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Prediction of the Dependence of Phase Velocity on Porosity in Cancellous Bone

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Abstract

In recent years, quantitative ultrasound (QUS) technologies have played a growing role in the diagnosis of osteoporosis. Most of the commercial bone sonometers measure speed of sound (SOS) and/or broadband ultrasonic attenuation (BUA) at peripheral skeletal sites. However, the QUS parameters are purely empirical measures that have not yet been firmly linked to physical parameters such as bone strength or porosity. In the present study, the theoretical models for wave propagation in cancellous bone, such as the Biot model, the stratified model, and the modified Biot-Attenborough (MBA) model, were applied to predict the dependence of phase velocity on porosity in cancellous bone. The optimum values for the input parameters of the three models in cancellous bone were determined by comparing the predictions with the previously published measurements in human cancellous bone *in vitro*. This modeling effort is relevant to the use of QUS in the diagnosis of osteoporosis because SOS is negatively correlated to the fracture risk of bone, and also advances our understanding of the relationship between phase velocity and porosity in cancellous bone.

Keywords: *Osteoporosis, Cancellous bone, Porosity, Ultrasound, Phase velocity, Biot model, Stratified model, Modified Biot-Attenborough model*

I. Introduction

Osteoporosis is a skeletal disease characterized by two factors: reduced bone mass and the disruption of the microstructure of the bone tissue. These symptoms increase bone fragility and can contribute to eventual fracture. If diagnosed sufficiently early, osteoporosis may be treated effectively to reduce the risk of fracture. Previous studies have shown that bone mass is the most important determinant of bone strength and accounts for up to 80% of its variance [1]. Currently, dual energy x-ray energy x-ray absorptiometry (DEXA) providing wo-dimensional representation of bone mineral density (BMD) is the preferred diagnostic technique for identifying individuals at risk of osteoporosis and monitoring response to treatment.

In recent years, quantitative ultrasound (QUS) technologies have played a growing role in the diagnosis of osteoporosis. Most of the commercial bone somometers measure speed of sound (SOS) and/orbroadband ultrasonic attenuation (BUA) at peripheral skeletal sites such as the calcaneus [2]. Although QUS is relatively inexpensive, is easily utilized, and is a close second to DEXA in the indicating hip fracture risk, the QUS parameters are purely empirical measures that have not yet been firmly linked to physical parameters such as bone strength or porosity. Establishing such relations

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through a validated predictive model for ultrasonic wave propagation in bone would be a significant advance.

Since the initial suggestion that the inversion of ultrasonic measurements might be used to infer material parameters of cancellous bone and thereby potentially indicate bone health [3], a range of propagation models have been proposed [4–12]. There have been numerous papers comparing the predictions of one or more of these models with measurements of propagation through cancellous bone and through phantoms. It is not uncommon, for example, to plot the phase velocity as a function of porosity and then to compare which model best predicts the experimental observations. Such a procedure involves a number of constraints, such as ensuring that only physically realistic values are allowable for fitting parameters. A key issue is in identifying whether any observed variations in the predictions of different models are the result of differences in the physics of the models, or due to inconsistency in the input values used for each model.

In the present study, the theoretical models for wave propagation in cancellous bone, such as the Biot model [4–6], the stratified model [7–10], and the modified Biot–Attenborough (MBA) model [11–13], were applied to predict the dependence of phase velocity on porosity in cancellous bone. The optimum values for the input parameters of the three models in cancellous bone were determined by comparing the predictions with the previously published measurements in human cancellous bone *in vitro*. In Section II, the Biot model, the stratified model, and the MBA model will be summarized without repeating the derivations.

II. Theoretical Models for Wave Propagation in Cancellous Bone

The Biot model [4] for elastic wave propagation in porous media has attracted the most attention with regard to modeling wave propagation in cancellous bone. This application of the Biot model was reviewed by Haire and Langton [5]. Recently, Wear *et al.* [6] successfully applied the Biot model to predict the dependence of phase velocity on porosity in human calcaneus samples *in vitro*. The Biot model predicts the existence of two compressional waves, called the fast and the slow waves, and one shear wave. The fast wave is a bulk wave where the fluid and the solid are locked together and move in phase. It usually exhibits negligible dispersion. The slow wave corresponds to a bulk wave where the fluid and the solid move out of phase. It is usually highly attenuated. The greatest difficulty in the application of the Biot model is that it depends on a large number of input parameters that are not necessarily easily measured, including elastic and structural parameters.

For an alternative propagation model in cancellous bone, Hughes *et al.* [7] first adopted the stratified model consisting of periodically alternating parallel solid–fluid layers, based on a work by Schoenberg [8], to predict the angular dependence of phase velocities for the fast and the slow waves in bovine cancellous bone *in vitro*. Wear [9] successfully applied the stratified model to predict negative dispersion of phase velocity in human cancellous bone *in vitro*. Lin *et al.* [10] also employed the stratified model to predict measurements of phase velocity and attenuation in sheep cancellous bone *in vitro*. The complex architecture of cancellous bone is modeled as a simple layered structure of alternating parallel bone–marrow plates. The stratified model requires only six input parameters, which compared with the Biot model, is an advantage for computational issues. The stratified model predicts two compressional waves that are equivalent to the fast and the slow wave of the Biot model but it is essentially an anisotropic model. This is a very interesting alternative approach to the Biot model for propagation in porous media.

Roh and Yoon [11] proposed the MBA model for acoustic wave propagation in fluid–saturated porous media such as cancellous bone and water–saturated sediments. Recently, Lee *et al.* [12,13] successfully applied the MBA model to predict the dependences of phase velocity and attenuation on frequency and porosity in bovine and human cancellous bones *in vitro*. The MBA model is based on separate treatments of the viscous and the thermal effects of the fluid since this simplifies the derivation according to Attenborough [14]. Zwikker and Kosten [15] showed that, at least in the limiting cases of low and high frequency, such independent treatments give the correct results for motion in a viscous, conducting fluid contained within a cylindrical pore when expressed in terms of a complex density and a complex compressibility. The Biot model has the merit of including the viscous effect of the interstitial fluid, but it does not take into account the thermal effect. In contrast, the MBA model includes the thermal effect specified by an analytic solution and also allows for an elastic solid and fluid medium by means of a parametric fit. However, the thermal effect is relatively small for wave propagation in cancellous bone.

III. Results and Discussion

Table 1 lists the input parameters of the Biot model, the stratified model, and the MBA model for cancellous bone. The intrinsic ultrasonic and physical parameters for cancellous bone tissue are assumed to be the same as those for solid bone (or cortical bone) comprising the skeletal frame. The parameters for fat are normally used for the pore fluid because bone marrow is mainly composed of fat with very little blood and tissue fluid. In the present study, however, all parameter values for the pore fluid are equal to those for water because the predictions are compared with the experimental data for defatted, water-saturated bone samples of Wear *et al.* [6].

Figure 1 shows the phase velocities at 0.5 MHz as functions of porosity predicted by the Biot model (solid curve), the stratified model (dashed curve), and the MBA model (dotted curve) with the input parameters listed in Table 1. The experimental data for the 53 human calcaneus samples (with porosities from 86 to 98%) in the figure were taken from Wear *et al.* [6]. The 23 circles denote the samples for which porosity was directly measured by using micro computed tomography (micro CT). The 30 asterisks denote the samples for which porosity was estimated from DEXA measurements. The phase velocity at 0.5 MHz in all 53 bone samples was measured in a water tank using a pair of coaxially aligned Panametrics (Waltham, MA) 1 in. diameter, focused (focal length = 1.5 in.), broadband transducers with center frequencies of 0.5 MHz [6]. The relative orientation between the ultrasound beam and the human calcanea was the mediolateral (ML) direction, which is the same as with *in vivo* measurements performed with commercial bone sonometers. It can be found that the model predictions agree reasonably well with the experimental data, even if the data are limited to a narrow range of porosities (from 86 to 98%).

In the Biot model, the exponent n of the power law for the elastic moduliis a fitting parameter, which is optimized by curve fitting to the experimental data of phase velocity as a function of porosity. As seen in Figure 1, The Biot model is well fitted to the experimental data with an optimized exponent of n = 1.75 (Table 1). All of the additional input parameters of the Biot model were taken from Wear *et al.* [6]. In predicting the phase velocity in cancellous bone, it may be regarded that the phase velocity parameter s_2 of the MBA model plays a role equivalent to that of the exponent n of the Biot model. As with n, s_2 can also be optimized by curve fitting to the experimental data of phase velocity as a function of porosity. The value of s_2 obtained by curve fitting to the data for all 53 samples was 1.23 (Table 1). The values of the common input parameters of the MBA model were taken by our previous work [13]. The values of the input parameters required by the stratified model were chosen to be consistent with those used in the Biot model and the MBA model.

As Figure 1 shows, a good agreement can be found between the predictions of the Biot model and the MBA model. This may be attributable to the use of consistent values for the input parameters required

Table 1. Input parameters of the Biot model, the stratified model, and the MBA model for cancellous bone.

Parameter	Biot model	Stratified model	MBA model
Density of solid (ρ_s)	1800 kg/m^3	1800 kg/m^3	1800 kg/m^3
Compressional speed of solid (c_s)		2500 m/s	2500 m/s
Shear speed of solid (c_{sh})		1800 m/s	
Young's modulus of solid (E_s)	8.3 GPa		
Poisson's ratio of solid (ν_s)	0.3		
Poisson's ratio of frame (ν_b)	0.23		
Density of fluid (ρ_f)	1000 kg/m^3	1000 kg/m^3	1000 kg/m^3
Compressional speed of fluid (c_f)		1483 m/s	1483 m/s
Bulk modulus of fluid (B_f)	2.2 GPa		
Viscosity of fluid (η)	0.001 Pa s		
Kinematic viscosity of fluid (ν)			1×10^{-6} m^2/s
Specific heat ratio of fluid (γ)			1.004
Prandtl number of fluid (N_{Pr})			7
Permeability (k)	5×10^{-9} m^{-2}		
Variable (r)	0.25		
Frequency (f)	0.5 MHz		0.5 MHz
Porosity (β)	Variable	Variable	Variable
Pore radius (a)	Depend on β		0.5 mm
Tortuosity (α)	Depend on β		1
Exponent (n)	1.75		
Boundary condition parameter (s_1)			1.5
Phase velocity parameter (s_2)			1.23

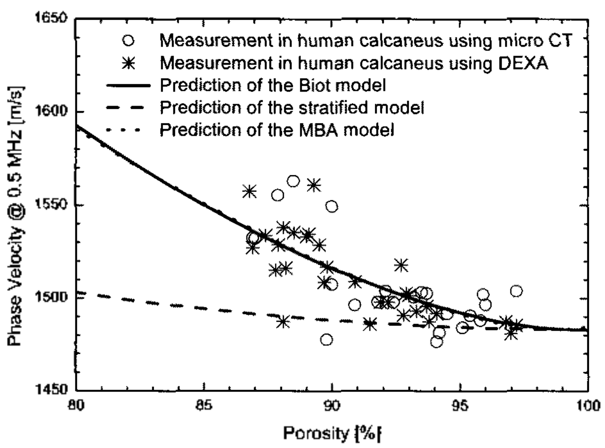

Fig. 1. Phase velocities at 0.5 MHz as functions of porosity predicted by the Biot model, the stratified model, and the MBA model with the input parameters listed in Table 1. The experimental data for the 53 human calcaneus samples (with porosities from 86 to 98%) in the figure were taken from Wear *et al.* [6].

by these two models. One feature of this range of models is the numerous parameters used as input or as fitting parameters. For example, the Biot model requires fourteen input parameters that are not necessarily easily measured, including one fitting parameter. The MBA model also requires fourteen input parameters including three empirical parameters determined from experimental data [11]. Thus, the two models can be considered to be equally efficient in terms of the number of input parameters. However, the MBA model uses commonly known input parameters, at the cost of introducing simplifications such as an enforced tortuosity of unity. As previously applied to bovine cancellous bone [12], the tortuosity α is set equal to unity based on the assumption that all of the cylindrical pores in cancellous bone have identical orientation normal to the surface and are parallel to the wave propagation direction.

Significant differences can be observed between the Biot (or the MBA) model and the stratified model, particularly at the lower porosities. The discrepancy gets wider as the porosity decreases. This discrepancy may be attributable to the fact that they were derived from fundamentally different perspectives.

Although the stratified model has been relatively successful in predicting the anisotropic nature of the fast and the slow wave velocities [7], it clearly gives an oversimplification of the structure of cancellous bone. Bone consisting of solid, parallel plates exists in very few skeletal sites over areas of less than a few centimeters, and is more usually found having arching plates filled with perforations. Moreover, the omission of fluid viscosity by Schoenberg's approach prevents it from accounting for viscous effects; similarly thermal effects are neglected [8]. By contrast, the Biot (or the MBA) model provides a comprehensive and more realistic description of the porous structure and fluid dynamics. Those different features may result in substantial discrepancy between the Biot (or the MBA) model and the stratified model.

The porosities of β>80% are the key region, and that fitting here is more important than for other porosities. This importance arises because, in terms of porosity, in human cancellous bone a 'normal' sample might reasonably be expected to exhibit β>90%, while greater porosities might be suggestive of the possibility of osteoporosis. The inset dataset taken from the work of Wear *et al.* [6] illustrates this point well. When the Biot model and the MBA model fitted to this data set, these require that the exponent n takes a value of n = 1.75 in the Biot model and the phase velocity parameter s_2 takes a value of $s_2 = 1.23$ in the MBA model. These optimum values of n and s_2 were obtained by observing the minimum root −mean−square error (RMSE) of curve fits of the models (varying the values of n and s_2 as free parameters) to the experimental data of phase velocity as a function of porosity, respectively. The stratified model, which requires only six input parameters without fitting parameters, follows the spread of data to some success. Considering the scatter of the data, if there were no other information available (such as on the anisotropy of the measurement), then the measurement of SOS as a function of porosity would present significant challenges in differentiating between normal and osteoporotic bone.

IV. Conclusions

In the present study, the theoretical models for wave propagation in cancellous bone, such as the Biot model, the stratified model, and the MBA model, were applied to predict the dependence of phase velocity on porosity in cancellous bone. The optimum values for the input parameters of the three models in cancellous bone were determined by comparing the predictions with the previously published measurements in human cancellous bone *in vitro*. This modeling effort is relevant to the use of QUS in the diagnosis of osteoporosis because SOS is negatively correlated to the fracture risk of bone, and also advances our understanding of the relationship between phase velocity and porosity in cancellous bone.

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