing method than those by ARIMA model for Fuji Electric Holdings, Pioneer,

Kyocera and Canon. Variance for both methods is nearly the same for TDK, whereas accuracy of forecasting is slightly better by ARIMA model than by exponential smoothing method for Oki Electric Industry. Considering the numerical order, both methods are nearly the same level for Oki Electric Industry.

Table 2. Variance of forecasting error

Company	Exponential Smoothing Method	ARIMA model
Fuji Electric Holdings	0.056942	0.252980
Oki Electric Industry	0.001280	0.000913
TDK	0.000033	0.000033
Pioneer	0.000933	0.096168
Kyocera	0.000169	0.003074
Canon	0.000016	0.000500

Therefore, the example shows relatively similar results between the two methods. From the practical point of view, the newly proposed method is much simpler to handle than ARIMA model. Therefore, the newly proposed method would, in general, be better in this case.

5. DISCUSSION

The proposed method is applicable for general time series applications such as sales forecasting, demand forecasting as well as stock market price forecasting. This method does not necessarily forecast for financial time series. For further application to financial time series, expansion to ARCH-type models or stochastic volatility model should be examined.

The stochastic volatility model is used to describe financial time series (Engle, 1982; Hyndman, *et al.*, 2002; Jacquier, 2004; Makridakis, 1998; Meyer and Yu, 2000; Barndorff-Nielsen and Shephard, 2001), which may be of significance for future work.

6. CONCLUSION

Many methods for time series analysis have been presented such as autoregressive model (AR model), autoregressive moving average model (ARMA model) and exponential smoothing method. Among these, exponential smoothing method was recognized to be a practical simple method.

Focusing on the equation of exponential smoothing method being equivalent to (1, 1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed, which satisfies minimum variance of forecasting error. Instead of arbitrarily selecting a smoothing constant, this

study utilized the discussed theoretical solution.

The new method was applied to the stock market price of electrical machinery industry (6 major companies in Japan). Variance of forecasting error of the new method was compared with those of ARIMA model.

Results show that the proposed model obtained better results than ARIMA model. The result of new method was apparently good in 4 company data sets and was nearly the same in 2 other companies. The application further shows that the new method was much simpler to handle than ARIMA model. Therefore, proposed method would be better in general cases. The effectiveness of this method should be examined in various cases.

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