A New H_2 Bound for H_∞ Entropy

Hui Zhang and Youxian Sun

Abstract: The H_∞ entropy in H_∞ control theory is discussed based on investigating information transmission in continuous-time linear stochastic systems. It is proved that the stabilizing feedback does not change the time-average information transmission between system input and output, and the H_∞ entropies of open- and closed-loop stable transfer functions are bounded by mutual information rate between input and output in the open-loop system. Furthermore, a new H_2 upper bound for H_∞ entropy is introduced with a numerical example. Thus the H_∞ entropy of a stable transfer function is sandwiched between H_2 norms of the original system and a static feedback system.

Keywords: Feedback, H_∞ entropy, H_2 bound, information transmission, linear stochastic system, mutual information rate.

1. INTRODUCTION

As a suboptimal robust control design method, the minimum entropy H_∞ control method has been developed in the past [1-3]. It adopts an unintuitive function of system closed-loop transfer matrix as its auxiliary performance index. For $F(s) \in \mathbb{RH}_\infty$, where $\mathbb{RH}_\infty$ denotes the set of all proper, rational and stable transfer functions, when the H_∞ norm satisfies $\|F\|_\infty < \lambda$, the entropy of $F(s)$ in H_∞ control is defined as:

$$E(F,\lambda) \triangleq \frac{-\lambda^2}{2\pi} \int_{-\infty}^{\infty} \ln \det[I - \lambda^{-2} F^*(i\omega) F(i\omega)] \mathrm{d}\omega, \quad (1)$$

where $F^*(i\omega) = F^{\mathrm{T}}(-i\omega)$. Function (1) is different from the concept of Shannon entropy. We refer to it as the H_∞ entropy. The minimum entropy H_∞ control method is to find an admissible controller minimizing the H_∞ entropy of a closed-loop system when its H_∞ norm is constrained. And the deduced minimum entropy H_∞ controller is in fact the 'central controller' [1]. It was known that the H_∞ entropy of a stable system is lower bounded by its H_2 norm, i.e.,

$$E(F,\lambda) \geq \| F \|_2^2 \quad (2)$$

with equality holds when $\lambda \to \infty$, and hence an index measuring the tradeoff between the H_∞ optimality and the H_2 optimality [10]. Although the H_∞ entropy is different from Shannon entropy, it was proved to be connected with information theoretic measures firmly [5]. In the authors' opinion, the physical meaning of H_∞ entropy still needs to be discussed, especially from the viewpoint of information theory [6].

In this note, a further discussion on H_∞ entropy of continuous-time, linear time invariant (LTI) control system is made based on investigating information transmission. It is shown that the stabilizing feedback does not change the information transmission from system input to output, and H_∞ entropies of stable open- and closed-loop systems are bounded by the mutual information rate between input and output in open-loop system. Furthermore, based on the relation between mutual information and the minimum mean-square error (MMSE) of state estimation, a new H_2 upper bound for H_∞ entropy is introduced. Thus the H_∞ entropy is sandwiched between H_2 norms of the original system and a static feedback system.

Conceptions and lemmas concerning information transmission are introduced in Section 2. In Section 3, the H_∞ entropies of open- and closed-loop systems are proved to be bounded by mutual information rates, and the new H_2 bound for H_∞ entropy is formulated. Section 4 gives an illustrative example. Section 5 is the conclusion. In this note, we use w_0^T to represent the path of a continuous-time process $w(t)$ over $0 \leq t \leq T$, $t \in \mathbb{R}$, use $w(s)$ and $\Phi_w(\omega)$ to represent its Laplace transform and spectrum density, respectively, and similarly for others.

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2. INFORMATION TRANSMISSION

In this section, information transmission in continuous-time LTI systems will be discussed by adopting the measure of mutual information rate which describes time average transmitted information. The mutual information rate between continuous-time processes $\alpha(t)$ and $\beta(t)$ is defined as [8]:

$$\bar{I}(\alpha;\beta) \triangleq \lim_{T\to\infty}\frac{1}{T}I(\alpha_0^T;\beta_0^T), \tag{3}$$

if the limit exists, where $I(\alpha_0^T;\beta_0^T)$ denotes the mutual information between α_0^T and β_0^T. When the processes are stationary, the limit in (3) exists [8]. If they are additionally joint Gaussian [9],

$$\bar{I}(\alpha;\beta) = \frac{1}{4\pi}\int_{-\infty}^{\infty}\ln\frac{\det\Phi_\alpha(\omega)\det\Phi_\beta(\omega)}{\det\Phi_\theta(\omega)}\mathrm{d}\omega, \tag{4}$$

where $\theta(t) \triangleq [\alpha^{\mathrm{T}}(t)\,\beta^{\mathrm{T}}(t)]^{\mathrm{T}}$.

Lemma 1: Suppose μ, η be random vectors and $f(\cdot)$ be a deterministic map. Then

$$H(\mu + f(\eta)\,|\,\eta) = H(\mu\,|\,\eta), \tag{5}$$

where $H(\cdot)$ and $H(\cdot|\cdot)$ denote entropy and conditional entropy, respectively.

Proof: This conclusion can be derived from basic facts in information theory [8].

Consider the following multivariable LTI system with standard state space model [11]:

$$\begin{cases}\dot{x}(t) = Ax(t) + Bw(t),\\ y(t) = Cx(t) + v(t),\end{cases} \tag{6}$$

where $x(t)\in\mathbb{R}^n$, $y(t)\in\mathbb{R}^r$, $w(t)\in\mathbb{R}^r$ and $v(t)\in\mathbb{R}^r$ are respectively system state, output, input process and measurement noise; A, B, and C are constant matrices with corresponding dimensions. Denote $z(t) = Cx(t)$. The system is illustrated by Fig. 1. The transfer matrix from $w(s)$ to $z(s)$ or $y(s)$ is

$$H(s) \triangleq C(sI - A)^{-1}B. \tag{7}$$

From the viewpoint of communication, system (6) can be considered as an information transmission channel, where $w(t)$ is the source message, $x(t)$ or $z(t)$ is the encoded channel input, $v(t)$ is the channel noise, and $y(t)$ is the channel output.

We will also consider a closed-loop system with a stabilizing feedback controller $C(s)$ connected around $H(s)$, as shown in Fig. 2, where $y_C(t)$

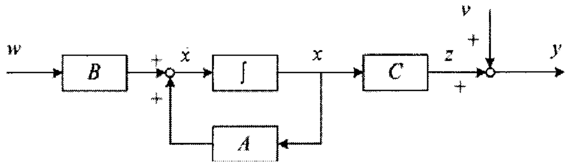


Fig. 1. Linear state-space model.

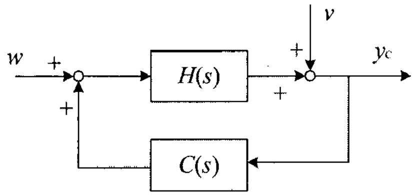


Fig. 2. The feedback system.

denotes the closed-loop measurement output. It can be considered as a channel with feedback. Let

$$T(s) \triangleq H(s)S(s) \tag{8}$$

denote the closed-loop transfer function from $w(t)$ to $y_C(t)$, where $S(s) \triangleq (I - H(s)C(s))^{-1}$.

The above systems will be discussed under the following assumptions.

Assumption 1: Suppose in Figs. 1 and 2, $w(t)$ and $v(t)$ are mutual independent stationary Gaussian white processes with zero means and covariances Q and R, respectively; $x(0)$ is a zero mean random vector and independent of $w(t)$ and $v(t)$. The realization (6) is completely controllable and observable. The feedback system is asymptotically stable and well-posed [10].

We do not assume $H(s)$ is stable. If it is stable, all variables in Fig. 1 are stationary and the mutual information rates (e.g., $\bar{I}(w;y)$, $\bar{I}(x;y)$) always exist. If it is unstable, the following lemma verifies the existence of mutual information rates and gives an interesting relation between mutual information and the MMSE of state estimation. This relation had recently gotten considerable interests in the field of communication [12].

Let the MMSE estimation of state be denoted as $\hat{x}(t)$. Under Assumption 1, the Kalman-Bucy filter for system (6) approaches to the time invariant implementation [11]:

$$\begin{cases}\dot{\hat{x}}(t) = (A - KC)\hat{x}(t) + Ky(t),\\ \tilde{y}(t) = y(t) - C\hat{x}(t),\end{cases} \tag{9}$$

where $\tilde{y}(t) = C\tilde{x}(t) + v(t)$ is the innovation process, $\tilde{x}(t) = x(t) - \hat{x}(t)$ is the estimation error,

$$K = PC^{\mathrm{T}}R^{-1}, \tag{10}$$

and the estimation error covariance $P(t) \triangleq \mathbb{E}\{\tilde{x}(t)\tilde{x}^{\mathrm{T}}(t)\}$ converges to a constant matrix P, which is the unique positive-definite solution to the Riccati equation:

$$AP + PA^{\mathrm{T}} + BQB^{\mathrm{T}} - PC^{\mathrm{T}}R^{-1}CP = 0. \quad (11)$$

Lemma 2 [13]: Under conditions in Assumption 1, the mutual information rate between state $x(t)$ and output $y(t)$ exists and is

$$\bar{I}(x;y) = \frac{1}{2}\mathrm{tr}[CPC^{\mathrm{T}}], \quad (12)$$

where tr[·] denotes the trace of a matrix.

We have the following conclusions on information transmission.

Lemma 3: Under conditions in Assumption 1, mutual information rates in system (6) shown in Fig. 1 exist and,

$$\bar{I}(x;y) = \bar{I}(z;y) = \bar{I}(w;y), \quad (13)$$

whether the open-loop system is stable or not. Furthermore, the mutual information rate between the extraneous input and the output in the closed-loop system shown in Fig. 2 is equivalent to that in the open-loop system shown in Fig. 1, i.e.,

$$\bar{I}(w;y_{\mathrm{C}}) = \bar{I}(w;y). \quad (14)$$

Proof: Let $\mathcal{C}$ denote the map corresponding to output matrix C, so that $y_0^T = \mathcal{C}(x_0^T) + v_0^T$. Based on Lemma 1 and the fact that in the open-loop system, $v(t)$ is independent of $x(t)$ and $z(t)$, $I(x_0^T; y_0^T) = H(y_0^T) - H(\mathcal{C}(x_0^T) + v_0^T \mid H(x_0^T)) = H(y_0^T) - H(v_0^T)$. On the other hand, $I(z_0^T; y_0^T) = H(y_0^T) - H(v_0^T)$. Then we have $\bar{I}(x;y) = \bar{I}(z;y)$ if they exist. Using the same method, we can get $\bar{I}(w;y) = \bar{I}(z;y)$ if they exist. Moreover, $\bar{I}(x;y)$ exists by Lemma 2. Then $\bar{I}(w;y)$, $\bar{I}(z;y)$ exist, and (13) is gotten.

Note that the well-posedness implies $S(s)$, $S^{-1}(s) \in \mathbb{RL}_\infty$, where $\mathbb{RL}_\infty$ denotes the set of all proper and real-rational transfer functions, are causal. It can be seen $y_{\mathrm{C}}(s) = S(s)y(s)$. Let $\mathcal{S}$ and $\mathcal{S}^{-1}$ denote respectively maps corresponding to $S(s)$ and $S^{-1}(s)$, so that $y_{\mathrm{C}0}^T = \mathcal{S}(y_0^T)$ and $y_0^T = \mathcal{S}^{-1}(y_{\mathrm{C}0}^T)$. By the property of mutual information [8],

$$\begin{aligned} I(w_0^T; y_{\mathrm{C}0}^T) &= I(w_0^T; \mathcal{S}(y_0^T)) \le I(w_0^T; y_0^T) \\ &= I(w_0^T; \mathcal{S}^{-1}(y_{\mathrm{C}0}^T)) \le I(w_0^T; y_{\mathrm{C}0}^T). \end{aligned}$$

Hence $\bar{I}(w;y_{\mathrm{C}}) = \bar{I}(w;y)$. □

Remark 1: Lemma 3 states that feedback has no effect on information transmission from extraneous input to output in the discussed system, where $v(t)$ is assumed to be white. However, it can be seen from the proof that even if $v(t)$ is not white, (14) is still true when system (6) is stable. This is different from the conclusion on communication, which states that for continuous stationary Gaussian channels with colored noise, the capacity (defined as the maximum mutual information rate between message and channel output) is increased by feedback [7,8].

3. A NEW H_2 BOUND OF H_∞ ENTROPY

Based on the above discussion, we will give a new mean-square interpretation for H_∞ entropies of systems shown in Figs. 1 and 2. Besides Assumption 1, suppose systems in Figs. 1 and 2 are driven by a standard Gaussian white noise, i.e., $\Phi_w = I$.

Consider the case that (6) is asymptotically stable. By the spectral factorization, there exists a rational matrix function $F_y(s)$ with all zeros and poles of $\det|F_y(s)|$ in the left hand side of the complex plane, such that $\Phi_y(\omega) = F_y^*(j\omega)F_y(j\omega)$. Let

$$\kappa \triangleq \| F_y(s) \|_\infty .$$

For $\Phi_v(\omega) > 0$, $\Phi_y(\omega) > H(j\omega)H^*(j\omega)$, then

$$\| H(s) \|_\infty < \kappa. \quad (15)$$

The H_∞ entropy of $H(s)$ under constraint (15) is

$$E(H,\kappa) \triangleq \frac{-\kappa^2}{2\pi}\int_{-\infty}^{\infty} \ln\det[\mathbf{I} - \kappa^{-2}H(j\omega)H^*(j\omega)]\, d\omega. \quad (16)$$

For the case that $H(s)$ is not necessary stable, we consider the stabilizing feedback system shown in Fig. 2. There exists a rational matrix function $F_{\mathrm{C}}(s)$ with all zeros and poles of $\det|F_{\mathrm{C}}(s)|$ in the left hand side of the complex plane, such that $\Phi_{y_{\mathrm{C}}}(\omega) = F_{\mathrm{C}}^*(j\omega)F_{\mathrm{C}}(j\omega)$. Let

$$\gamma \triangleq \| F_{\mathrm{C}}(s) \|_\infty . \quad (17)$$

Then $\Phi_{y_{\mathrm{C}}}(\omega) \le \gamma^2 I$. For $S(j\omega)\Phi_v(\omega)S^*(j\omega) > 0$, so $\Phi_{y_{\mathrm{C}}}(\omega) > T(j\omega)T^*(j\omega)$. Hence,

$$\gamma^{-2}T(j\omega)T^*(j\omega) \le \Phi_{y_{\mathrm{C}}}^{-1}(\omega)T(j\omega)T^*(j\omega) < I. \quad (18)$$

So,

$$\|T(s)\|_\infty < \gamma. \tag{19}$$

The H_∞ entropy of $T(s)$ under (19) is

$$E(T,\gamma) \triangleq \frac{-\gamma^2}{2\pi}\int_{-\infty}^{\infty} \ln\det[I-\gamma^{-2}T(j\omega)T^*(j\omega)]\,\mathrm{d}\omega. \tag{20}$$

We have the following conclusions.

Proposition 1: Suppose $\Phi_w(\omega)=I$ besides Assumption 1. The H_∞ entropy of the closed-loop transfer function $T(s)$ and the information transmission in systems shown in Figs. 1 and 2 possess the following relation,

$$E(T,\gamma) \le 2\gamma^2 \bar{I}(w;y_C) = 2\gamma^2 \bar{I}(w;y), \tag{21}$$

with the first equality holds if and only if $y_C(t)$ in Fig. 2 is a Gaussian white noise with spectrum $\gamma^2 I$. If the open-loop system $H(s)$ is asymptotically stable, the H_∞ entropy of $H(s)$ and the information transmission in system (6) possess the following relation,

$$E(H,\kappa) \le 2\kappa^2 \bar{I}(w;y), \tag{22}$$

with the equality holds if and only if $y(t)$ in Fig. 1 is a Gaussian white noise with spectrum $\kappa^2 I$.

Proof: Let $\xi(t) \triangleq [w^{\mathrm T}(t)\ y_C^{\mathrm T}(t)]^{\mathrm T}$ with spectrum $\Phi_\xi(\omega)=\begin{bmatrix}\Phi_w(\omega) & \Phi_{wy_C}(\omega)\\ \Phi_{wy_C}^*(\omega) & \Phi_{y_C}(\omega)\end{bmatrix}$, where $\Phi_{wy_C}(\omega)$ is the mutual spectrum of $w(t)$, $y_C(t)$. From (4),

$$\bar{I}(w;y_C) = \frac{-1}{4\pi}\int_{-\infty}^{\infty} \ln\det[I-\Phi_{y_C}^{-1}(\omega)T(j\omega)T^*(j\omega)]\,\mathrm{d}\omega.$$

From (18),

$$\frac{-1}{4\pi}\int_{-\infty}^{\infty} \ln\det[I-\Phi_{y_C}^{-1}(\omega)T(j\omega)T^*(j\omega)]\,\mathrm{d}\omega \ge \frac{-1}{4\pi}\int_{-\infty}^{\infty} \ln\det[I-\gamma^{-2}T(j\omega)T^*(j\omega)]\,\mathrm{d}\omega.$$

Then combining with Lemma 3 we get (21). When $H(s)$ is stable,

$$\bar{I}(w;y) = \bar{I}(x;y) = \frac{-1}{4\pi}\int_{-\infty}^{\infty} \ln\det[I-\Phi_y^{-1}(\omega)H(j\omega)H^*(j\omega)]\,\mathrm{d}\omega.$$

For $\Phi_y(\omega) \le \kappa^2 I$, we can get (22) by combining the above equation with (16). □

Remark 2: Proposition 1 states an interesting property of H_∞ entropies: they are lower bounds of mutual information rates between inputs and outputs. Note that the H_∞ entropy of closed-loop system is related to the mutual information rate in open-loop system. This is an extension of conclusion in [5].

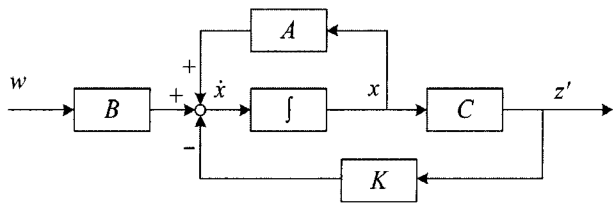


Fig. 3. A static output feedback system.

Corresponding to $H(s)$ we construct a static output feedback system without measurement noise, as shown in Fig. 3. The transfer function from $w(t)$ to output $z'(t)$ is

$$\bar{H}(s) = C(sI-A+KC)^{-1}B. \tag{23}$$

Assumption 2: Besides conditions in Assumption 1, suppose the static feedback system $\bar{H}(s)$ shown in Fig. 3 is closed-loop stable, i.e., all the eigenvalues of $A-KC$ are in the left hand side of the complex plane. Systems in Figs. 1, 2, and 3 are driven by a standard Gaussian white noise $w(t)$ with covariance $Q=I$.

Proposition 2: Consider systems shown in Figs. 1, 2 and 3 under Assumption 2. If the static feedback matrix K in system $\bar{H}(s)$ is chosen as

$$K = \Pi C^{\mathrm T} R^{-1}, \tag{24}$$

where Π is the steady-state covariance of state of the feedback system $\bar{H}(s)$, R is the covariance of the measurement noise $v(t)$ in system (6), then the H_∞ entropy of closed-loop transfer function $T(s)$ and the H_2 norm of $\bar{H}(s)$ shown in Fig. 3 possess the following relation,

$$E(T,\gamma) \le \gamma^2 \|\bar{H}(s)\|_2^2, \tag{25}$$

with the equality holds if and only if $y_C(t)$ in Fig. 2 is a Gaussian white noise with spectrum $\gamma^2 I$. If the open-loop transfer function $H(s)$ is asymptotically stable, and the feedback matrix K is chosen as (24), then the H_∞ entropy of $H(s)$ and the H_2 norm of $\bar{H}(s)$ possess the following relation,

$$E(H,\kappa) \le \kappa^2 \|\bar{H}(s)\|_2^2, \tag{26}$$

with the equality holds if and only if $y(t)$ in Fig. 1 is a Gaussian white noise with spectrum $\kappa^2 I$.

Proof: When $\bar{H}(s)$ is stable, Π is the unique positive solution of the Lyapunov equation $(A-KC)\Pi+\Pi(A-KC)^{\mathrm{T}}+BB^{\mathrm{T}}=0.$ It can be seen from the Kalman filtering theory [11] that when K is chosen as (24), $\bar{H}(s)$ is stable and the above equation is identical to (11), $\Pi=P.$ Moreover, under Assumption 2, the H_2 norm of $\bar{H}(s)$ is $\|\bar{H}(s)\|_2^2=\lim_{t\to\infty}\mathbb{E}\{(z'(t))^{\mathrm{T}}z'(t)\}=\mathrm{tr}[C\Pi C^{\mathrm{T}}].$ Then by Lemma 2,

$$\bar{I}(x;y)=\frac{1}{2}\|\bar{H}(s)\|_2^2. \tag{27}$$

With Proposition 1 and Lemma 3 we have $E(T,\gamma)\le 2\gamma^2\bar{I}(x;y),$ with the equality holds if and only if $y_C(t)$ is a Gaussian white noise with spectrum $\gamma^2 I.$ Combining with (27), we get (25). When $H(s)$ is stable, we can get (26) by combining (27) with (13) and (22). □

Remark 3: It was known that the H_2 norm of a stable transfer function is a lower bound of its H_∞ entropy [1] as stated by (2). In (26), however, an upper H_2 norm bound for the H_∞ entropy is formulated with a static feedback. Thus the H_∞ entropy of a stable transfer function is sandwiched between H_2 norms of the original and static feedback systems:

$$\|H(s)\|_2^2\le E(H,\kappa)\le\kappa^2\|\bar{H}(s)\|_2^2. \tag{28}$$

4. ILLUSTRATIVE EXAMPLE

To illustrate inequalities (28), we give here a numerical example. The computing results are obtained by using Matlab. Let parameters in realization (6) be $A=\begin{bmatrix}-2 & 1\\ 0 & -2\end{bmatrix},$ $B=\begin{bmatrix}0 & 1\end{bmatrix}^{\mathrm{T}},$ $C=\begin{bmatrix}1 & 0\end{bmatrix},$ the covariances of $w(t)$ and $v(t)$ in (6) are $Q=1,$ $R=1,$ respectively. We get that the steady-state covariance of the filtering error is $P=\begin{bmatrix}0.0307 & 0.0618\\ 0.0618 & 0.2490\end{bmatrix}.$ $\bar{H}(s)=C(sI-A_1)^{-1}B,$ where $A_1=A-KC=\begin{bmatrix}-2.0307 & 1\\ -0.0618 & -2\end{bmatrix}.$ The H_2 norms of $H(s)$ and $\bar{H}(s)$ are calculated to be

$$\|H(s)\|_2=0.1768,\quad \|\bar{H}(s)\|_2=0.1735, \tag{29}$$

respectively. To get the bound κ in (15), we need the spectral factorization realization $F_y(s)$ of system (6) satisfying $F_y^*(j\omega)F_y(j\omega)=\Phi_y(\omega).$ This can be obtained by Kalman-Bucy filter [11], i.e., $F_y(s)=C(sI-A)^{-1}B_2+D_2,$ where $D_2=R^{1/2}=1,$ $B_2=PC^{\mathrm{T}}R^{-1/2}=\begin{bmatrix}0.0307\\ 0.0618\end{bmatrix}.$ The H_∞ norm of $F_y(s)$ is then calculated to be

$$\kappa=\|F_y(s)\|_\infty=1.0312. \tag{30}$$

Because in this example $y(t)$ is not white, and $\kappa=1.0312<\infty,$ then from (28), (29), and (30) we get

$$0.0313<E(H,\kappa)<0.0320. \tag{31}$$

On the other hand, by using the state space computing method of the H_∞ entropy [4], we get that under $\|H(s)\|_\infty<\kappa=1.0312$ the H_∞ entropy of $H(s),$ defined by (16), is $E(H,\kappa)=0.0317.$ This coincides with inequalities (31).

5. CONCLUSIONS

By investigating information transmission in continuous-time multivariable LTI systems, relations between mutual information rates and H_∞ entropies were formulated. The H_∞ entropies of stable open- and closed-loop transfer functions were proved to be upper bounded by mutual information rates between system inputs to outputs. Especially, the H_∞ entropy of closed-loop system is related to mutual information rate in open-loop system. A new H_2 bound for H_∞ entropy was introduced with an illustrative example. These relations formulate information descriptive property of the H_∞ entropy, and provide potential instruments for analysis and design of stochastic control systems.

REFERENCES

[1] D. Mustafa, K. Glover, *Minimum Entropy H_∞ Control*, Springer-Verlay, Berlin, 1990.

[2] P. A. Iglesias and M. A. Peters, "An entropy formula for nonlinear systems," *International Journal of Franklin Institute*, vol. 337, pp. 859-874, 2000.

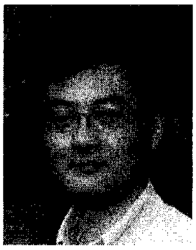
[3] S.-H. Lee and J.-S. Kim, "Mixed H_2/H_∞ -controller realization with entropy integral," *International Journal of Control, Automation, and Systems*, vol. 1, no. 2, pp. 206-209, 2003.

[4] S. Boyd, *Linear Controller Design - Limits of Performance*, Prentice-Hall, New Jersey, 1991.

[5] A. A. Stoorvogel and J. H. Van Schuppen, "System identification with information theoretic criteria," *Identification, Adaptation, Learning* (S. Bittanti, G. Picc, Eds.), pp. 289-338, Springer, Berlin, 1996.

[6] H. Zhang and Y. X. Sun, "Information theoretic

interpretations for H_∞ entropy," *Proc. of the 16th IFAC World Congress*, Prague, July 3-8, 2005.
[7] T. Cover and S. Pombra, "Gaussian feedback capacity," *IEEE Trans. on Information Theory*, vol. 35, no. 1, pp. 37-43, 1989.
[8] S. Ihara, *Information Theory for Continuous Systems,* World Scientific Publishing Co. Pte. Ltd., Singapore, 1993.
[9] M. S. Pinsker, *Information and Information Stability of Random Variables and Processes*, Holden-Day, Inc., 1964.
[10] K. Zhou, *Essential of Robust Control*, Prentice-Hall, New Jersey, 1998.
[11] T. Kailath, *Linear Estimation*, Prentice Hall, New Jersey, 2000.
[12] D. P. Palomar and S. Verdú, "Representation of mutual information via input estimates," *IEEE Trans. on Information Theory*, vol. 53, no. 2, pp. 453-470, 2007.
[13] T. E. Duncan, "On the calculation of mutual information," *SIAM Journal on Applied Mathematics*, vol. 19, no. 1, pp. 215-220, 1970.



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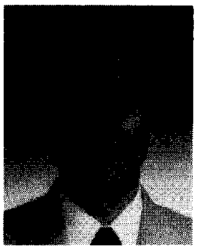

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