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Expansion of Measured Static and Dynamic Data as Basic Information for Damage Detection

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Abstract

The number of measured degrees of freedom for detecting the damage of any structures is usually less than the number of model degrees of freedom. It is necessary to expand the measured data to full set of model degrees of freedom for updating modal data. This study presents the expansion methods to estimate all static displacements and dynamic modal data of finite element model from the measured data. The static and dynamic methods are derived by minimizing the variation of the potential energy and the Gauss's function, respectively. The applications illustrate the validity of the proposed methods. It is observed that the numerical results obtained by the static approach correspond with the Guyan condensation method and the derived static and dynamic approaches provide the fundamental idea for damage detection.

Keywords : Data Expansion, Modal Parameter, Damage Detection

1. INTRODUCTION

The damage detection and assessment algorithms have been proposed based on measured displacement data from a static test or measured modal data from a dynamic test. Static displacement responses and modal parameters such as eigenvalues and eigenvectors have been widely utilized as reference data for damage detection. The number of measured degrees of freedom does not usually match the number of model degrees of freedom. The data obtained from vibration or static testing is incomplete not to be able to measure all the modes or all static displacements.

The problem can be overcome by modal reduction or data expansion. This means either to reduce the system matrices to the number of measured degrees of freedom or conversely expand the measured displacement or modal vectors to full size of the finite element model matrices. The model reduction methods depend on the determination of transformation operators which reduce the analytical model to the same degrees of freedom measured in test. And the expansion methods compute the unmeasured displacements by assuming certain equilibrium conditions.

The spatially sampled field measured during dynamic or static testing has been an active area of research for many years. Comparing just the measured partition of the full analytic displacement shapes to the test displacement shapes, most analysis procedures are possible only when there is one-to-one correspondence between the modal degrees of freedom and the test measurements. Most expansion techniques utilized involve the use of the finite element model as a mechanism to complete the unmeasured degrees of freedom from the experimental model.

Guyan (1965) presented a transformation matrix to establish the relationship between the expanded eigenvector and the measured components. The Improved Reduced System (O'Callanhan 1989) is an improvement on the static reduction method and provides a perturbation to the transformation from the static case by including the inertia terms as pseudo static forces. Utilizing mass normalized real mode shapes determined by finite element models, O'Callahan et al. (1989) proposed the System Equivalent Reduction-Expansion Process (SEREP). Based on a dynamic modal expansion which minimizes the residual error in the eigenvalue equation for each measured mode, Kenneth and Francois (2000) presented an algorithm for expanding measured mode shapes obtained from modal testing to the full set of degrees of freedom of a corresponding finite element model. Ewins (2000) presented some definitions and a very concise summary of each of the major algorithms for updating problem together with some discussion of how and when each of these might be considered for use in practice.

Although an initial theoretical model should be refined, corrected or updated based on measured data on the actual structure, the existing expansion methods are limited to the methods to expand the experimentally measured DOF over all finite element DOF under the assumption that the static or dynamic properties of the initial system are invariant. It is necessary to investigate the uniqueness of the mathematical solution because the transformation matrix is rank deficient.

Starting from the displacement data or dynamic modal data obtained from the structural analysis of initial structural system, this study focuses on the data expansion based on the measured displacement data and modal data, and aims to develop the static and dynamic expansion techniques for providing damage detection and assessment algorithm of damaged structural systems. Minimizing the objective function formulated by the difference between the computed and measured displacements or dynamic modal parameters, and assuming that the system exhibits the constrained static or dynamic behavior restricted by the measured data, the static and dynamic expansion methods are presented. It is expected that the proposed method can be widely utilized as basic information for damage detection algorithm by introducing the concept of the displacement or mode shape curvature provided by Pandey et al. (1991).

2. EXPANSION OF STATIC DISPLACEMENT DATA

In order to identify the physical model parameters, the parameters of the initial analysis model must be described based on the measured structural behavior. The measured structural behavior involves the displacement data and dynamic modal data from the static and dynamic testing, respectively. First, we consider the static expansion originated from the measured displacement responses and expand the measured deflection data to estimate the data at unmeasured locations.

Consider a discrete structural system with n degrees of freedom having a stiffness matrix $\mathbf{K}$ characterized by n constitutive parameters $\mathbf{u}$. The governing equation of static equilibrium is

$$\mathbf{K}_0\mathbf{u}_0 = \mathbf{f}_0 \tag{1}$$

where $\mathbf{f}_0$ is a vector of applied forces and $\mathbf{u}_0$ denotes the corresponding response. If the structural stiffness matrix exactly captures the properties of the system, and if the measured responses were free from error, then Eqn. (1) would exactly satisfy and the measured data can be expanded to the all response data by the expansion techniques. Considering the difference between the computed and measured displacements due to the damage of the member, the output error is

$$e = \mathbf{K}^{-1}\mathbf{f}_0 - \mathbf{u} \tag{2}$$

where $\mathbf{u}$ is displacement vector in the finite element model of a structure, $\mathbf{K} = \mathbf{K}_0 + \delta\mathbf{K}$, $\mathbf{K}$ is the stiffness matrix of the damaged structure to be expressed by the sum of the initial stiffness $\mathbf{K}_0$ and the variation of the stiffness matrix $\delta\mathbf{K}$ caused by the difference between the computed and measured displacements due to measurement errors and modeling errors.

Considering the stiffness effects only, Eqn. (1) can be written as

$$(\mathbf{K}_0 + \delta\mathbf{K})\mathbf{u} = \mathbf{f}_0 \tag{3}$$

where we assume that the external forces are invariant in the analysis process and $\mathbf{u}$ is the corresponding responses of the damaged structure. Taking the first-order approximation of the displacement vector $\mathbf{u}$, it can be written by

$$\begin{aligned}\mathbf{u} = (\mathbf{K}_0 + \delta\mathbf{K})^{-1}\mathbf{f}_0 &\approx \left(\mathbf{K}_0^{-1} - \mathbf{K}_0^{-1}\delta\mathbf{K}\mathbf{K}_0^{-1}\right)\mathbf{f}_0 \\ &= \left(\mathbf{I} - \mathbf{K}_0^{-1}\delta\mathbf{K}\right)\mathbf{u}_0\end{aligned} \tag{4}$$

The displacement change due to the damage can be defined as

$$\delta\mathbf{u} = \mathbf{u} - \mathbf{u}_0 \tag{5}$$

The damage leads to the deterioration of the stiffness and the displacement change calculated from Eqn. (5) should be positive.

The unmeasured displacements of the damaged structure are obtained by minimizing an objective function formulated by the displacement change vector defined by Eqn. (5) and a weighting matrix $\mathbf{K}_0$ of the initial stiffness matrix, which is expressed as

$$\underset{\min}{\delta V} = \frac{1}{2}(\delta\mathbf{u})^T\mathbf{K}_0\delta\mathbf{u} \tag{6}$$

which expresses the variation of the potential energy due to the damage.

We can express the displacement vector at the n_d measured degrees of freedom in the finite element model as

$$\mathbf{u}_d = \mathbf{A}\mathbf{u} \tag{7}$$

where $\mathbf{A}$ is an $n_d \times n$ Boolean matrix that extracts the measured response $\mathbf{u}_d$ from the complete displacement vector $\mathbf{u}$. The substitution of Eqn. (5) into Eqn. (7) yields

$$\mathbf{A}(\mathbf{u}_0 + \delta\mathbf{u}) = \mathbf{u}_d \quad \text{or} \quad \mathbf{A}\delta\mathbf{u} = \mathbf{u}_d - \mathbf{A}\mathbf{u}_0 \tag{8}$$

In order to minimize the cost function of Eqn. (6), Eqn. (8) is modified as

$$\mathbf{A}\mathbf{K}^{-1/2}\mathbf{K}^{1/2}\delta\mathbf{u} = \mathbf{u}_d - \mathbf{A}\mathbf{u}_0 \tag{9}$$

Finding the general solution of Eqn. (9) with respect to $\mathbf{K}_0^{1/2}\delta\mathbf{u}$, it is obtained as

$$\mathbf{K}_0^{1/2}\delta\mathbf{u} = \left(\mathbf{A}\mathbf{K}_0^{-1/2}\right)^+ (\mathbf{u}_d - \mathbf{A}\mathbf{u}_0) + \left[\mathbf{I} - \left(\mathbf{A}\mathbf{K}_0^{-1/2}\right)^+ \left(\mathbf{A}\mathbf{K}_0^{-1/2}\right)\right]\mathbf{y} \tag{10}$$

where $\mathbf{y}$ is an arbitrary and the solution of Eqn. (10) becomes infinite. The condition to satisfy the variation of the potential energy of Eqn. (6) is

$$\left(\mathbf{A}\mathbf{K}^{-1/2}\right)^+ (\mathbf{u}_d - \mathbf{A}\mathbf{u}_0) + \left[\mathbf{I} - \left(\mathbf{A}\mathbf{K}^{-1/2}\right)^+ \left(\mathbf{A}\mathbf{K}^{-1/2}\right)\right]\mathbf{y} = \mathbf{0} \tag{11}$$

Introducing $\mathbf{S} = \mathbf{A}\mathbf{K}_0^{-1/2}$ into Eqn. (11) and solving it with respect to the arbitrary vector **y,** it follows that

$$\mathbf{y} = -\left[\mathbf{I} - \mathbf{S}^+\mathbf{S}\right]\mathbf{S}^+ (\mathbf{u}_d - \mathbf{A}\mathbf{u}_0) + \left[\mathbf{I} - \left(\mathbf{I} - \mathbf{S}^+\mathbf{S}\right)^+ \left(\mathbf{I} - \mathbf{S}^+\mathbf{S}\right)\right]\mathbf{z} \tag{12}$$

where $\mathbf{z}$ is another arbitrary vector. Utilizing the properties of the generalized inverse matrix of $\mathbf{S}^+\mathbf{S}\mathbf{S}^+ = \mathbf{S}^+$ and $\left(\mathbf{I} - \mathbf{S}^+\mathbf{S}\right)^+ = \left(\mathbf{I} - \mathbf{S}^+\mathbf{S}\right)$ into Eqn. (12), the vector $\mathbf{y}$ is derived as

$$\mathbf{y} = \mathbf{S}^+\mathbf{S}\mathbf{z} \tag{13}$$

Subsituting Eqn. (13) into Eqn. (10) and premultiplying $\mathbf{K}_0^{-1/2}$ at both sides, the displacement variation is obtained as

$$\delta\mathbf{u} = \mathbf{K}_0^{-1/2}\left(\mathbf{A}\mathbf{K}_0^{-1/2}\right)^+ (\mathbf{u}_d - \mathbf{A}\mathbf{u}_0) \tag{14}$$

Thus, the displacement vector of the damaged structure can be expressed as

$$\mathbf{u} = \mathbf{u}_0 + \mathbf{K}_0^{-1/2}\left(\mathbf{A}\mathbf{K}_0^{-1/2}\right)^+ (\mathbf{u}_d - \mathbf{A}\mathbf{u}_0) \tag{15}$$

The static displacements of the full size of the finite element model can be calculated by Eqn. (15) from the information on the initial displacements of intact structure.

3. EXPANSION OF DYNAMIC MODAL DATA

An initial theoretical model constructed for analyzing the dynamics of a structure can be refined, corrected or updated, using measured on the actual structure, has become one of the most demanding and demanded applications for modal testing. The number of measured degrees of freedom does not usually match the number of model degrees of freedom, so expansion of the mode shapes is necessary.

The dynamic behaviour of a structure which is assumed to be linear and approximately discretized for n degrees of freedom can be described by the equations of motion

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}(t) \tag{16}$$

where $\mathbf{u} = [u_1 \quad u_2 \quad \cdots \quad u_n]^T$, and $\mathbf{M} \in R^{n\times n}$, $\mathbf{C} \in R^{n\times n}$ and $\mathbf{K} \in R^{n\times n}$ are the mass, damping and stiffness matrices, respectively. And $\mathbf{f}(t)$ is the $n\times 1$ load excitation vector. Without loss of generality, Rayleigh damping is adopted as

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{17}$$

where α and β are the two proportionality constants which can be related to the damping ratios of the first and second natural modes.

In order to expand the measured mode shapes to estimate the data of unmeasured mode shapes, this study introduces an analytical method by minimizing the performance index as a function of the changes of the mode shape. In this process, we can utilize the measured mode shapes as the dynamic constraints to describe the modal characteristics of the damaged structure.

Let us define the i-th mode shapes of undamaged and damaged structures as $\boldsymbol{\varphi}_{i,0}$ and $\boldsymbol{\varphi}_{i,d}$ ($i = 1,2,...n$), respectively, and the corresponding natural frequencies as $\omega_{i,0}$ and $\omega_{i,d}$, respectively. If the dynamic system of Eqn. (16) is undamped or only lightly damped, the characteristic features of the system are natural frequencies $\omega_{i,0}$ and the normal modes $\boldsymbol{\varphi}_{i,0}$, which can be calculated from an eigenvalue problem.

In order to expand the measured mode shapes to the full degrees of freedom, we utilize the Gauss's principle. The Gauss's principle indicates that the accelerations $\ddot{\mathbf{u}}$ are such that the Gaussian function G defined as

$$G = [\ddot{\mathbf{u}} - \mathbf{a}]^T \mathbf{M}[\ddot{\mathbf{u}} - \mathbf{a}] \tag{18}$$

is minimized over all $\ddot{\mathbf{u}}$ which satisfy the constraint equation, where $\ddot{\mathbf{u}}$ and $\mathbf{a}$ represent the acceleration vectors for constrained and unconstrained systems, respectively. From the modal relationship, the Gauss's function is modified as

$$G = \left[\omega_{i,0}{}^2 \boldsymbol{\varphi}_{i,0} - \omega_{i,d}^2 \boldsymbol{\varphi}_{i,d}\right]^T \mathbf{M}\left[\omega_{i,0}{}^2 \boldsymbol{\varphi}_{i,0} - \omega_{i,d}^2 \boldsymbol{\varphi}_{i,d}\right], \quad i = 1,2,\ldots,n_d \tag{19}$$

The measured mode shapes represent a part of mode shapes of damaged structure and restrict the dynamic characteristics. Let us assume the n_d measured mode shapes at the i-th mode as the dynamic constraints to be

$$\mathbf{A}\omega_{i,d}^2 \boldsymbol{\varphi}_{i,d} = \mathbf{b}, \qquad n_d < n \tag{20}$$

where the matrix $\mathbf{A}$ is an $n_d \times n$ Boolean matrix to exhibit the measuring positions. Modifying Eqn. (20) as $\mathbf{AM}^{-1/2}\omega_{i,d}^2 \mathbf{M}^{1/2}\boldsymbol{\varphi}_{i,d} = \mathbf{b}$, defining $\mathbf{Q} = \mathbf{AM}^{-1/2}$ and solving it with respect to $\omega_{i,d}^2 \mathbf{M}^{1/2}\boldsymbol{\varphi}_{i,d}$, the result can be derived as

$$\omega_{i,d}^2 \mathbf{M}^{1/2}\boldsymbol{\varphi}_{i,d} = \mathbf{Q}^+\mathbf{b} + \left(\mathbf{I} - \mathbf{Q}^+\mathbf{Q}\right)\mathbf{h} \tag{21}$$

where $\mathbf{h}$ is an arbitrary vector. Utilizing Eqn. (21) into Eqn. (19) and minimizing the result, it follows that

$$\mathbf{Q}^+\mathbf{b} + \left(\mathbf{I} - \mathbf{Q}^+\mathbf{Q}\right)\mathbf{h} = \omega_{i,0}^2 \mathbf{M}^{1/2}\boldsymbol{\varphi}_{i,0} \tag{22}$$

Solving Eqn. (22) with respect to the arbitrary vector $\mathbf{h}$, we obtain

$$\mathbf{h} = \left(\mathbf{I} - \mathbf{Q}^+\mathbf{Q}\right)\omega_{i,0}^2 \mathbf{M}^{1/2}\boldsymbol{\varphi}_{i,0} + \mathbf{Q}^+\mathbf{Q}\mathbf{x} \tag{23}$$

where $\mathbf{x}$ denotes another arbitrary vector. Substituting Eqn. (23) into Eqn. (21) and arranging the result, it follows that

$$\omega_{i,d}^2 \mathbf{M}^{1/2}\boldsymbol{\varphi}_{i,d} = \omega_{i,0}^2 \mathbf{M}^{1/2}\boldsymbol{\varphi}_{i,0} + \left(\mathbf{AM}^{-1/2}\right)^+ \left(\mathbf{b} - \omega_{i,0}^2 \mathbf{A}\boldsymbol{\varphi}_{i,0}\right) \tag{24}$$

Finally, all mode shapes of the damaged dynamic system can be derived as

$$\boldsymbol{\varphi}_{i,d} = \left(\frac{\omega_{i,0}}{\omega_{i,d}}\right)^2 \boldsymbol{\varphi}_{i,0} + \frac{1}{\omega_{i,d}^2}\mathbf{M}^{-1/2}\left(\mathbf{AM}^{-1/2}\right)^+ \left(\mathbf{b} - \omega_{i,0}^2 \mathbf{A}\boldsymbol{\varphi}_{i,0}\right) \tag{25}$$

It is observed that the mode shapes at unmeasured degrees of freedom can be estimated based on the measured mode shapes and the initial mode shapes of the intact system by Eqn. (25).

4. APPLICATIONS

As an example, we consider a complicated structural system shown in Fig. 1. The static equilibrium equation of the system can be written in matrix form of

$$\mathbf{Ku} = \mathbf{F} \tag{26}$$

where $\mathbf{u} = [u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5]^T$,
$\mathbf{F} = [F_1 \quad F_2 \quad F_3 \quad F_4 \quad F_5]^T$, and

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 + k_3 & -k_2 & -k_3 & 0 & 0 \\ -k_2 & k_2 + k_4 + k_6 + k_7 & 0 & -k_4 - k_6 & -k_7 \\ -k_3 & 0 & k_3 + k_5 & -k_5 & 0 \\ 0 & -k_4 - k_6 & -k_5 & k_4 + k_5 + k_6 + k_8 & -k_8 \\ 0 & -k_7 & 0 & -k_8 & k_7 + k_8 + k_9 \end{bmatrix} \tag{27}$$

Assuming that the system is damaged and the stiffness k_3 is reduced to $0.8k_3$, the static displacements of the damaged structure with the following numerical values are calculated by

$$\begin{gathered} k_1 = 500\text{N/mm}, \; k_2 = 450\text{N/mm}, \\ k_3 = 600\text{N/mm}, \; k_4 = 300\text{N/mm}, \\ k_5 = 900\text{N/mm}, \; k_6 = 500\text{N/mm}, \; k_7 = 800\text{N/mm}, \\ k_8 = 400\text{N/mm}, \; k_9 = 770\text{N/mm} \\ F_1 = F_2 = F_3 = F_5 = 0, \quad F_4 = 1000\text{N} \end{gathered} \tag{28}$$

Assuming that the static displacements at two nodal points 2 and 3 are measured, the displacements at the unmeasured nodal points can be estimated by utilizing the expansion equation of Eqn. (15), the initial stiffness matrix of Eqn. (27), $\mathbf{K}_0$ and the measured displacements.

$$\mathbf{u}_0 = [0.8149 \quad 1.0998 \quad 1.2802 \quad 1.5904 \quad 0.7696]^T \text{(mm)}$$
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$
$$u_{2,d} = 1.1121\text{mm}, \quad u_{3,d} = 1.3343\text{mm} \tag{29}$$

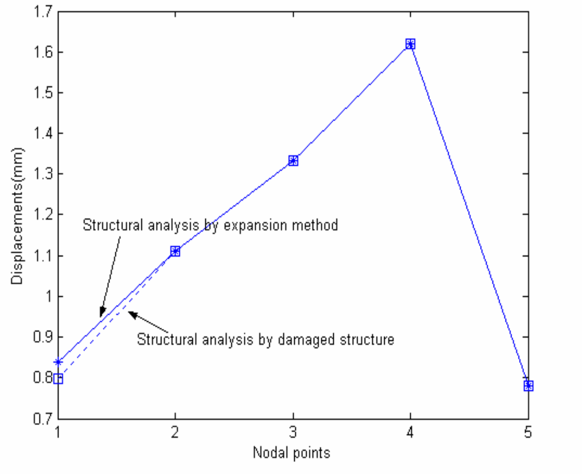


Figure 2. Displacement comparison

Figure 2 compares the actual displacements of the damaged structure and the estimated displacements by the expansion method. As shown by the plots, the expansion method exactly estimates the nodal displacements at nodes 2, 3, 4 and 5 except node 1. Although the displacement at node 1 exhibits the displacement difference of about 5%, it can be concluded that the proposed expansion method describes properly the static displacements at unmeasured nodal points.

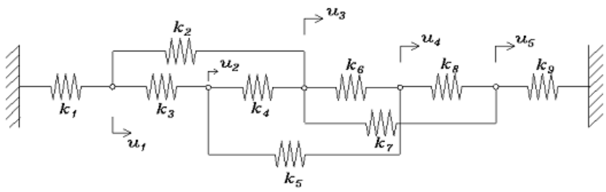


Figure 1. A five degrees of freedom system

As another example, let us consider a dynamic system without damping shown in Fig. 3. The dynamic equation of motion for the system can be derived as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F}(t) \tag{30}$$

where $\mathbf{u} = [u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5 \quad u_6]^T$, $\mathbf{M} = diag([m_1 \quad m_2 \quad m_3 \quad m_4 \quad m_5 \quad m_6])$, and

$$\mathbf{K} = \begin{bmatrix} k_1+k_2 & 0 & 0 & -k_2 & 0 & 0 \\ 0 & k_3+k_4 & -k_4 & 0 & 0 & -k_3 \\ 0 & -k_4 & k_4+k_7+k_8 & 0 & -k_7 & -k_8 \\ -k_2 & 0 & 0 & k_2+k_3+k_6 & -k_6 & 0 \\ 0 & 0 & -k_7 & -k_6 & k_6+k_7 & 0 \\ 0 & -k_3 & -k_8 & 0 & 0 & k_3+k_8 \end{bmatrix} \tag{31}$$

Let us assume the following values for obtaining the modal parameters of the initial system.

$$m_1 = 1, \quad m_2 = 1.3, \quad m_3 = 1.0, \quad m_4 = 1.6, \quad m_5 = 1.4, \quad m_6 = 1.9\text{N}\cdot\text{sec}^2/\text{mm}$$

$$k_1 = 500\text{N/mm}, \quad k_2 = 450\text{N/mm},$$
$$k_3 = 600\text{N/mm}, \quad k_4 = 300\text{N/mm},$$
$$k_5 = 900\text{N/mm}, \quad k_6 = 500\text{N/mm},$$
$$k_7 = 800\text{N/mm}, \quad k_8 = 400\text{N/mm} \tag{32}$$

The first two eigenvalues and the corresponding eigenvectors of the initial system can be calculated as

$$\omega_{1,o}^2 = 38.7(rad./\sec.)^2, \quad \omega_{2,o}^2 = 387.9(rad./\sec.)^2$$
$$\boldsymbol{\varphi}_{1,o} = [0.0575 \quad 0.4682 \quad 0.3860 \quad 0.1164 \quad 0.2946 \quad 0.4699]^T$$
$$\boldsymbol{\varphi}_{1,o} = [0.0575 \quad 0.4682 \quad 0.3860 \quad 0.1164 \quad 0.2946 \quad 0.4699]^T \tag{33}$$

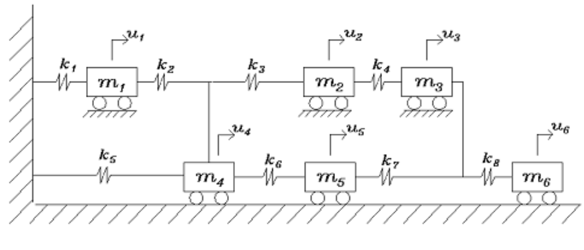


Figure 3. A structural system

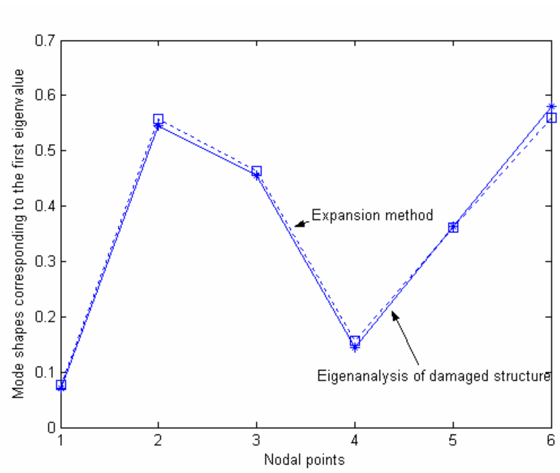
Assuming that the system is damaged and the stiffness of k_3 is reduced by 80%, the modal parameters of the initial system must be changed. Assume that the first two eigenvalues and the eigenvectors corresponding to nodes 2 and 3 of the damaged structure are measured.

$$\omega_{1,d}^2 = 36.9(rad./\sec.)^2, \quad \omega_{2,d}^2 = 363.9(rad./\sec.)^2$$
$$(\varphi_{1,d})_2 = 0.4642, \quad (\varphi_{1,d})_3 = 0.3867$$
$$(\varphi_{2,d})_2 = -0.2438, \quad (\varphi_{2,d})_3 = 0.1692 \tag{34}$$

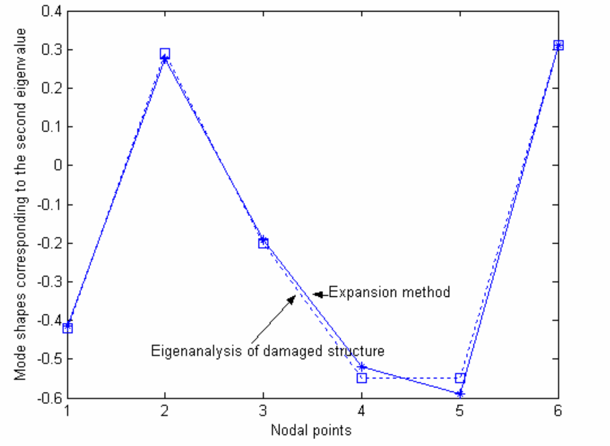
The mode shape values at the unmeasured nodes can be calculated from Eqn. (25) utilizing the measured modal parameters and the following values.

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad i = 1,2$$
$$\mathbf{b}_1 = \begin{bmatrix} 17.14 \\ 14.28 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} -88.73 \\ 61.59 \end{bmatrix} \tag{35}$$

The first two mode shape vectors can be estimated by Eqn. (25). Figure 4 exhibits that the proposed method properly describes the unmeasured mode shapes although the numerical results locally exhibited the maximum deviation of 6.4 and 13.4% from the actual first and second mode shapes, respectively. The modal data of the full set of finite element degrees of freedom to be obtained by the proposed method should provide the fundamental idea for damage detection.



(a)



(b)

Figure 4. Numerical comparison of eigenanalysis and expansion method of damage structure; (a) the first mode, (b) the second mode

5. CONCLUSIONS

This study provided the expansion method to estimate the static and dynamic data at the unmeasured nodes based on the measured static displacement or measured modal parameters. Utilizing the measured data as the constraints for static and dynamic behavior, the expansion methods were derived by minimizing the variation of the potential energy for static approach and the Gauss's function for dynamic approach. Although the proposed methods exhibit some deviation from the actual data, it can be concluded that they properly describe the static displacements and dynamic modes of the full set of finite element degrees of freedom. It is recognized that the proposed methods can be widely utilized in the damage detection methods.

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