Delay-dependent Stabilization of Singular Systems with Multiple Internal and External Incommensurate Constant Point Delays

Yong-Fang Xie, Wei-Hua Gui, and Zhao-Hui Jiang*

Abstract: In this paper, the problem of delay-dependent stabilization for singular systems with multiple internal and external incommensurate constant point delays is investigated. The condition when a singular system subject to point delays is regular independent of time delays is given and it can be easily test with numerical or algebraic methods. Based on Lyapunov-Krasovskii functional approach and the descriptor integral-inequality lemma, a sufficient condition for delay-dependent stability is obtained. The main idea is to design multiple memoryless state feedback control laws such that the resulting closed-loop system is regular independent of time delays, impulse free, and asymptotically stable via solving a strict linear matrix inequality (LMI) problem. An explicit expression for the desired memoryless state feedback control laws is also given. Finally, a numerical example illustrates the effectiveness and the availability for the proposed method.

Keywords: Delay-dependent stabilization, descriptor integral-inequality, point delays, singular systems, state feedback control.

1. INTRODUCTION

Singular systems contain a mixture of differential and algebraic equations, where the algebraic equations represent the constraints imposed on the solution of the differential part. Such systems (sometimes also referred to as descriptor, degenerate, implicit or semi state and generalized state space systems) describe a broad class of systems which are not only of theoretical interest but also have great practical significance. Models of chemical processes, for example, typically consist of differential equations describing the dynamic balances of mass and energy, and additional algebraic equations giving thermodynamic equilibrium relations, steady-state assumptions, empirical correlations, etc. [1]. In mechanical engineering, singular system descriptions result from homonymic and non-homonymic constraints [2]. Also in electronics and even in economics, singular systems descriptions exist [3].

It is difficulty to deal with such singular systems analytically and numerically due to their nature complex, particularly in control system design for them. In the past years, analysis and synthesis problems of singular systems have been extensively studied due to the fact that the singular systems provide a more complicated, yet richer, description of dynamical systems than the standard state-space systems [4-9]. Furthermore, the study of the dynamic performance of singular systems is much more difficult than that of standard state-space systems since singular systems usually have three types of modes, namely, finite dynamic modes, impulsive modes and non-dynamic modes [5], while the latter two do not appear in the state space systems.

Delay systems have attracted many attentions over past decades since time delay is one of the main causes for instability and poor performance of many control systems and are frequently encountered in many industrial processes such as in the steel industry, oil industry, etc. [10]. Delays may be classified as point delays or distributed delays according to their nature, and can also be classified as internals (i.e., in the state) or externals (i.e., in the input or output) according to the signals that they influence. Point delays may be commensurate if each delay is an integer multiple of a base delay or, more generally, incommensurate if they are arbitrary real numbers [11]. The presence of internal delays leads to a large complexity in the resulting system dynamics since the

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whole dynamical systems become infinite-dimensional. In addition, this fact increases the difficulty when studying basic properties, such as, controllability, observability, stability and stabilization, compared to the delay-free case since the transfer functions consist of a transcendent numerator and denominator quasi-polynomials [12]. Due to these reasons, the stabilization of multiple internal incommensurate constant point delay systems is much more challenging than that of the delay-free case [13].

Time delays have naturally an effect on the dynamics of singular systems. It is concluded that their dynamics would be more complex because time delays may exist in the differential equations and / or algebraic equations. Singular delay systems, which are those where the delays influence the system's behaviours such as regularity, impulse, asymptotical stability and so on, present even a higher analysis and design difficulty. It should be pointed out that the stability problem for singular time delay systems is much more complicated than that for regular systems because it requires to consider not only the asymptotical stability, but also regularity and impulse-free at the same time [14]. The latter two need not be considered in regular systems. Very recently, a great effort has been devoted to the investigation of the stability and stabilization problem of singular time-delay systems [15-26], which are the general form of delay systems. However, to the best of authors' knowledge, there are few contributions on the problems of delay-dependent stabilization of singular linear continuous-time systems subject to multiple internal and external incommensurate constant point delays (SLCS-MIEID). This fact motivated the authors to develop delay-dependent controllers for SLCS-MIEID based on state feedback control.

The present paper provides an approach for stability analysis of SLCS-MIEID. To prove the stability, we introduce the descriptor integral-inequality which can be used to study the delay-dependent stabilization problems of singular time-delay systems. Using the Lyapunov-Krasovskii functional technique combined with LMI techniques, we design a state feedback controller for singular time-delay system, which guarantees that the closed-loop system is regular, impulse free and asymptotically stable. The delay-dependent stability criterion is derived in terms of LMIs, and the solutions provide a parameterized representation of the state feedback controller. The LMIs can be easily solved by various efficient convex optimization algorithms.

The rest of the paper is organized as follows. In the second section, a mathematical model for SLCS-MIEID is introduced and at the same time the necessary background for the stability of such systems is also given. The regular independent of time delays problem is also discussed in this section. In the section 3, by employing the descriptor integral-inequality lemma, sufficient conditions for delay-dependent stabilization are deduced based on nonlinear matrix inequality (NLMI), which are associated with the time delays and can not be solved directly. From the viewpoint of LMI, the design method of state feedback control law is summarized. An example is presented in the section 4 to illustrate the effectiveness of the proposed method.

Notation: Throughout the paper the subset of the integers $j=\{0,1,\cdots,n\}$ represents any integer $n\geq 0$, the superscript 'T' stands for the transpose of a matrix. $\mathbb{R}$ and $\mathbb{C}$ are the sets of real and complex numbers, respectively. $\mathcal{R}^n$ denotes the n-dimensional Euclidean space, $\mathcal{R}^{n\times m}$ is the set of all $n\times m$ real matrices, and the notation $P>0$ for $P\in\mathcal{R}^{n\times n}$ means that P is symmetric and positive definite. I and 0 denote the appropriately dimensioned identity matrix and zero matrix, respectively. $\operatorname{diag}\{\cdots\}$ denotes a block-diagonal matrix. The symmetric terms in a symmetric matrix are denoted by $*$, e.g., $\begin{bmatrix} X & Y \\ * & Z\end{bmatrix}=\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z\end{bmatrix}$.

2. PROBLEM STATEMENT

Consider the following singular linear continuous time system with q internal and q' external incommensurate constant point delays described by

$$\begin{cases} E\dot{x}(t)=\sum_{j=0}^{q} A_j x(t-h_j)+\sum_{j=0}^{q'} B_j u_j(t-h'_j), \\ x(t)=\phi(t), \quad t\in[-\max\{h,h'\},0], \end{cases} \tag{1}$$

where $x(t)\in\mathcal{R}^n$ and $u_j(t)\in\mathcal{R}^m$ are the state and control input vectors, respectively, and $A_j\,(j=0,1\cdots,q)$, $B_j\,(j=0,1,\cdots,q')$ are real matrices of compatible orders with the dimensions of those vectors, and $h_j\,(j=1,\cdots,q)$, $h'_j\,(j=1,\cdots,q')$ are the q internal and q' external point delays respectively. The zero delays $h_0=h'_0=0$ corresponding to the delay-free dynamics and current delay-free input are added for notational simplification convenience. $\phi(t)$ is a compatible vector valued continuous different initial function. The maximum delays h and h' are defined as $h=\max_{1\leq j\leq q}(h_j)$ and $h'=\max_{1\leq j\leq q'}(h'_j)$. The singular matrix $E\in\mathcal{R}^{n\times n}$ with $\operatorname{rank}(E)=r<n$ gives the singular character to system (1)

compared to the case $E = I_n$ (standard system).

The unforced singular delay system of (1) can be written as

$$E\dot{x}(t) = A_0 x(t) + \sum_{j=1}^{q} A_j x(t-h_j). \tag{2}$$

It is well known that a singular system has a more complicated structure and contains not only finite dynamical modes (exponential modes), but also infinite frequency modes, including infinite non-dynamical and dynamical modes that may generate undesired impulse behaviours, they should be eliminated. In order to guarantee that the system (2) is regular and impulse-free, the following definition and lemmas are given.

Definition 1: The system (2) is said to be regular if there exists a constant $s \in \mathbb{C}$ such that $\left| sE - \sum_{j=0}^{q} A_j e^{-h_j s} \right| \neq 0.$

The Definition 1 is not easy to test because the term $\left| sE - \sum_{j=0}^{q} A_j e^{-h_j s} \right| \neq 0$ depends on the internal point delays $h_j (j = 1, \cdots, q)$. An alternative characterization of regularity is given as follows. First, the generic rank (g.r.) in $\mathbb{C}$ of a complex matrix $Q(s)$ is defined as $\text{g.r.}(Q()s) = \max_{s \in \mathbb{C}}(\text{rank}[Q(s)])$. The following theorem presents a condition for regularity of the system (2) which is equivalent to Definition 1.

Theorem 1: The system (2) is said to be regular independent of the delays $h_j (j = 1, \cdots, q)$ if the $\text{rank}\left[E, \bar{A}_j\right] = n$ where $\bar{A}_j = \left[A_0, \bar{A}_{j1}\right]$ with $\bar{A}_{j1} = \left[A_1, A_2, \cdots, A_q\right]$.

Proof: From Definition 1, a straight forward calculation yields

$$\begin{aligned} & sE - \sum_{j=0}^{q} A_j e^{-h_j s} \\ & = \left[E, \bar{A}_j\right] \times \left[sI_n, -I_n, -e^{-h_1 s} I_n, \cdots, -e^{-h_q s} I_n\right]^{\mathrm{T}}. \end{aligned} \tag{3}$$

Thus, from (3) we can get that $\exists s \in \mathbb{C}$: $\left| sE - \sum_{j=0}^{q} A_j e^{-h_j s} \right| \neq 0$ is equivalent to $\text{rank}[E, \bar{A}_j] = n$. Since $\text{rank}[sI_n, -I_n, -e^{-h_1 s} I_n, \cdots, -e^{-h_q s} I_n] = n, \forall s \in \mathbb{C}$ then $\text{g.r.}_{s \in \mathbb{C}}\left(sE - \sum_{j=0}^{q} A_j e^{-h_j s} \right) = \text{rank}[E, \bar{A}_j]$.

So, $\text{rank}\left[E, \bar{A}_j\right] = n \Rightarrow \text{g.r.}_{s \in \mathbb{C}}\left(sE - \sum_{j=0}^{q} A_j e^{-h_j s} \right) = n$ $\Rightarrow \left| sE - \sum_{j=0}^{q} A_j e^{-h_j s} \right| \neq 0$ for $s \in \mathbb{C}$ and Theorem 1 is proved.

Lemma 1 [16]: Suppose the pair (E, A_0) is said to be regular and impulse free, then the solution to (2) exists and is impulse free and unique on $[0, \infty)$.

Lemma 2 [27]: The singular system $E\dot{x}(t) = A_0 x(t)$ is regular, impulse free and stable if and only if there exists a matrix P such that $EP^{\mathrm{T}} = PE^{\mathrm{T}} \geq 0$ and $P^{\mathrm{T}} A_0 + A_0^{\mathrm{T}} P < 0$.

Lemma 3 [28]: For a given matrix $A \in R^{m \times n}$, there exists any full rank matrix $P \in R^{m \times m}$ or $Q \in R^{n \times n}$ such that $\text{rank}(A) = \text{rank}(PA) = \text{rank}(AQ)$.

Lemma 4 [29]: For any $a \in \mathcal{R}^{n_a}$, $b \in \mathcal{R}^{n_b}$, $N \in \mathcal{R}^{n_a \times n_b}$, $X \in \mathcal{R}^{n_a \times n_a}$, $Y \in \mathcal{R}^{n_a \times n_b}$, and $Z \in \mathcal{R}^{n_b \times n_b}$, the following inequality holds

$$-2a^{\mathrm{T}} N b \leq \begin{bmatrix} a \\ b \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y - N \\ * & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix},$$

where

$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \geq 0.$$

For system (1), we suppose that the state is available for feedback and the following memoryless state feedback controllers are adopted:

$$u_j(t) = K_j x(t), \quad j = 0, 1, \cdots, q'. \tag{4}$$

When we apply control law (4) to system (1), the resulting closed-loop system is given by

$$\begin{cases} E\dot{x}(t) = \sum_{j=0}^{q} A_j x\left(t - h_j\right) + \sum_{j=0}^{q'} \bar{B}_j x\left(t - h_j'\right), \\ x(t) = \phi(t), \quad t \in [-\max\{h, h'\}, 0], \end{cases} \tag{5}$$

where $\bar{B}_j = B_j K_j (j = 0, 1, \cdots, q')$.

The aim of this paper is to develop a new delay-dependent stabilization method that provides the control gain, K_j, of the control law (4) such that the closed-loop system (5) is regular, impulse free and asymptotically stable. For this purpose, the following lemma is introduced.

Lemma 5: Let $x(t) \in \mathcal{R}^n$ be a vector-valued function with first-order continuous-derivative entries.

Then, the following descriptor integral-inequality holds for any matrices E, M_1, M_2, Y, and $X = X^{\mathrm{T}} > 0$, and a scalar $h \geq 0$

$$-\int_{t-h}^{t} \dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}XE\dot{x}(s)\mathrm{d}s \leq \xi^{\mathrm{T}}(t)\Upsilon\xi(t) + h\xi^{\mathrm{T}}(t)Y^{\mathrm{T}}X^{-1}Y\xi(t), \tag{6}$$

where

$$\Upsilon := \begin{bmatrix} M_1^{\mathrm{T}}E + E^{\mathrm{T}}M_1 & -M_1^{\mathrm{T}}E + E^{\mathrm{T}}M_2 \\ * & -M_2^{\mathrm{T}}E - E^{\mathrm{T}}M_2 \end{bmatrix},$$

$$\xi(t) := \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}, \quad Y := \begin{bmatrix} M_1 & M_2 \end{bmatrix}.$$

Proof: From the Leibniz-Newton formula

$$0 = x(t) - x(t-h) - \int_{t-h}^{t} \dot{x}(s)\mathrm{d}s. \tag{7}$$

So, for any N_1, $N_2 \in \mathcal{R}^{n\times n}$ the following equation holds

$$\begin{aligned} 0 &= 2[x^{\mathrm{T}}(t)E^{\mathrm{T}}N_1^{\mathrm{T}} + x^{\mathrm{T}}(t-h)E^{\mathrm{T}}N_2^{\mathrm{T}}]E \\ &\quad\times\left[x(t) - x(t-h) - \int_{t-h}^{t} \dot{x}(s)\mathrm{d}s\right] \\ &= 2\xi^{\mathrm{T}}(t)\bar{N}^{\mathrm{T}}[E \;\; -E]\xi(t) - 2\int_{t-h}^{t} \xi^{\mathrm{T}}(t)\bar{N}^{\mathrm{T}}E\dot{x}(t)\mathrm{d}s, \end{aligned} \tag{8}$$

where $\bar{N} := [N_1E \quad N_2E]$. Applying Lemma 4 with $a := E\dot{x}(s), b := \xi(t)$, and $Z = Y^{\mathrm{T}}X^{-1}Y$ yields

$$\begin{aligned} &-2\int_{t-h}^{t} \xi^{\mathrm{T}}(t)\bar{N}^{\mathrm{T}}E\dot{x}(s)\mathrm{d}s \\ &\leq \int_{t-h}^{t} \begin{bmatrix} E\dot{x}(s) \\ \xi(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y-\bar{N} \\ * & Y^{\mathrm{T}}X^{-1}Y \end{bmatrix} \begin{bmatrix} E\dot{x}(s) \\ \xi(t) \end{bmatrix} \mathrm{d}s \\ &= \int_{t-h}^{t} \dot{x}^{\mathrm{T}}E^{\mathrm{T}}(s)XE\dot{x}(s)\mathrm{d}s + h\xi^{\mathrm{T}}(t)Y^{\mathrm{T}}X^{-1}Y\xi(t) \\ &\quad + 2\xi^{\mathrm{T}}(t)\left(Y^{\mathrm{T}} - \bar{N}^{\mathrm{T}}\right)[E \;\; -E]\xi(t). \end{aligned} \tag{9}$$

Substituting (9) into (8) gives us

$$-\int_{t-h}^{t} \dot{x}^{\mathrm{T}}E^{\mathrm{T}}(s)XE\dot{x}(s)\mathrm{d}s \leq h\xi^{\mathrm{T}}(t)Y^{\mathrm{T}}X^{-1}Y\xi(t) + 2\xi^{\mathrm{T}}(t)Y^{\mathrm{T}}\left[E \;\; -E\right]\xi(t). \tag{10}$$

After a simple rearrangement, (10) yields (6). This completes the proof.

Remark 1: (6) is called a descriptor integral-inequality. It plays a key role in deriving of a criterion for delay-dependent stabilization in this paper. When $E = I$, the descriptor integral inequality (6) is equivalent to the integral inequality in [30]. Therefore Lemma 5 is an extension of Proposition 3 in [30].

3. MAIN RESULT

This section addresses the delay-dependent stabilization sufficient conditions obtained by means of the descriptor integral-inequality method. The following theorem is obtained for system (5).

Theorem 2: For given scalars $h > 0$ and $h' > 0$, if there exist symmetric and positive definite matrices $X = X^{\mathrm{T}} > 0$, $Y_j = Y_j^{\mathrm{T}} > 0\left(j = 1,\cdots,q\right)$, $Y_j' = Y_j'^{\mathrm{T}} > 0\left(j = 1,\cdots,q'\right)$ $R = R^{\mathrm{T}} > 0$, $R' = R'^{\mathrm{T}} > 0$, and any matrices $M_{1j}, M_{2j}\left(j = 1,\cdots,q\right)$ and $M_{1j}', M_{2j}'(j = 1, \cdots, q')$ such that

$$\operatorname{rank}\left[E, \mathrm{T}_0\right] = n, \tag{11}$$

$$EX^{\mathrm{T}} = XE^{\mathrm{T}} \geq 0, \tag{12}$$

$$\Psi = \begin{bmatrix} \Xi & qh\mathrm{T}_0^{\mathrm{T}} & q'h'\mathrm{T}_0^{\mathrm{T}} & h\mathrm{H}_1^{\mathrm{T}} & \cdots & h\mathrm{H}_q^{\mathrm{T}} & h'\mathrm{H}_1'^{\mathrm{T}} & \cdots & h'\mathrm{H}_{q'}'^{\mathrm{T}} \\ * & -qhR^{-1} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & -q'h'R'^{-1} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & * & -hR & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & \cdots & -hR & 0 & \cdots & 0 \\ * & * & * & * & \cdots & * & -h'R' & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & \cdots & * & * & * & -h'R' \end{bmatrix} < 0, \tag{13}$$

where

$$\Xi = \begin{bmatrix} (1,1) & \Pi_1 & \cdots & \Pi_q & \Pi_1' & \cdots & \Pi_{q'}' \\ * & -\amalg_1 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & -\amalg_q & 0 & \cdots & 0 \\ * & * & * & * & -\amalg_1' & \cdots & 0 \\ * & * & * & * & * & \ddots & \vdots \\ * & * & * & * & * & * & -\amalg_{q'}' \end{bmatrix},$$

$$(1,1) = X\left(A_0+\bar{B}_0\right)+\left(A_0+\bar{B}_0\right)^{\mathrm{T}} X + \sum_{j=1}^{q} Y_j + \sum_{j=1}^{q'} Y_j' + \left(\sum_{j=1}^{q} M_{1j}^{\mathrm{T}} + \sum_{j=1}^{q'} M_{1j}'^{\mathrm{T}}\right) E + E^{\mathrm{T}}\left(\sum_{j=1}^{q} M_{1j} + \sum_{j=1}^{q'} M_{1j}'\right),$$

$$\Pi_j = XA_j - M_{1j}^{\mathrm{T}}E + E^{\mathrm{T}}M_{2j},$$

$$\amalg_j = Y_j + M_{2j}^{\mathrm{T}}E + E^{\mathrm{T}}M_{2j}, \quad j=1,\cdots,q,$$

$$\Pi_j' = X\bar{B}_j - M_{1j}'^{\mathrm{T}}E + E^{\mathrm{T}}M_{2j}',$$

$$\amalg_j' = Y_j' + M_{2j}'^{\mathrm{T}}E + E^{\mathrm{T}}M_{2j}', \quad j=1,\cdots,q',$$

$$\mathrm{T}_0 = \left[A_0+\bar{B}_0 \quad A_1 \quad \cdots \quad A_q \quad \bar{B}_1 \quad \cdots \quad \bar{B}_{q'}\right],$$

$$\mathrm{H}_j = \left[M_{1j} \quad L \quad \underbrace{0 \quad \cdots \quad 0}_{q'}\right],$$

$$L = \left[r_{1j} \quad \cdots \quad r_{ij} \quad \cdots \quad r_{qj}\right],$$

$$\mathrm{H}_j' = \left[M_{1j}' \quad \underbrace{0 \quad \cdots \quad 0}_{q} \quad L'\right],$$

$$L' = \left[r_{1j}' \quad \cdots \quad r_{ij}' \quad \cdots \quad r_{q'j}'\right],$$

$$r_{ij} = \begin{cases} M_{2j}, j=i \\ 0, \ j \neq i \end{cases}, \ i,j=1,\cdots,q,$$

$$r_{ij}' = \begin{cases} M_{2j}', j=i \\ 0, \ j \neq i \end{cases}, \ i,j=1,\cdots,q',$$

then the closed-loop system (5) is regular independent of the delays and impulse free and asymptotically stable.

Proof: Suppose (11)-(13) hold for $X=X^{\mathrm{T}}>0$, $Y_j=Y_j^{\mathrm{T}}>0$, $Y_j'=Y_j'^{\mathrm{T}}>0$, $R=R^{\mathrm{T}}>0$, $R'=R'^{\mathrm{T}}>0$, M_{1j}, M_{2j}, M_{1j}', M_{2j}' then from (13) it is easy to see that

$$X\left(A_0+\bar{B}_0\right)+\left(A_0+\bar{B}_0\right)^{\mathrm{T}} X<0. \tag{14}$$

By Theorem 1, Lemma 1 and Lemma 2, it follows from (11), (12) and (14) that system (5) is regular independent of the delays and the pair $\left(E, A_0+\bar{B}_0\right)$ is regular and impulse free.

Next, we shall examine the stability of system (5). To this end, we choose a Lyapunov-Krasovskii functional candidate as

$$V(t) = V_1(t) + V_2(t),$$

with

$$V_1(t) = x^{\mathrm{T}}(t)XEx(t) + \sum_{j=1}^{q}\int_{t-h_j}^{t} x^{\mathrm{T}}(s)Y_jx(s)\mathrm{d}s + \sum_{j=1}^{q'}\int_{t-h_j'}^{t} x^{\mathrm{T}}(s)Y_j'x(s)\mathrm{d}s,$$

$$V_2(t) = \sum_{j=1}^{q}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}RE\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \sum_{j=1}^{q'}\int_{-h'}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}R'E\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,$$

where $X=X^{\mathrm{T}}>0$, $Y_j=Y_j^{\mathrm{T}}>0$, $Y_j'=Y_j'^{\mathrm{T}}>0$, $R=R^{\mathrm{T}}>0$ and $R'=R'^{\mathrm{T}}>0$ are to be determined.

Then, the time derivative of $V(t)$ along the trajectory (5) satisfies

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t),$$

where

$$\begin{aligned}\dot{V}_1(t) &= 2x^{\mathrm{T}}(t)XE\dot{x}(t) + x^{\mathrm{T}}(t)\left(\sum_{j=1}^{q}Y_j + \sum_{j=1}^{q'}Y_j'\right)x(t) \\ &\quad - \sum_{j=1}^{q}x^{\mathrm{T}}\left(t-h_j\right)Y_jx\left(t-h_j\right) \\ &\quad - \sum_{j=1}^{q'}x^{\mathrm{T}}\left(t-h_j'\right)Y_j'x\left(t-h_j'\right) \\ &= \eta^{\mathrm{T}}(t)\Xi_0\eta(t),\end{aligned} \tag{15}$$

$$\begin{aligned}\dot{V}_2(t) &= qh\dot{x}^{\mathrm{T}}(t)E^{\mathrm{T}}RE\dot{x}(t) + q'h'\dot{x}^{\mathrm{T}}(t)E^{\mathrm{T}}R'E\dot{x}(t) \\ &\quad - \sum_{j=1}^{q}\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}RE\dot{x}(s)\mathrm{d}s \\ &\quad - \sum_{j=1}^{q'}\int_{t-h'}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}R'E\dot{x}(s)\mathrm{d}s \\ &= \eta^{\mathrm{T}}(t)\left(\mathrm{T}_0^{\mathrm{T}}qhR\mathrm{T}_0 + \mathrm{T}_0^{\mathrm{T}}q'h'R'\mathrm{T}_0\right)\eta(t) \\ &\quad - \sum_{j=1}^{q}\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}RE\dot{x}(s)\mathrm{d}s \\ &\quad - \sum_{j=1}^{q'}\int_{t-h'}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}R'E\dot{x}(s)\mathrm{d}s,\end{aligned} \tag{16}$$

with

$$\eta(t) = \left[x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-h_1) \quad \cdots \quad x^{\mathrm{T}}\left(t-h_q\right) \quad x^{\mathrm{T}}(t-h_1') \quad \cdots \quad x^{\mathrm{T}}\left(t-h_q'\right)\right],$$

$$\Xi_0 = \begin{bmatrix} (1,1)_0 & XA_1 & \cdots & XA_q & X\bar{B}_1 & \cdots & X\bar{B}_{q'} \\ * & -Y_1 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & -Y_q & 0 & \cdots & 0 \\ * & * & * & * & -Y_1' & \cdots & 0 \\ * & * & * & * & * & \ddots & \vdots \\ * & * & * & * & * & * & -Y_{q'}' \end{bmatrix},$$

$$(1,1)_0 = X\left(A_0+\bar{B}_0\right)+\left(A_0+\bar{B}_0\right)^{\mathrm{T}} X+\sum_{j=1}^{q} Y_j+\sum_{j=1}^{q'} Y_j',$$

$$\mathrm{T}_0=[A_0+\bar{B}_0 \quad A_1 \quad \cdots \quad A_q \quad \bar{B}_1 \quad \cdots \quad \bar{B}_{q'}].$$

It is clear that the following is true

$$\begin{aligned} &-\sum_{j=1}^{q}\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}RE\dot{x}(s)\mathrm{d}s \\ &\quad -\sum_{j=1}^{q'}\int_{t-h'}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}R'E\dot{x}(s)\mathrm{d}s \\ &\le -\sum_{j=1}^{q}\int_{t-h_j}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}RE\dot{x}(s)\mathrm{d}s \\ &\quad -\sum_{j=1}^{q'}\int_{t-h_j'}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}R'E\dot{x}(s)\mathrm{d}s. \end{aligned} \tag{17}$$

Applying Lemma 5 to the term on the right-hand side of (17) for any $M_{1j}, M_{2j}, M_{1j}', M_{2j}' \in \mathcal{R}^{n\times n}$, yields

$$\begin{aligned} &-\sum_{j=1}^{q}\int_{t-h_j}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}RE\dot{x}(s)\mathrm{d}s \\ &\quad -\sum_{j=1}^{q'}\int_{t-h_j'}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}R'E\dot{x}(s)\mathrm{d}s \\ &\le \sum_{j=1}^{q}\xi_j^{\mathrm{T}}(t)\Upsilon_j\xi_j(t)+h\xi_j^{\mathrm{T}}(t)P_j^{\mathrm{T}}R^{-1}P_j\xi_j(t) \\ &\quad +\sum_{j=1}^{q'}\xi_j'^{\mathrm{T}}(t)\Upsilon_j'\xi_j'(t)+h'\xi_j'^{\mathrm{T}}(t)P_j'^{\mathrm{T}}R'^{-1}P_j'\xi_j'(t) \\ &=\eta^{\mathrm{T}}(t)\left(\Xi_1+\sum_{j=1}^{q}\mathrm{H}_j^{\mathrm{T}}hR^{-1}\mathrm{H}_j+\sum_{j=1}^{q'}\mathrm{H}_j'^{\mathrm{T}}h'R'^{-1}\mathrm{H}_j'\right)\eta(t), \end{aligned} \tag{18}$$

where

$$\Upsilon_j := \begin{bmatrix} M_{1j}^{\mathrm{T}}E+E^{\mathrm{T}}M_{1j} & -M_{1j}^{\mathrm{T}}E+E^{\mathrm{T}}M_{2j} \\ * & -M_{2j}^{\mathrm{T}}E-E^{\mathrm{T}}M_{2j} \end{bmatrix},$$

$$\Upsilon_j' := \begin{bmatrix} M_{1j}'^{\mathrm{T}}E+E^{\mathrm{T}}M_{1j}' & -M_{1j}'^{\mathrm{T}}E+E^{\mathrm{T}}M_{2j}' \\ * & -M_{2j}'^{\mathrm{T}}E-E^{\mathrm{T}}M_{2j}' \end{bmatrix},$$

$$P_j := \begin{bmatrix} M_{1j} & M_{2j} \end{bmatrix},\ \xi_j(t)=\begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-h_j) \end{bmatrix}^{\mathrm{T}},$$

$$P_j' := \begin{bmatrix} M_{1j}' & M_{2j}' \end{bmatrix},\ \xi_j'(t)=\begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-h_j') \end{bmatrix}^{\mathrm{T}},$$

$$\Xi_1 = \begin{bmatrix} (1,1)_1 & \mathrm{T}_1 & \cdots & \mathrm{T}_q & \mathrm{T}_1' & \cdots & \mathrm{T}_{q'}' \\ * & \Gamma_1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \Gamma_q & 0 & \cdots & 0 \\ * & * & \cdots & * & \Gamma_1' & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & * & * & \cdots & \Gamma_{q'}' \end{bmatrix},$$

$$(1,1)_1 = \left(\sum_{j=1}^{q}M_{1j}^{\mathrm{T}}+\sum_{j=1}^{q'}M_{1j}'^{\mathrm{T}}\right)E+E^{\mathrm{T}}\left(\sum_{j=1}^{q}M_{1j}+\sum_{j=1}^{q'}M_{1j}'\right),$$

$$\mathrm{T}_j=-M_{1j}^{\mathrm{T}}E+E^{\mathrm{T}}M_{2j},$$

$$\Gamma_j=-M_{2j}^{\mathrm{T}}E-E^{\mathrm{T}}M_{2j},\quad j=1,\cdots,q,$$

$$\mathrm{T}_j'=-M_{1j}'^{\mathrm{T}}E+E^{\mathrm{T}}M_{2j}',$$

$$\Gamma_j'=-M_{2j}'^{\mathrm{T}}E-E^{\mathrm{T}}M_{2j}',\quad j=1,\cdots,q'.$$

Substituting (17) and (18) into (16) gives us

$$\begin{aligned} \dot{V}_2(t)\le \eta^{\mathrm{T}}(t)\Bigg(&\Xi_1+\mathrm{T}_0^{\mathrm{T}}qhR\mathrm{T}_0+\mathrm{T}_0^{\mathrm{T}}q'h'R'\mathrm{T}_0 \\ &+\sum_{j=1}^{q}\mathrm{H}_j^{\mathrm{T}}hR^{-1}\mathrm{H}_j+\sum_{j=1}^{q'}\mathrm{H}_j'^{\mathrm{T}}h'R'^{-1}\mathrm{H}_j'\Bigg)\eta(t). \end{aligned} \tag{19}$$

Combining (15)-(19) yields

$$\begin{aligned} V(t)\le \eta^{\mathrm{T}}(t)\Bigg(&\Xi+\mathrm{T}_0^{\mathrm{T}}qhR\mathrm{T}_0+\mathrm{T}_0^{\mathrm{T}}q'h'R'\mathrm{T}_0 \\ &+\sum_{j=1}^{q}\mathrm{H}_j^{\mathrm{T}}hR^{-1}\mathrm{H}_j+\sum_{j=1}^{q'}\mathrm{H}_j'^{\mathrm{T}}h'R'^{-1}\mathrm{H}_j'\Bigg)\eta(t), \end{aligned} \tag{20}$$

where $\Xi=\Xi_0+\Xi_1$.

From (20), we find that, if (13) holds, then applying the Schur complement [31] yields $\dot{V}(t)<0$. Thus, according to the Lyapunov-Krasovskii functional theorem [32], we can conclude that (5) is asymptotically stable. This completes the proof of Theorem 2.

As $K_j (j=0,1,\cdots,q')$ are design matrixes, Ψ is nonlinear in the design parameters K_j and X, the nonlinearities also come from R and R^{-1}, R' and R'^{-1}. Thus in this case (13) can not be solved directly by LMI toolbox. In order to obtain a controller gain, K_j, from the nonlinear matrix inequality (13), we give the following theorem.

Theorem 3: For given numbers $h>0$, $h'>0, \lambda_j$, $\mu_j \neq 0 (j=1,\cdots,q)$ and λ'_j, $\mu'_j \neq 0 (j=1,\cdots,q')$, if there exist symmetric and positive matrices $\bar{Y}_j$ $(j=1,\cdots,q)$, $\bar{Y}'_j (j=0,1,\cdots,q')$, $\bar{R}$, $\bar{R}'$ and any matrices $\bar{K}_j (j=0,1,\cdots,q')$ such that

$$\operatorname{rank}[E, A_0\bar{Y}'_0 + B_0\bar{K}_0, A_1,\cdots,A_q, B_1\bar{K}_1, \cdots, B_{q'}\bar{K}_{q'}] = n, \tag{21}$$

$$\bar{Y}'_0 E^{\mathrm{T}} = E\bar{Y}'_0 \geq 0 \tag{22}$$

$$\begin{bmatrix}
\Omega_1 & qh\Omega_2^{\mathrm{T}} & q'h'\Omega_2^{\mathrm{T}} & h\mathrm{N}_1^{\mathrm{T}} & \cdots & h\mathrm{N}_q^{\mathrm{T}} & h'\mathrm{N}_1'^{\mathrm{T}} & \cdots & h'\mathrm{N}_{q'}'^{\mathrm{T}} & \bar{Y}'_0 & \cdots & \bar{Y}'_0 & \bar{Y}'_0 & \cdots & \bar{Y}'_0 \\
* & -qh\bar{R} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
* & * & -q'h'\bar{R}' & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
* & * & * & -h\bar{R} & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
* & * & * & * & \cdots & -h\bar{R} & * & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
* & * & * & * & \cdots & * & -h'\bar{R}' & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
* & * & * & * & \cdots & * & * & \cdots & -h'\bar{R}' & 0 & \cdots & 0 & 0 & \cdots & 0 \\
* & * & * & * & \cdots & * & * & \cdots & * & -\bar{Y}_1 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
* & * & * & * & \cdots & * & * & \cdots & * & * & \cdots & -\bar{Y}_q & 0 & \cdots & 0 \\
* & * & * & * & \cdots & * & * & \cdots & * & * & \cdots & * & -\bar{Y}'_1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & * & * & \ddots & \vdots \\
* & * & * & * & \cdots & * & * & \cdots & * & * & \cdots & * & * & \cdots & -\bar{Y}'_{q'}
\end{bmatrix} < 0, \tag{23}$$

where

$$\Omega_1 = \begin{bmatrix}
\Omega_{11} & \Sigma_{11} & \cdots & \Sigma_{1q} & \Sigma'_{11} & \cdots & \Sigma'_{1q'} \\
* & -\Sigma_{21} & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
* & * & \cdots & -\Sigma_{2q} & 0 & \cdots & 0 \\
* & * & \cdots & * & -\Sigma'_{21} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
* & * & \cdots & * & * & \cdots & -\Sigma'_{2q'}
\end{bmatrix},$$

$$\Sigma_{1j} = \mu_j^{-1}A_j\bar{Y}_j + \left(\bar{Y}'_0 + \lambda_j\mu_j^{-1}\bar{Y}_j\right)E^{\mathrm{T}} + \lambda_j\mu_j^{-2}\bar{Y}_j, \Sigma_{2j} = \mu_j^{-1}\left(E\bar{Y}_j + \bar{Y}_jE^{\mathrm{T}}\right) + \mu_j^{-2}\bar{Y}_j, j=1,\cdots,q,$$

$$\Sigma'_{1j} = \mu_j'^{-1}B_j\bar{K}_j + \left(\bar{Y}'_0 + \lambda'_j\mu_j'^{-1}\bar{Y}'_j\right)E^{\mathrm{T}} + \lambda'_j\mu_j'^{-2}\bar{Y}'_j, \Sigma'_{2j} = \mu_j'^{-1}\left(E\bar{Y}'_j + \bar{Y}'_jE^{\mathrm{T}}\right) + \mu_j'^{-2}\bar{Y}'_j, j=1,\cdots,q',$$

$$\Omega_{11} = \left(A_0\bar{Y}'_0 + B_0\bar{K}_0\right) - \sum_{j=1}^{q}\lambda_j\mu_j^{-1}A_j\bar{Y}_j - \sum_{j=1}^{q'}\lambda'_j\mu_j'^{-1}B_j\bar{K}_j + \left(\left(A_0\bar{Y}'_0 + B_0\bar{K}_0\right) - \sum_{j=1}^{q}\lambda_j\mu_j^{-1}A_j\bar{Y}_j - \sum_{j=1}^{q'}\lambda'_j\mu_j'^{-1}B_j\bar{K}_j\right)^{\mathrm{T}} - \sum_{j=1}^{q}\lambda_j\mu_j^{-1}\bar{Y}_j - \sum_{j=1}^{q'}\lambda'_j\mu_j'^{-1}\bar{Y}'_j,$$

$$\Omega_2 = \left[\Omega_{21} \quad \mu_1^{-1}A_1\bar{Y}_1 \ \cdots \ \mu_q^{-1}A_q\bar{Y}_q \quad \mu_1'^{-1}B_1\bar{K}_1 \ \cdots \ \mu_{q'}'^{-1}B_{q'}\bar{K}_{q'}\right],$$

$$\Omega_{21} = \left(A_0\bar{Y}'_0 + B_0\bar{K}_0\right) - \sum_{j=1}^{q}\lambda_j\mu_j^{-1}A_j\bar{Y}_j - \sum_{j=1}^{q'}\lambda'_j\mu_j'^{-1}B_j\bar{K}_j,$$

$$\mathrm{N}_j = \left[0 \quad \bar{L} \quad \underbrace{0 \ \cdots \ 0}_{q'}\right], \bar{L} = \left[\bar{r}_{1j} \ \cdots \ \bar{r}_{ij} \ \cdots \ \bar{r}_{qj}\right],$$

$$\mathrm{N}'_j = \left[0 \quad \underbrace{0 \ \cdots \ 0}_{q} \quad \bar{L}'\right], \bar{L}' = \left[\bar{r}'_{1j} \ \cdots \ \bar{r}'_{ij} \ \cdots \ \bar{r}'_{q'j}\right],$$

$$\bar{r}_{ij} = \begin{cases}\bar{R}, j=i \\ 0, \ j\neq i,\end{cases} i,j=1,\cdots,q, \quad \bar{r}'_{ij} = \begin{cases}\bar{R}', j=i \\ 0, \ j\neq i,\end{cases} i,j=1,\cdots,q',$$

then the memoryless state feedback controller (4) guarantees that the closed-loop system (5) is regular independent of the delays, impulse free as well as asymptotically stable. The controller gains K_j can be obtained by solving (21)-(23) and $K_j = \bar{K}_j\bar{Y}_j'^{-1}$ $j=0,1,\cdots,q'$.

Proof: To cast the problem of designing a stabilising controller (4) into the LMI formulation, we let $\bar{Y}_0' = X^{-1}$, $\bar{Y}_j = Y_j^{-1}(j=1,\cdots,q)$, $\bar{Y}_j' = Y_j'^{-1}(j=1,\cdots,q')$, $\bar{K}_j = K_j\bar{Y}_j'(j=0,1,\cdots,q')$, $\bar{R}=R^{-1}$, $\bar{R}' = R'^{-1}$. Define a full rank matrix $P = \mathrm{diag}\left\{I \;\; \bar{Y}_0' \;\; \underbrace{I \;\cdots\; I}_{q} \;\; \bar{Y}' \;\cdots\; \bar{Y}_{q'}'\right\}$, then $\left[E,\ \mathrm{T}_0\right]P = \left[E,\ A_0\bar{Y}_0' + B_0\bar{K}_0,\ A_1,\cdots,A_q,\ B_1\bar{K}_1,\cdots,B_{q'}\bar{K}_{q'}\right]$. From Lemma 3, we can conclude that $\mathrm{rank}\left(\left[E,\ \mathrm{T}_0\right]P\right) = \mathrm{rank}\left(\left[E,\ \mathrm{T}_0\right]\right) = n.$ That is to say (21) is equivalent to (11). By pre- and post-multiplying (12) by $\bar{Y}_0'$, we can conclude that (22) is hold. Due to $EX^{\mathrm{T}} = XE^{\mathrm{T}} \ge 0$ and define the following matrices

$$W = \begin{bmatrix} X & 0 \\ W_1 & W_2 \end{bmatrix},\quad \bar{A} = \begin{bmatrix} A_0 + \bar{B}_0 & \bar{A}_{11} \\ \bar{A}_{12} & \bar{A}_{13} \end{bmatrix},$$

$$\mathrm{U} = \mathrm{diag}\left\{\sum_{j=1}^{q} Y_j + \sum_{j=1}^{q'} Y_j', -Y_1,\cdots,-Y_q,-Y_1',\cdots,-Y_{q'}'\right\},$$

$$W_1 = \left[M_{11}^{\mathrm{T}} \;\cdots\; M_{1q}^{\mathrm{T}} \;\; M_{11}'^{\mathrm{T}} \;\cdots\; M_{1q'}'^{\mathrm{T}}\right]^{\mathrm{T}},$$

$$W_2 = \mathrm{diag}\left\{M_{21},\cdots,M_{2q},M_{21}',\cdots,M_{2q'}'\right\},$$

$$\bar{A}_{11} = \left[A_1 \;\cdots\; A_q \;\; \bar{B}_1 \;\cdots\; \bar{B}_{q'}\right],$$

$$\bar{A}_{12} = \left[\underbrace{E^{\mathrm{T}} \;\cdots\; E^{\mathrm{T}}}_{q} \;\; \underbrace{E^{\mathrm{T}} \;\cdots\; E^{\mathrm{T}}}_{q'}\right]^{\mathrm{T}},$$

$$\bar{A}_{13} = \mathrm{diag}\left\{\underbrace{-E \;\cdots\; -E}_{q} \;\; \underbrace{-E \;\cdots\; -E}_{q'}\right\},$$

then

$$\Xi = W^{\mathrm{T}}\bar{A} + \bar{A}^{\mathrm{T}}W + \mathrm{U},$$

$$\mathrm{H}_j = \left[0 \;\; V \;\; \underbrace{0 \;\cdots\; 0}_{q'}\right]W,\quad V = \left[v_{1j} \cdots v_{ij} \cdots v_{qj}\right],$$

$$\mathrm{H}_j' = \left[0 \;\; \underbrace{0 \;\cdots\; 0}_{q} \;\; V'\right]W,\quad V' = \left[v_{1j}' \cdots v_{ij}' \cdots v_{q'j}'\right],$$

$$v_{ij} = \begin{cases} I, & j=i \\ 0, & j\neq i,\end{cases} \; i,j=1,\cdots,q,\quad v_{ij}' = \begin{cases} I, & j=i \\ 0, & j\neq i,\end{cases} \; i,j=1,\cdots,q'.$$

Now, we consider the case where $M_{1j} = \lambda_j X$, $M_{2j} = \mu_j Y_j$, $M_{1j}' = \lambda_j' X$, $M_{2j}' = \mu_j' Y_j'$, $\mu_j \neq 0$, and $\mu_j' \neq 0$. In this condition, W is invertible, and

$$W^{-1} = \begin{bmatrix} \bar{X} & 0 \\ \bar{W}_1 & \bar{W}_2 \end{bmatrix},$$

$$\bar{W}_1 = \left[-\lambda_1\mu_1^{-1}\bar{Y}_1 \;\cdots\; -\lambda_q\mu_q^{-1}\bar{Y}_q \;\; -\lambda_1'\mu_1'^{-1}\bar{Y}_1' \;\cdots\; -\lambda_{q'}'\mu_{q'}'^{-1}\bar{Y}_{q'}'\right]^{\mathrm{T}},$$

$$\bar{W}_2 = \mathrm{diag}\left\{\mu_1^{-1}\bar{Y}_1,\cdots,\mu_q^{-1}\bar{Y}_q,\mu_1'^{-1}\bar{Y}_1',\cdots,\mu_{q'}'^{-1}\bar{Y}_{q'}'\right\}.$$

Define

$$T = \mathrm{diag}\left\{W^{-1},I,I,R^{-1},\cdots,R^{-1},R'^{-1},\cdots,R'^{-1}\right\},$$

then

$$T^{\mathrm{T}}\Psi T = \begin{bmatrix} (1,1)_2 & qh\Omega_2^{\mathrm{T}} & q'h'\Omega_2^{\mathrm{T}} & h\mathrm{N}_1^{\mathrm{T}} & \cdots & h\mathrm{N}_q^{\mathrm{T}} & h'\mathrm{N}_1'^{\mathrm{T}} & \cdots & h'\mathrm{N}_{q'}'^{\mathrm{T}} \\ * & -qh\bar{R} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & -q'h'\bar{R}' & 0 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & * & -h\bar{R} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & \cdots & -h\bar{R} & 0 & \cdots & 0 \\ * & * & * & * & \cdots & * & -h'\bar{R}' & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & \cdots & * & * & \cdots & -h'\bar{R}' \end{bmatrix} < 0, \quad (24)$$

where

$$(1,1)_2 = \begin{bmatrix} (1,1)_3 & \Sigma_{11} & \cdots & \Sigma_{1q} & \Sigma_{11}' & \cdots & \Sigma_{1q'}' \\ * & -\Sigma_{21} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & -\Sigma_{2q} & 0 & \cdots & 0 \\ * & * & \cdots & * & -\Sigma_{21}' & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & * & * & \cdots & -\Sigma_{2q'}' \end{bmatrix},$$

$$(1,1)_3 = \left(A_0\bar{Y}_0' + B_0\bar{K}_0\right) - \sum_{j=1}^{q}\lambda_j\mu_j^{-1}A_j\bar{Y}_j$$

$$-\sum_{j=1}^{q'}\lambda_j'\mu_j'^{-1}B_j\bar{K}_j+\Bigg(\left(A_0\bar{Y}_0'+B_0\bar{K}_0\right)$$
$$-\sum_{j=1}^{q}\lambda_j\mu_j^{-1}A_j\bar{Y}_j-\sum_{j=1}^{q'}\lambda_j'\mu_j'^{-1}B_j\bar{K}_j\Bigg)^{\mathrm{T}}$$
$$-\sum_{j=1}^{q}\lambda_j\mu_j^{-1}\bar{Y}_j-\sum_{j=1}^{q'}\lambda_j'\mu_j'^{-1}\bar{Y}_j'+\bar{Y}_0'\left(\sum_{j=1}^{q}Y_j+\sum_{j=1}^{q'}Y_j'\right)\bar{Y}_0',$$

and Σ_{1j}, Σ_{2j}, Σ_{1j}', Σ_{2j}', N_j, N_j', Ω_2 are defined in Theorem 3.

Therefore, (24) follows from (23). From this derivation, we conclude that, if there exist symmetric and positive matrices $\bar{Y}_j\left(j=1,\cdots,q\right)$, $\bar{Y}_j'(j=0,1,\cdots,q')$, $\bar{R}$, $\bar{R}'$ and any matrices $\bar{K}_j(j=0,1,\cdots,q')$ satisfying (21)-(23), then the following symmetric and positive matrices X, $Y_j\left(j=1,\cdots,q\right)$, $Y_j'(j=1,\cdots,q')$, R, R', any matrices M_{1j}, $M_{2j}\left(j=1,\cdots,q\right)$ and M_{1j}', $M_{2j}'\left(j=1,\cdots,q'\right)$ satisfy (11)-(13). So, the resulting closed-loop system (5) is regular independent of the delays, impulse free and asymptotically stable. The desired controller is defined by (4) with $K_j=\bar{K}_j\bar{Y}_j'^{-1}\left(j=0,1,\cdots,q'\right)$. This completes the proof of Theorem 3.

4. NUMERAL EXAMPLE

This section presents a numerical example that demonstrates the validity of the results described above. Consider a SLCS-MIEID system (1), with parameters as

$$E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\ A_0=\begin{bmatrix}0.2&0.1&-0.5\\0.25&-0.3&-0.2\\0&1&-0.8\end{bmatrix},$$
$$A_1=\begin{bmatrix}-0.4&0.2&-0.6\\0&-0.5&0\\0&0&-2\end{bmatrix},\ B_0=\begin{bmatrix}1\\0\\0\end{bmatrix},$$
$$B_1=\begin{bmatrix}0\\1\\0\end{bmatrix},\ B_2=\begin{bmatrix}0\\0\\1\end{bmatrix}.$$

In this example, we assume that the maximum point time delays are $h=2.46$ and $h'=0.4$. The purpose is to design a state feedback control law such that the resulting closed-loop system is regular independent of the delays, impulse free and asymptotically stable. To this end, we choose

$$\lambda_1=-1.8953,\ \lambda_1'=-1.4451,\ \lambda_2'=-0.3451,$$
$$\mu_1=14.7388,\ \mu_1'=0.3654,\ \ \mu_2'=31.3654.$$

Using Matlab LMI Control Toolbox to solve the feasible problems (22) and (23), we obtain the solutions described as

$$\bar{Y}_1=\begin{bmatrix}0.2780&0.0105&-0.0012\\0.0105&0.2547&0.0594\\-0.0012&0.0594&0.0176\end{bmatrix},$$
$$\bar{Y}_0'=\begin{bmatrix}212.3690&-0.3692&0\\-0.3692&0.6366&0\\0&0&1.5235\end{bmatrix},$$
$$\bar{Y}_1'=\begin{bmatrix}0.2466&-0.0001&0.0011\\-0.0001&0.1960&-0.0084\\0.0011&-0.0084&0.0309\end{bmatrix},$$
$$\bar{Y}_2'=\begin{bmatrix}0.1226&-0.0019&0.0002\\-0.0019&0.1290&-0.0059\\0.0002&-0.0059&0.1644\end{bmatrix},$$
$$\bar{R}=\begin{bmatrix}0.5044&-0.0019&-0.0012\\-0.0019&0.4762&-0.0012\\-0.0012&-0.0012&0.3984\end{bmatrix},$$
$$\bar{R}'=\begin{bmatrix}0.1483&-0.0006&0.0003\\-0.0006&0.1430&-0.0006\\0.0003&-0.0006&0.1142\end{bmatrix},$$
$$\bar{K}_0=\left[-10.1349\quad -15.0689\quad -5.0311\right],$$
$$\bar{K}_1=\left[-8.9865\quad -1.0730\quad -3.0083\right],$$
$$\bar{K}_2=\left[-7.0335\quad -4.6161\quad -9.0921\right].$$

Therefore, by Theorem 3, a stabilizing state feedback control law can be obtained as:

$$u_0\left(t\right)=\left[-35.3014\quad -70.9122\quad -77.1641\right]x\left(t\right),$$
$$u_1=\left[-36.0052\quad -9.7236\quad -98.7176\right]x\left(t\right),$$
$$u_2=\left[-57.8850\quad -39.2269\quad -56.6421\right]x\left(t\right).$$

It is easy to check that the conditions given by (11), (12), and (14) are satisfied. Hence, according to Theorem 1-Theorem3, the closed system (5) is regular independent of the delays and impulse free.

5. CONCLUSION

In this paper, the delay-dependent stabilization problem of SLCS-MIEID has been studied. The main contribution of this study is to obtain the control law, which can delay-dependent stabilises singular time-delay time-invariant systems. This is done by using the Lyapunov-Krasovskii functional approach combined with a descriptor integral-inequality. The numerical example shows that the proposed controller can achieve desired design purposes.

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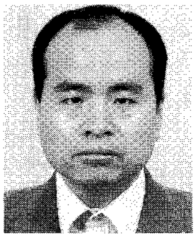

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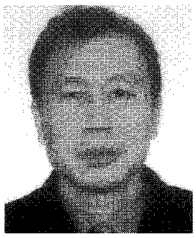

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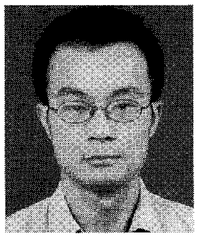

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