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COMMON FIXED POINT THEOREM FOR MULTIVALUED MAPPINGS IN INTUITIONISTIC FUZZY METRIC SPACE

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Abstract. The purpose of this paper is to prove common fixed point theorem for multivalued mappings satisfying some conditions in intuitionistic fuzzy metric space in the sense of Alaca et al.

1. Introduction

In 1965, the concept of fuzzy sets was introduced initially by Zadeh [27]. Since then, many authors have expansively developed the theory of fuzzy sets and applications. Especially, Deng [5], Erceg [7], Kaleva and Seikkala [13], Kramosil and Michalek [15] have introduced the concept of fuzzy metric spaces in different ways. Grabiec [10] followed Kramosil and Michalek [15] and obtained the fuzzy version of Banach contraction principle. Fang [8] proved some fixed point theorems in fuzzy metric spaces, which improve, generalize, unify and extend some main results of Banach [2], Edelstein [6], Mishra et al. [19] obtained common fixed point theorems for compatible maps on fuzzy metric spaces. Sharma [23] proved common fixed point theorems for six mappings satisfying some conditions in fuzzy metric spaces. Many authors have also studied the fixed point theory in these fuzzy metric spaces [3], [4], [9]. Fixed point for multivalued mappings in fuzzy metric space is studied by Kubiaczyk and Sharma [17]. Alaca et al. [1] using the idea of intuitionistic fuzzy sets, they defined the notion of intuitionistic fuzzy metric space as Park [20] with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [15]. Further, they introduced the notion of Cauchy sequences in an intuitionistic fuzzy metric space and proved the well-known fixed point theorems of Banach [2] and Edelstein [6], for intuitionistic fuzzy metric spaces with the help of Grabiec [10]. Turkoglu et al. [24] gave generalization of Jungck's common fixed point theorem [12] to intuitionistic fuzzy metric spaces. Turkoglu et al. [25] defined compatible mappings of type (α) and (β) in intuitionistic fuzzy metric spaces. Gregori et al. [11],

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Saadati and Park [21] studied the concept of intuitionistic fuzzy metric space and its applications.

In this paper we prove common fixed point theorem for multivalued mappings satisfying some conditions in intuitionistic fuzzy metric space in the sense of Alaca et al. [1].

2. Preliminaries

Definition 1 ([22])**.** A binary operation $*\colon [0,1]\times[0,1]\to[0,1]$ is *continuous t-norm* if $*$ is satisfying the following conditions:

(i) $*$ is commutative and associative,
(ii) $*$ is continuous,
(iii) $a * 1 = a$ for all $a \in [0,1]$,
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, $a, b, c, d \in [0,1]$.

Definition 2 ([22])**.** A binary operation $\diamond\colon [0,1]\times[0,1]\to[0,1]$ is *continuous t-conorm* if $\diamond$ is satisfying the following conditions:

(i) $\diamond$ is commutative and associative,
(ii) $\diamond$ is continuous,
(iii) $a \diamond 0 = a$ for all $a \in [0,1]$,
(iv) $a \diamond b \le c \diamond d$ whenever $a \le c$ and $b \le d$, $a, b, c, d \in [0,1]$.

Remark 3. The concept of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. These concepts were originally introduced by Menger [18] in his study of statistical metric spaces. Several examples for these concepts were proposed by many authors including [14],[26].

Definition 4 ([1])**.** A 5-tuple $(X, M, N, *, \diamond)$ is said to be an *intuitionistic fuzzy metric spaces* if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0,\infty)$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$,

(i) $M(x, y, t) + N(x, y, t) \le 1$,
(ii) $M(x, y, 0) = 0$,
(iii) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
(iv) $M(x, y, t) = M(y, x, t)$,
(v) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,
(vi) $M(x, y, .)\colon [0,\infty) \to [0,1]$ is left continuous,
(vii) $\lim_{t\to\infty} M(x, y, t) = 1$ for all x, y in X,
(viii) $N(x, y, 0) = 1$,
(ix) $N(x, y, t) = 0$ for all $t > 0$ if and only if $x = y$,
(x) $N(x, y, t) = N(y, x, t)$,
(xi) $N(x, y, t) \diamond N(y, z, s) \ge N(x, z, t + s)$,
(xii) $N(x, y, .)\colon [0,\infty) \to [0,1]$ is right continuous,
(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all x, y in X.

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 5. Every fuzzy metric space $(X,M,*)$ is an intuitionistic fuzzy metric space of the form $(X,M,1-M,*,\diamond)$ such that t-norm $*$ and t-conorm $\diamond$ are associated [16], i.e., $x\diamond y = 1-((1-x)*(1-y))$ for all $x,y\in X$.

Example 6. Let (X,d) be a metric space. Define t-norm $a*b=\min\{a,b\}$ and t-conorm $a\diamond b=\max\{a,b\}$ and for all $x,y\in X$ and $t>0$,

$$M_d(x,y,t)=\frac{t}{t+d(x,y)},\qquad N_d(x,y,t)=\frac{d(x,y)}{t+d(x,y)}$$

Then $(X,M,N,*,\diamond)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M,N) induced by the metric d the standard intuitionistic fuzzy metric.

Remark 7. In intuitionistic fuzzy metric space X, $M(x,y,.)$ is non-decreasing and $N(x,y,.)$ is non-increasing for all $x,y\in X$.

Definition 8 ([1])**.** Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space. Then

(i) A sequence $\{x_n\}$ in X is said to be *convergent* to a point $x\in X$ (denoted by $lim_{n\to\infty}x_n=x$) if, for all $t>0$,

$$\lim_{n\to\infty} M(x_n,x,t)=1,\qquad \lim_{n\to\infty} N(x_n,x,t)=0.$$

(ii) A sequence $\{x_n\}$ in X is said to be *Cauchy sequence* if, for all $t>0$ and $p>0$,

$$\lim_{n\to\infty} M(x_{n+p},x_n,t)=1,\qquad \lim_{n\to\infty} N(x_{n+p},x_n,t)=0.$$

Remark 9. Since $*$ and $\diamond$ are continuous, the limit is uniquely determined from (v) and (xi), respectively.

Definition 10 ([1])**.** An intuitionistic fuzzy metric space $(X,M,N,*,\diamond)$ is said to be *complete* if and only if every Cauchy sequence in X is convergent.

Lemma 11 ([1])**.** *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and $\{x_n\}$ be a sequence in X. If there exists a number $k\in(0,1)$ such that*

$$M(x_{n+2},x_{n+1},kt)\ge M(x_{n+1},x_n,t),\qquad N(x_{n+2},x_{n+1},kt)\le N(x_{n+1},x_n,t)$$

for all $t>0$ and $n=1,2,\ldots$, then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 12 ([1])**.** *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and for all $x,y\in X$, $t>0$ and if for a number $k\in(0,1)$,*

$$M(x,y,kt)\ge M(x,y,t)\quad \text{and}\quad N(x,y,kt)\le N(x,y,t)$$

then $x=y$.

Kubiaczyk and Sharma [17] introduced the following concept of multivalued mappings in the sence of Kramosil and Michalek [15]. We denote by $CB(X)$ the set of all non-empty, bounded and closed subset of X. We have

$$M^{\bigtriangledown}(B,y,t) = \max\{M(b,y,t) : b \in B\}$$
$$N^{\bigtriangleup}(B,y,t) = \min\{N(b,y,t) : b \in B\}$$
$$M_{\bigtriangledown}(A,B,t) \geq \min\{\min_{a\in A} M^{\bigtriangledown}(a,B,t), \min_{b\in B} M^{\bigtriangledown}(A,b,t)\}$$
$$N_{\bigtriangleup}(A,B,t) \leq \max\{max_{a\in A} N^{\bigtriangleup}(a,B,t), \max_{b\in B} N^{\bigtriangleup}(A,b,t)\}$$

for all A, B in X and $t > 0$.

Now we prove common fixed point theorem for multivalued mappings satisfying some conditions in intuitionistic fuzzy metric space in the sense of Alaca et al. [1].

3. Main Result

Theorem 13. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\diamond$ defined by $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$, for all $t \in [0,1]$. Let $F_1, F_2, F_3 : X \to CB(X)$ satisfying, (3.1) there exists a number $k \in (0,1)$ such that*

$$\begin{aligned} M_{\bigtriangledown}(F_1x, F_2y, kt) \geq \min\{&M(x,y,t), M^{\bigtriangledown}(x,F_1x,t), M^{\bigtriangledown}(y,F_2y,t),\\ &M^{\bigtriangledown}(x,F_2y,(2-\alpha)t), M^{\bigtriangledown}(y,F_1x,t),\\ &M^{\bigtriangledown}(x,F_3y,(2-\alpha)t), M^{\bigtriangledown}(y,F_3y,t)\}\end{aligned}$$

and

$$\begin{aligned} N_{\bigtriangleup}(F_1x, F_2y, kt) \leq \max\{&N(x,y,t), N^{\bigtriangleup}(x,F_1x,t), N^{\bigtriangleup}(y,F_2y,t),\\ &N^{\bigtriangleup}(x,F_2y,(2-\alpha)t), N^{\bigtriangleup}(y,F_1x,t),\\ &N^{\bigtriangleup}(x,F_3y,(2-\alpha)t), N^{\bigtriangleup}(y,F_3y,t)\}\end{aligned}$$

for all $x, y \in X$, $\alpha \in (0,2)$ and $t > 0$. Then F_1, F_2 and F_3 have a common fixed point .

Proof. Let x_0 be an arbitrary point in X and $x_1 \in X$ is such that $x_1 \in F_1x_0$ and

$$M(x_0,x_1,kt) \geq M^{\bigtriangledown}(x_0,F_1x_0,kt) - \epsilon,$$
$$N(x_0,x_1,kt) \leq N^{\bigtriangleup}(x_0,F_1x_0,kt) + \epsilon,$$

$x_2 \in X$ is such that $x_2 \in F_2x_1$ and

$$M(x_1,x_2,kt) \geq M^{\bigtriangledown}(x_1,F_2x_1,kt) - \frac{\epsilon}{2},$$
$$N(x_1,x_2,kt) \leq N^{\bigtriangleup}(x_1,F_2x_1,kt) + \frac{\epsilon}{2},$$

$x_3 \in X$ is such that $x_3 \in F_3 x_2$ and

$$M(x_2, x_3, kt) \geq M^{\bigtriangledown}(x_2, F_3 x_2, kt) - \frac{\epsilon}{2^2},$$
$$N(x_2, x_3, kt) \leq N^{\bigtriangleup}(x_2, F_3 x_2, kt) + \frac{\epsilon}{2^2}.$$

Inductively $x_{2n+1} \in X$ is such that $x_{2n+1} \in F_1 x_{2n}$ and

$$M(x_{2n}, x_{2n+1}, kt) \geq M^{\bigtriangledown}(x_{2n}, F_1 x_{2n}, kt) - \frac{\epsilon}{2^{2n}},$$
$$N(x_{2n}, x_{2n+1}, kt) \leq N^{\bigtriangleup}(x_{2n}, F_1 x_{2n}, kt) + \frac{\epsilon}{2^{2n}},$$

$x_{2n+2} \in X$ is such that $x_{2n+2} \in F_2 x_{2n+1}$ and

$$M(x_{2n+1}, x_{2n+2}, kt) \geq M^{\bigtriangledown}(x_{2n+1}, F_2 x_{2n+1}, kt) - \frac{\epsilon}{2^{2n+1}},$$
$$N(x_{2n+1}, x_{2n+2}, kt) \leq N^{\bigtriangleup}(x_{2n+1}, F_2 x_{2n+1}, kt) + \frac{\epsilon}{2^{2n+1}},$$

$x_{2n+3} \in X$ is such that $x_{2n+3} \in F_3 x_{2n+2}$ and

$$M(x_{2n+2}, x_{2n+3}, kt) \geq M^{\bigtriangledown}(x_{2n+2}, F_3 x_{2n+2}, kt) - \frac{\epsilon}{2^{2n+2}},$$
$$N(x_{2n+2}, x_{2n+3}, kt) \leq N^{\bigtriangleup}(x_{2n+2}, F_3 x_{2n+2}, kt) + \frac{\epsilon}{2^{2n+2}},$$

Now we show that $\{x_n\}$ is a Cauchy sequence.

By (3.1), for all $t > 0$ and $\alpha = 1 - q$ with $q \in (0, 1)$, we have

$$\begin{aligned}
&M(x_{2n+1}, x_{2n+2}, kt)\\
&\quad \geq M^{\bigtriangledown}(x_{2n+1}, F_2 x_{2n+1}, kt) - \frac{\epsilon}{2^{2n+1}}\\
&\quad \geq M_{\bigtriangledown}(F_1 x_{2n}, F_2 x_{2n+1}, kt) - \frac{\epsilon}{2^{2n+1}}\\
&\quad \geq \min\{M(x_{2n}, x_{2n+1}, t), M^{\bigtriangledown}(x_{2n}, F_1 x_{2n}, t), M^{\bigtriangledown}(x_{2n+1}, F_2 x_{2n+1}, t),\\
&\qquad M^{\bigtriangledown}(x_{2n}, F_2 x_{2n+1}, (2-\alpha)t), M^{\bigtriangledown}(x_{2n+1}, F_1 x_{2n}, t),\\
&\qquad M^{\bigtriangledown}(x_{2n}, F_3 x_{2n+1}, (2-\alpha)t), M^{\bigtriangledown}(x_{2n+1}, F_3 x_{2n+1}, t)\} - \frac{\epsilon}{2^{2n+1}}\\
&\quad \geq \min\{M(x_{2n}, x_{2n+1}, t), M(x_{2n}, x_{2n+1}, t), M(x_{2n+1}, x_{2n+2}, t),\\
&\qquad M(x_{2n}, x_{2n+2}, (1+q)t), M(x_{2n+1}, x_{2n+1}, t),\\
&\qquad M(x_{2n}, x_{2n+2}, (1+q)t), M(x_{2n+1}, x_{2n+2}, t)\} - \frac{\epsilon}{2^{2n+1}}\\
&\quad \geq \min\{M(x_{2n}, x_{2n+1}, t), M(x_{2n}, x_{2n+1}, t), M(x_{2n+1}, x_{2n+2}, t),\\
&\qquad M(x_{2n}, x_{2n+1}, t), M(x_{2n+1}, x_{2n+2}, qt), 1, M(x_{2n}, x_{2n+1}, t),\\
&\qquad M(x_{2n+1}, x_{2n+2}, qt), M(x_{2n+1}, x_{2n+2}, t)\} - \frac{\epsilon}{2^{2n+1}}\\
&\quad \geq \min\{M(x_{2n}, x_{2n+1}, t), M(x_{2n+1}, x_{2n+2}, t), M(x_{2n+1}, x_{2n+2}, qt)\}\\
&\qquad - \frac{\epsilon}{2^{2n+1}}
\end{aligned} \tag{3.2}$$

and

$$
\begin{aligned}
&N(x_{2n+1}, x_{2n+2}, kt)\\
&\quad \leq N^{\triangle}(x_{2n+1}, F_2x_{2n+1}, kt) + \frac{\epsilon}{2^{2n+1}}\\
&\quad \leq N_{\triangle}(F_1x_{2n}, F_2x_{2n+1}, kt) + \frac{\epsilon}{2^{2n+1}}\\
&\quad \leq \max\{N(x_{2n}, x_{2n+1}, t), N^{\triangle}(x_{2n}, F_1x_{2n}, t), N^{\triangle}(x_{2n+1}, F_2x_{2n+1}, t),\\
&\qquad N^{\triangle}(x_{2n}, F_2x_{2n+1}, (2-\alpha)t), N^{\triangle}(x_{2n+1}, F_1x_{2n}, t),\\
&\qquad N^{\triangle}(x_{2n}, F_3x_{2n+1}, (2-\alpha)t), N^{\triangle}(x_{2n+1}, F_3x_{2n+1}, t)\} + \frac{\epsilon}{2^{2n+1}}\\
&\quad \leq \max\{N(x_{2n}, x_{2n+1}, t), N(x_{2n}, x_{2n+1}, t), N(x_{2n+1}, x_{2n+2}, t),\\
&\qquad N(x_{2n}, x_{2n+2}, (1+q)t), N(x_{2n+1}, x_{2n+1}, t),\\
&\qquad N(x_{2n}, x_{2n+2}, (1+q)t), N(x_{2n+1}, x_{2n+2}, t)\} + \frac{\epsilon}{2^{2n+1}}\\
&\quad \leq \max\{N(x_{2n}, x_{2n+1}, t), N(x_{2n}, x_{2n+1}, t), N(x_{2n+1}, x_{2n+2}, t),\\
&\qquad N(x_{2n}, x_{2n+1}, t), N(x_{2n+1}, x_{2n+2}, qt), 0, N(x_{2n}, x_{2n+1}, t),\\
&\qquad N(x_{2n+1}, x_{2n+2}, qt), N(x_{2n+1}, x_{2n+2}, t)\} + \frac{\epsilon}{2^{2n+1}}\\
&\quad \leq \max\{N(x_{2n}, x_{2n+1}, t), N(x_{2n+1}, x_{2n+2}, t), N(x_{2n+1}, x_{2n+2}, qt)\}\\
&\qquad + \frac{\epsilon}{2^{2n+1}}
\end{aligned} \tag{3.3}
$$

Since the t-norm $*$ and t-conorm $\diamond$ are continuous, $M(x, y, .)$ is left continuous and $N(x, y, .)$ is right continuous, letting $q \to 1$ in (3.2) and (3.3), we have

$$
\begin{aligned}
&M(x_{2n+1}, x_{2n+2}, kt)\\
&\qquad \geq \min\{M(x_{2n}, x_{2n+1}, t), M(x_{2n+1}, x_{2n+2}, t)\} - \frac{\epsilon}{2^{2n+1}}
\end{aligned} \tag{3.4}
$$

and

$$
\begin{aligned}
&N(x_{2n+1}, x_{2n+2}, kt)\\
&\qquad \leq \max\{N(x_{2n}, x_{2n+1}, t), N(x_{2n+1}, x_{2n+2}, t)\} + \frac{\epsilon}{2^{2n+1}}
\end{aligned} \tag{3.5}
$$

Similarly, we also have

$$
\begin{aligned}
&M(x_{2n+2}, x_{2n+3}, kt)\\
&\qquad \geq \min\{M(x_{2n+1}, x_{2n+2}, t), M(x_{2n+2}, x_{2n+3}, t)\} - \frac{\epsilon}{2^{2n+2}}
\end{aligned} \tag{3.6}
$$

and

$$
\begin{aligned}
&N(x_{2n+2}, x_{2n+3}, kt)\\
&\qquad \leq \max\{N(x_{2n+1}, x_{2n+2}, t), N(x_{2n+2}, x_{2n+3}, t)\} + \frac{\epsilon}{2^{2n+2}}
\end{aligned} \tag{3.7}
$$

Thus from (3.4), (3.5), (3.6) and (3.7), it follows that

$$M(x_{n+1}, x_{n+2}, kt) \geq \min\{M(x_n, x_{n+1}, t), M(x_{n+1}, x_{n+2}, t)\} - \frac{\epsilon}{2^{n+1}}$$

and

$$N(x_{n+1}, x_{n+2}, kt) \leq \max\{N(x_n, x_{n+1}, t), N(x_{n+1}, x_{n+2}, t)\} + \frac{\epsilon}{2^{n+1}}.$$

For $n = 1, 2, \ldots$, and so, for positive integers n, p

$$M(x_{n+1}, x_{n+2}, kt) \geq \min\{M(x_n, x_{n+1}, t), M(x_{n+1}, x_{n+2}, t/k^p)\} - \frac{\epsilon}{2^{n+1}}$$

and

$$N(x_{n+1}, x_{n+2}, kt) \leq \max\{N(x_n, x_{n+1}, t), N(x_{n+1}, x_{n+2}, t/k^p)\} + \frac{\epsilon}{2^{n+1}}.$$

Thus, since $\lim_{p\to\infty} M(x_{n+1}, x_{n+2}, t/k^p) = 0$ and $\lim_{p\to\infty} N(x_{n+1}, x_{n+2}, t/k^p) = 1$, we have

$$M(x_{n+1}, x_{n+2}, kt) \geq M(x_n, x_{n+1}, t) - \frac{\epsilon}{2^{n+1}}$$

and

$$N(x_{n+1}, x_{n+2}, kt) \leq N(x_n, x_{n+1}, t) + \frac{\epsilon}{2^{n+1}}.$$

Since ϵ is arbitrary, taking $\epsilon \to 0$, we have

$$M(x_{n+1}, x_{n+2}, kt) \geq M(x_n, x_{n+1}, t)$$

and

$$N(x_{n+1}, x_{n+2}, kt) \leq N(x_n, x_{n+1}, t).$$

By Lemma 11, $\{x_n\}$ is a Cauchy sequence in X. Since X is complete, $\{x_n\}$ converges to a point $z \in X$. By (3.1) with $\alpha = 1$, we have

$$\begin{aligned}
M^{\bigtriangledown}(x_{2n+2}, &F_1 z, kt) \\
&\geq M_{\bigtriangledown}(F_1 z, F_2 x_{2n+1}, kt) \\
&\geq \min\{M(z, x_{2n+1}, t), M^{\bigtriangledown}(z, F_1 z, t), M^{\bigtriangledown}(x_{2n+1}, F_2 x_{2n+1}, t), \\
&\quad M^{\bigtriangledown}(z, F_2 x_{2n+1}, t), M^{\bigtriangledown}(x_{2n+1}, F_1 z, t), \\
&\quad M^{\bigtriangledown}(z, F_3 x_{2n+1}, t), M^{\bigtriangledown}(x_{2n+1}, F_3 x_{2n+1}, t)\} \\
&\geq \min\{M(z, x_{2n+1}, t), M^{\bigtriangledown}(z, F_1 z, t), M(x_{2n+1}, x_{2n+2}, t), \\
&\quad M(z, x_{2n+2}, t), M^{\bigtriangledown}(x_{2n+1}, F_1 z, t), \\
&\quad M(z, x_{2n+2}, t), M(x_{2n+1}, x_{2n+2}, t)\}
\end{aligned}$$

and

$$\begin{aligned}N^{\triangle}(x_{2n+2}, &F_1z, kt)\\ &\leq N_{\triangle}(F_1z, F_2x_{2n+1}, kt)\\ &\leq \max\{N(z, x_{2n+1}, t), N^{\triangle}(z, F_1z, t), N^{\triangle}(x_{2n+1}, F_2x_{2n+1}, t),\\ &\quad N^{\triangle}(z, F_2x_{2n+1}, t), N^{\triangle}(x_{2n+1}, F_1z, t),\\ &\quad N^{\triangle}(z, F_3x_{2n+1}, t), N^{\triangle}(x_{2n+1}, F_3x_{2n+1}, t)\}\\ &\leq \max\{N(z, x_{2n+1}, t), N^{\triangle}(z, F_1z, t), N(x_{2n+1}, x_{2n+2}, t),\\ &\quad N(z, x_{2n+2}, t), N^{\triangle}(x_{2n+1}, F_1z, t),\\ &\quad N(z, x_{2n+2}, t), N(x_{2n+1}, x_{2n+2}, t)\}\end{aligned}$$

Letting $n \to \infty$, we have

$$M^{\triangledown}(z, F_1z, kt) \geq \min\{M(z, z, t), M^{\triangledown}(z, F_1z, t), M(z, z, t), M(z, z, t),\\ M^{\triangledown}(z, F_1z, t), M(z, z, t), M(z, z, t)\}$$

and

$$N^{\triangle}(z, F_1z, kt) \leq \max\{N(z, z, t), N^{\triangle}(z, F_1z, t), N(z, z, t), N(z, z, t),\\ N^{\triangle}(z, F_1z, t), N(z, z, t), N(z, z, t)\}$$

Then we have

$$M^{\triangledown}(z, F_1z, kt) \geq M^{\triangledown}(z, F_1z, t)$$

and

$$N^{\triangle}(z, F_1z, kt) \leq N^{\triangle}(z, F_1z, t).$$

Therefore by Lemma 12, we have $z \in F_1z$.

Now we prove that $z \in F_2z$.

By (3.1) with $\alpha = 1$, we have

$$\begin{aligned}M^{\triangledown}(z, &F_2x_{2n+1}, kt)\\ &\geq M_{\triangledown}(F_1z, F_2x_{2n+1}, kt)\\ &\geq \min\{M(z, x_{2n+1}, t), M^{\triangledown}(z, F_1z, t), M^{\triangledown}(x_{2n+1}, F_2x_{2n+1}, t),\\ &\quad M^{\triangledown}(z, F_2x_{2n+1}, t), M^{\triangledown}(x_{2n+1}, F_1z, t), M^{\triangledown}(z, F_3x_{2n+1}, t),\\ &\quad M^{\triangledown}(x_{2n+1}, F_3x_{2n+1}, t)\}\\ &\geq \min\{M(z, x_{2n+1}, t), M(z, z, t), M^{\triangledown}(x_{2n+1}, F_2x_{2n+1}, t),\\ &\quad M^{\triangledown}(z, F_2x_{2n+1}, t), M(x_{2n+1}, z, t), M(z, x_{2n+2}, t),\\ &\quad M(x_{2n+1}, x_{2n+2}, t)\}\end{aligned}$$

and

$$\begin{aligned}
N^{\triangle}(z, &F_2 x_{2n+1}, kt) \\
&\le N_{\triangle}(F_1 z, F_2 x_{2n+1}, kt) \\
&\le \max\{N(z, x_{2n+1}, t), N^{\triangle}(z, F_1 z, t), N^{\triangle}(x_{2n+1}, F_2 x_{2n+1}, t), \\
&\quad N^{\triangle}(z, F_2 x_{2n+1}, t), N^{\triangle}(x_{2n+1}, F_1 z, t), N^{\triangle}(z, F_3 x_{2n+1}, t), \\
&\quad N^{\triangle}(x_{2n+1}, F_3 x_{2n+1}, t)\} \\
&\le \max\{N(z, x_{2n+1}, t), N(z, z, t), N^{\triangle}(x_{2n+1}, F_2 x_{2n+1}, t), \\
&\quad N^{\triangle}(z, F_2 x_{2n+1}, t), N(x_{2n+1}, z, t), N(z, x_{2n+2}, t), \\
&\quad N(x_{2n+1}, x_{2n+2}, t)\}
\end{aligned}$$

Letting $n \to \infty$, we obtain

$$\begin{aligned}
M^{\triangledown}(z, F_2 z, kt) \ge \min\{&M(z, z, t), M(z, z, t), M^{\triangledown}(z, F_2 z, t), M^{\triangledown}(z, F_2 z, t), \\
&M(z, z, t), M(z, z, t), M(z, z, t)\}
\end{aligned}$$

and

$$\begin{aligned}
N^{\triangle}(z, F_2 z, kt) \le \max\{&N(z, z, t), N(z, z, t), N^{\triangle}(z, F_2 z, t), N^{\triangle}(z, F_2 z, t), \\
&N(z, z, t), N(z, z, t), N(z, z, t)\}
\end{aligned}$$

Then we have

$$M^{\triangledown}(z, F_2 z, kt) \ge M^{\triangledown}(z, F_2 z, t)$$

and

$$N^{\triangle}(z, F_2 z, kt) \le N^{\triangle}(z, F_2 z, t).$$

Therefore by Lemma 12, we have $z \in F_2 z$.

Similarly we can prove that $z \in F_3 z$. This completes the proof of the Theorem. □

References

[1] C. Alaca, D. Turkoglu and C. Yildiz, *Fixed points in intuitionistic fuzzy metric spaces*, Choas, Solitons & Fractals **29** (2006), 1073–1078.

[2] S. Banach, *Theorie les operations Lineaires*, Manograie Mathematyezne Warsaw, Poland, 1932.

[3] S. S. Chang, Y. J. Cho, B. S. Lee, J. S. Jung and S. M. Kang, *Coincidence point and minimization theorems in fuzzy metric spaces*, Fuzzy Sets and Systems **8** (1997), no. 1, 119–128.

[4] Y. J. Cho, H. K. Pathak, S. M. Kang and J. S. Jung, *Common fixed points of compatible maps of type (β) on fuzzy metric spaces*, Fuzzy Sets and Systems **93** (1988), 99–111.

[5] Z. Deng, *Fuzzy pseudo-metric space*, J. Math. Anal. Appl. **86** (1982), 74–95.

[6] M. Edelstein, *On fixed and periodic points under contractive mappings*, J. London Math. Soc. **37** (1962), 74–79.

[7] M. A. Erceg, *Metric space in fuzzy set theory*, J. Math. Anal. Appl. **69** (1979), 205–230.

[8] J. X. Fang, *On fixed point theorems in fuzzy metric spaces*, Fuzzy Sets and Systems **46** (1992), 107–113.

[9] A. George and P. Veeramani, *On some results of analysis for fuzzy metric spaces*, Fuzzy Sets and Systems **90** (1997), 365–368.
[10] M. Grabiec, *Fixed points in fuzzy metric space*, Fuzzy Sets and Systems **27** (1988), 385–389.
[11] V. Gregori, S. Romaguera and P. Veeramani, *A note on intuitionistic fuzzy metric spaces*, Choas, Solitons & Fractals **28** (2006), 902–905.
[12] G. Jungck, *Commuting maps and fixed points*, Amer. Math. Monthly **83** (1976), 261–263.
[13] O. Kaleva and S. Seikkala, *On fuzzy metric spaces*, Fuzzy Sets and Systems **12** (1984), 225–229.
[14] E. P. Klement, R. Mesiar and E. Pap, *Triangular Norms*, Kluwer Academic Pub. Trends in Logic **8**, Dordrecht 2000.
[15] O. Kramosil and J. Michalek, *Fuzzy metric and statistical metric spaces*, Kybernetica **11** (1975), 326–334.
[16] I. Kubiaczyk and Sushil Sharma, *Common coincidence point in fuzzy metric space*, J. Fuzzy Math. **11** (2003), 1–5.
[17] I. Kubiaczyk and Sushil Sharma, *Common fixed point, multimaps in fuzzy metric space*, East Asian Math. J. Korea **18** (2002), no. 2, 175–182.
[18] K. Menger, *Statistical metrices*, Proc. Nat. Acad. Sci. **28** (1942), 535–537.
[19] S. N. Mishra, N. Sharma and S. L. Singh, *Common fixed points of maps on fuzzy metric spaces*, Internat. J. Math. Math. Sci. **17** (1994), 253–258.
[20] J. H. Park, *Intuitionistic fuzzy metric spaces*, Choas, Solitons & Fractals **22** (2004), 1039–1046.
[21] R. Saadati and J. H. Park, *On the intuitionistic topological spaces*, Choas, Solitons & Fractals **27** (2006), 331–344.
[22] B. Schweizer and A. Sklar, *Statistical metric spaces*, Pacific Journal Math. **10** (1960), 314–334.
[23] Sushil Sharma, *Common fixed point theorems in fuzzy metric space*, Fuzzy Sets and Systems, Vol. 1 27 (2002), 345–352.
[24] D. Turkoglu, C. Alaca, Y. J. Cho and C. Yildiz, *Common fixed point theorems in intuitionistic fuzzy metric spaces*, J. Appl. Math. & Computing.
[25] D. Turkoglu, C. Alaca and C. Yildiz, *Compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces*, Demonstratio Math. **39** (2006), no. 3, 671–684.
[26] R. R. Yager, *On a class of weak triangular norm operators*, Information Sciences **96** (1997), no. 1-2, 47–78.
[27] L. A. Zadeh, *Fuzzy Sets*, Inform. and Control **8** (1965), 338–353.

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