A Study on Intelligent Decentralized Active Suspension Control System with Descriptor LMI Design Method

Jung-Hyen Park, *Member, KIMICS*

***Abstract*—**An Intelligent optimal control system design algorithm in active suspension equipment adopting linear matrix inequalities control system design theory with representing by descriptor system form is presented. The validity of the linear matrix inequalities intelligent decentralized control system design with representing by descriptor system form in active suspension system through the numerical examples is also investigated.

***Index Terms*—**Active Suspension, Intelligent Control, Linear Matrix Inequalities, Descriptor System.

I. INTRODUCTION

This paper proposes modeling and design methods in vehicle suspension system to analyze active suspension equipment by adopting linear matrix inequalities (LMI) theory to design intelligent decentralized robust control system. Recently, in the field of suspension system designs, it is general to adopt active control scheme for stiffness and damping. Connection with the other complicate vehicle stability control equipment is also intricate. It is required for the control system scheme to design more robust, fast response and high precision control equipment. It is known that the active suspension system is much better than passive spring-damper system in designing the suspension equipment [1]-[5].

In this paper, I deal with a design method based upon intelligent decentralized robust control solution which is obtained by descriptor LMI theory for improving vehicle performance and driver's ride comfort problems. In the problems to improve ride comfort, it is most important indicator to control bouncing displacement and pitching angle vibration on driving vehicle. The method to control bouncing displacement and pitching angle actively in this paper assures the robust performance and driver's ride comfort problem to the unpredictably added road disturbances under the steady speed driving condition.

The descriptor LMI robust h^{∞} controller is designed based on a steady speed driving real vehicle system into 4-DOF linear vehicle system model which represents the bouncing displacement and pitching angle of a vehicle concerned with front-rear parts bouncing displacements.

Manuscript received February 8, 2008; revised May 4, 2008. Jung-Hyen Park is Associate Professor with the Department of Automotive & Mechanical Engineering, Silla University, Pusan, 617-736, Korea

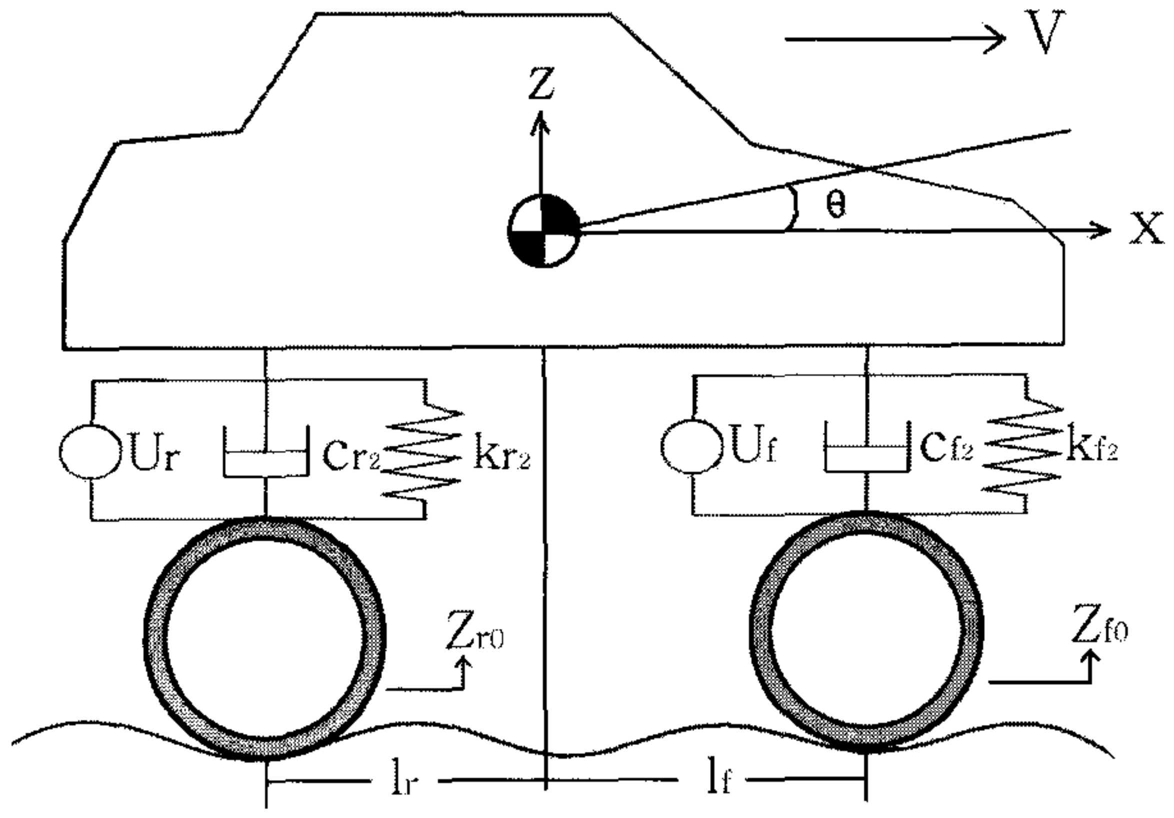


Fig. 1 Steady speed driving vehicle system.

The active suspension system with considering location of front-rear wheelbase and driving velocity by road disturbance input time delay is analyzed and the robust control system is also designed. The validity of the descriptor LMI intelligent decentralized robust control system design in active suspension system through the numerical examples and experiments is also investigated.

II. DESCRIPTOR LMI DESIGN METHOD

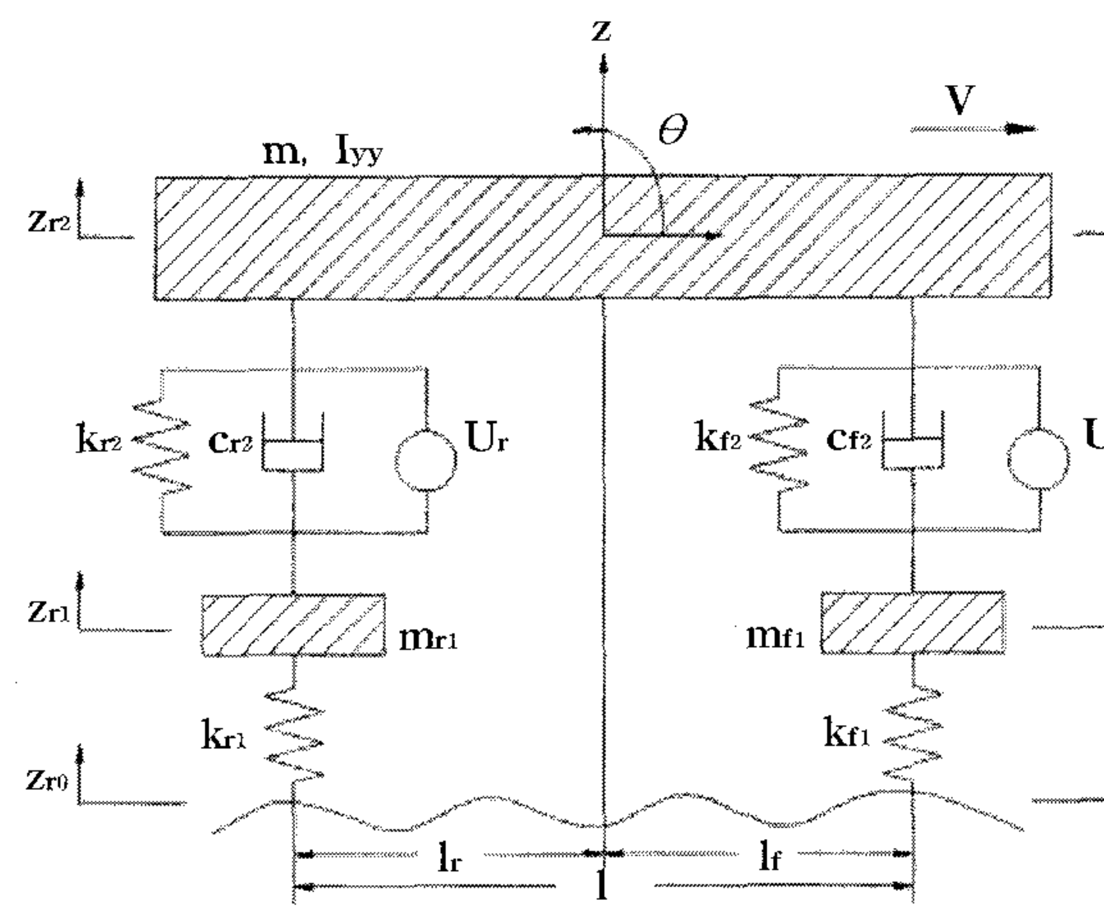


Fig. 2 4-DOF vehicle system model.

The analysis, modeling and design object used for this paper is a 4-DOF vehicle system model shown on figure 2 [6]-[7]. In Fig. 2, V denotes the velocity of the driving direction, i.e. the longitudinal velocity (x -direction); m and I_{yy} denote the mass of the vehicle and the pitch moment of inertia about its mass center in the lateral direction (y-direction); θ and z denote the pitching angle and the bouncing displacement, i.e. the upper and lower motion (z-direction); U_f and U_r denote the control inputs to the front part suspension and rear part suspension; z_{f0} and z_{r0} denote road displacement disturbances to the front part suspension and rear part suspension, respectively.

Based on the 4-DOF vehicle system model analysis frames shown in Fig. 2, the equations of the vehicle motion can be obtained as follows. The dynamic equations of the suspension upper part mass can be defined as

$$m\ddot{z} = F_f + F_r, \quad I_{yy}\ddot{\theta} = l_f F_f - l_r F_r$$
$$z = \frac{l_r z_{f2} + l_f z_{r2}}{l}, \quad \theta = \frac{z_{f2} - z_{r2}}{l} \tag{1}$$

where $F_f = -k_{f2}(z_{f2} - z_{f1}) - c_{f2}(\dot{z}_{f2} - \dot{z}_{f1})$ denotes the force transmitted through the front part suspension, and $F_r = -k_{r2}(z_{r2} - z_{r1}) - c_{r2}(\dot{z}_{r2} - \dot{z}_{r1})$ denotes the force transmitted through the rear part suspension; k and c denote stiffness and damping coefficient. The dynamic equations of the suspension lower part mass in the front-rear parts can be also defined as follows.

$$m_{f1}\ddot{z}_{f1} = -F_f - k_{f1}(z_{f1} - z_{f0}) + U_f$$
$$m_{r1}\ddot{z}_{r1} = -F_r - k_{r1}(z_{r1} - z_{r0}) + U_r \tag{2}$$

Assuming that $I_{yy} = ml_f l_r$ condition, from 2nd term of the Eq. (1) follows can be obtained.

$$m_{f2}\ddot{z}_{f2} = F_f \quad (m_{f2} = \frac{ml_r}{l})$$
$$m_{r2}\ddot{z}_{r2} = F_r \quad (m_{r2} = \frac{ml_f}{l}) \tag{3}$$

From Eqs. (2) and (3), the matrix representations of the equations of motion are expressed by

$$M_t\ddot{z}_t + C_t\dot{z}_t + K_t z_t = H_t w + F_{t1}u_1 \tag{4}$$
$$M_t\ddot{z}_t + C_t\dot{z}_t + K_t z_t = H_t w + F_{t2}u_2 \tag{5}$$
$$u_1 = U_f, \quad u_2 = U_r,$$

$$M_t = \begin{bmatrix} m_{f1} & 0 & 0 & 0 \\ 0 & m_{f2} & 0 & 0 \\ 0 & 0 & m_{r1} & 0 \\ 0 & 0 & 0 & m_{r2} \end{bmatrix}, \quad z_t = \begin{bmatrix} z_{f1} \\ z_{f2} \\ z_{r1} \\ z_{r2} \end{bmatrix},$$

$$C_t = \begin{bmatrix} c_{f2} & -c_{f2} & 0 & 0 \\ -c_{f2} & c_{f2} & 0 & 0 \\ 0 & 0 & c_{r2} & -c_{r2} \\ 0 & 0 & -c_{r2} & c_{r2} \end{bmatrix}, \quad w = \begin{bmatrix} z_{f0} \\ z_{r0} \end{bmatrix},$$

$$K_t = \begin{bmatrix} (k_{f1}+k_{f2}) & -k_{f2} & 0 & 0 \\ -k_{f2} & k_{f2} & 0 & 0 \\ 0 & 0 & (k_{r1}+k_{r2}) & -k_{r2} \\ 0 & 0 & -k_{r2} & k_{r2} \end{bmatrix},$$

$$H_t = \begin{bmatrix} k_{f1} & 0 \\ 0 & 0 \\ 0 & k_{r1} \\ 0 & 0 \end{bmatrix}, \quad F_{t1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad F_{t2} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

In modeling equations of motion Eq. (4) and (5), the control inputs with considering location of front-rear wheelbase and driving velocity by road disturbance inputs time delay are analyzed as follows,

$$z_{r0}(t) = z_{f0}(t - \frac{l}{V}) \tag{6}$$

and the intelligent decentralized robust control system is designed. In this paper, the control system is designed with the descriptor LMI robust h^{∞} control to suppress the effect of the road disturbance. Eqs. (4) and (5) can be modeled and expressed by descriptor system as

$$E\dot{x} = A_t x + B_{1t}w + B_t u_i$$
$$z = C_{1t}x + D_{12}u_i \tag{6}$$
$$y = C_{2t}x + D_{21}w$$

where x and u denote the descriptor system variables and control input; y and z denote measured output and controlled output, respectively; and w denotes the road disturbances input. System design variables and matrix parameters become as follows (i=1,2).

$$E = \begin{bmatrix} I & 0 \\ 0 & M_t \end{bmatrix}, \quad A_t = \begin{bmatrix} 0 & I \\ -K_t & -C_t \end{bmatrix}$$
$$B_{1t} = \begin{bmatrix} 0 \\ H_t \end{bmatrix}, \quad B_t = \begin{bmatrix} 0 \\ F_{ti} \end{bmatrix}$$
$$C_{2t} = \begin{bmatrix} F_{ti}^T & 0 \end{bmatrix}, \quad x = \begin{bmatrix} z_t \\ \dot{z}_t \end{bmatrix}$$

Because the descriptor equation can preserve physical variables and a physical structure of the control object system, it can be said that it will be an expression as natural as the system that exists. In descriptor system form, coefficient matrices of the motion equation are represented linearly, so it is possible to say that descriptor form is more excellent than state equations for system modeling. The state equation representation of motion equations Eqs. (4) and (5) becomes the following.

$$\dot{x} = A_{ts}x + B_{1ts}w + B_{ts}u$$

$$A_{ts}=\begin{bmatrix}0 & I\\ -M_t^{-1}K_t & -M_t^{-1}C_t\end{bmatrix}$$
$$B_{1ts}=\begin{bmatrix}0\\ M_t^{-1}H_t\end{bmatrix},\quad B_{ts}=\begin{bmatrix}0\\ M_t^{-1}F_{ti}\end{bmatrix}$$

In this case, mass matrix M_t appears in the form of inverse matrix M_t^{-1}. In general, the change in the parameter of the structure system appears complexly in the change of the procession of the coefficient matrices of the state equation because all elements of M_t^{-1} change even when only one of elements of M_t changes. For this case, adopting the descriptor equation form more than the case of the state equation expression can shorten the calculation processing time.

In this paper, h^∞ control problem is to find a controller such that the closed-loop system is internally stable and the following h^∞ norm condition is satisfied.

: $N=\|T_{zw}(s)\|_\infty<\gamma,\ \|T_{zw}(s)\|_\infty=\sup_\omega \sigma_{\max}(T_{zw}(j\omega))$

$T_{zw}(s)$ is the transfer function from the disturbance input w to the controlled output z in the closed-loop system, γ is a prescribed positive number, and $\sigma_{\max}(T_{zw})$ is the maximum singular value of T_{zw}. In order to design descriptor linear matrix inequalities robust h^∞ controller for controlled objective plant represented by Eq. (5), it is considered that robust controller can be expressed as follows.

$$\begin{aligned}\dot{x}_c&=A_cx_c+B_cy\\ u&=C_cx_c\end{aligned}\qquad(7)$$

The necessary and sufficient conditions for the existence of the descriptor LMI robust h^∞ controller are that there exist X and Y which satisfy the follows [8].

$$\begin{bmatrix}B_t^\perp & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}A_tX+X^TA_t^T & B_{1t} & X^TC_{1t}^T\\ B_{1t}^T & -\gamma I & 0\\ C_{1t}X & 0 & -\gamma I\end{bmatrix}\begin{bmatrix}B_t^{\perp T} & 0\\ 0 & I\end{bmatrix}<0$$
$$\begin{bmatrix}E & 0\\ 0 & E^T\end{bmatrix}\begin{bmatrix}X & I\\ I & Y\end{bmatrix}=\begin{bmatrix}X^T & I\\ I & Y^T\end{bmatrix}\begin{bmatrix}E^T & 0\\ 0 & E\end{bmatrix}\geq 0\qquad(8)$$
$$\begin{bmatrix}C_{2t}^{\perp T} & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}Y^TA_t+A_t^TY & YB_{1t} & C_{1t}^T\\ B_{1t}^TY & -\gamma I & 0\\ C_{1t} & 0 & -\gamma I\end{bmatrix}\begin{bmatrix}C_{2t}^\perp & 0\\ 0 & I\end{bmatrix}<0$$

where

$$B_t^\perp\left[B_t\ \ B_t^{\perp T}\right]=\left[0\ \ I\right],\ C_{2t}^{\perp T}\left[C_{2t}^T\ \ C_{2t}^\perp\right]=\left[0\ \ I\right]$$
$$P\cong A_tX+XA_t^T+XC_{1t}^TC_{1t}X+B_{1t}B_{1t}^T-B_tB_t^T<0$$
$$Q\cong YA_t+A^TY+YB_{1t}B_{1t}^TY+C_{1t}^TC_{1t}-C_{2t}^TC_{2t}<0\ .$$

Under the assumptions that the Eq. (8) linear matrix inequalities conditions designed by descriptor system are satisfied, one of the Eq. (6) robust controllers can be obtained as follows [9]-[10].

$$\begin{aligned}A_c&=A_t+B_tC_c-B_cC_{2t}+Y^{-1}C_{1t}^TC_{1t}-Y^{-1}Q(I-XY)^{-1}\\ B_c&=Y^{-1}C_{2t}^T,\ C_c=B_t^TY(I-XY)^{-1}\end{aligned}\qquad(9)$$

III. NUMERICAL SIMULATIONS

For the validities of the proposed modeling and design methods in this paper for vehicle suspension system to analyze active suspension equipment by adopting a descriptor LMI method algorithm to design an intelligent decentralized robust h^∞ control system, numerical simulations are carried out under the condition which added a projecting (unpredicted) road disturbance. Throughout the simulations, the vehicle was driven at steady speed V=60km/h. In detail numerical simulation specifications, those were set that the vehicle mass $m=$ 1790kg, the suspension front lower part mass $m_{f1}=$ 134.1kg, the suspension rear lower part mass $m_{r1}=$ 109.5kg, the pitch moment of inertia $I_{yy}=3523.6$ kgm^2, the tire and the suspension parts stiffness $k_{f1}=k_{r1}=$ 1411N/m and $k_{f2}=k_{r2}=$1376N/m, the suspension part damping coefficients $c_{f2}=c_{r2}=$ 118Ns/m, and l the wheel base parts are $l_f=1.27$m, $l_r=1.55$m.

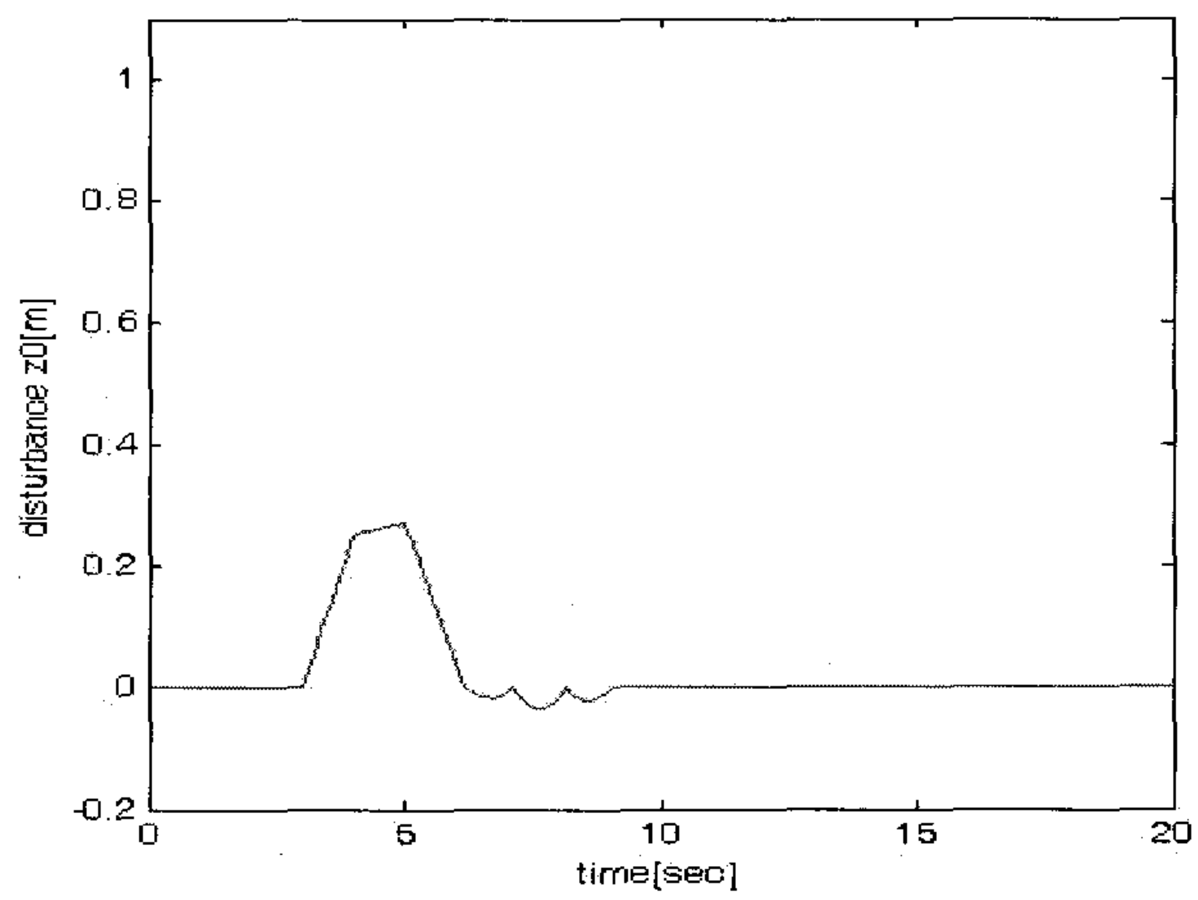


Fig. 3 Projecting road disturbance z_0.

Fig. 3 shows the projecting, uneven road disturbance which is added to the vehicle model. On the figures of throughout the results of simulations, a solid line represents the result of controlled one, and a dotted line represents that of uncontrolled one. The upper and lower motion displacement response of the suspension front lower part, the upper and lower motion displacement response of the suspension front upper part in cases of uncontrolled and controlled are shown in Fig. 4 and Fig. 5, respectively. Fig. 6 shows the front part actuator control input U_f (kgm/s^2).

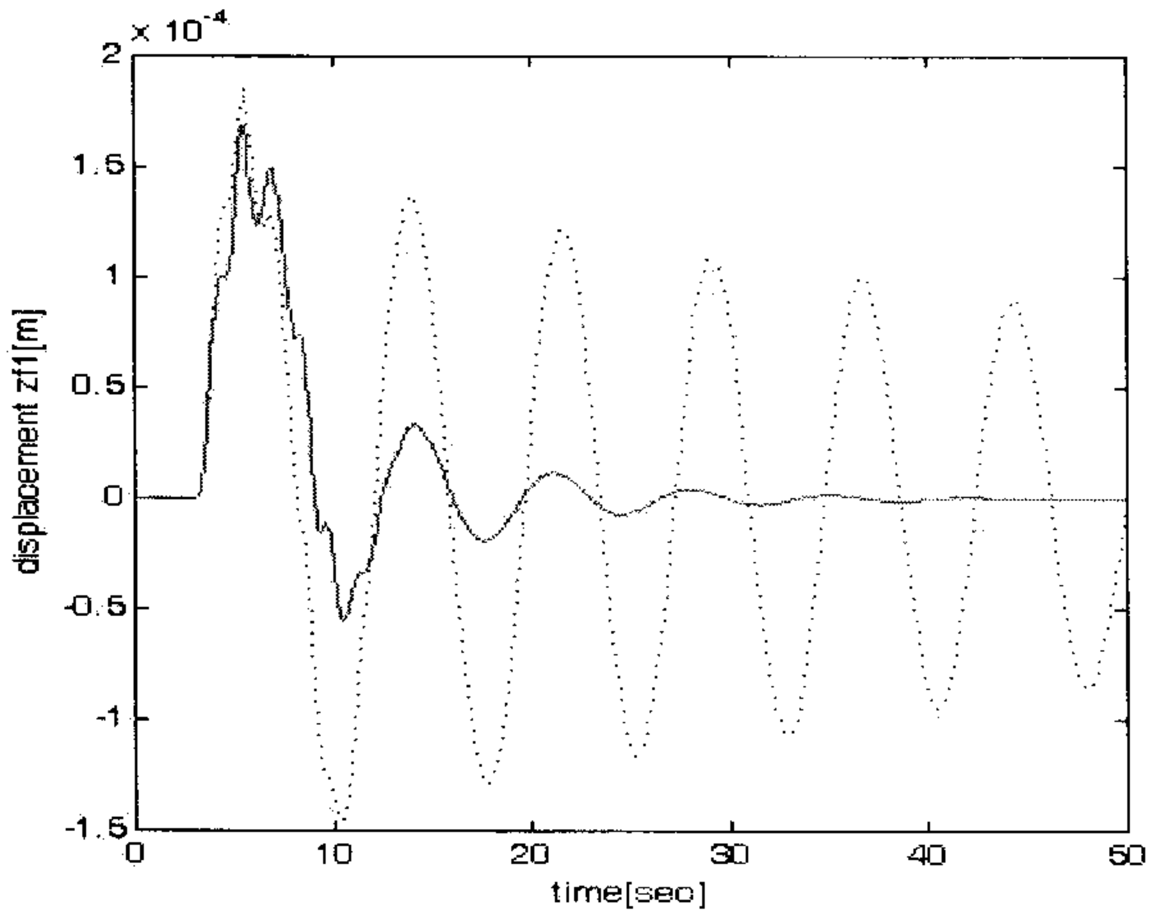


Fig. 4 Road disturbance response z_{f1}.

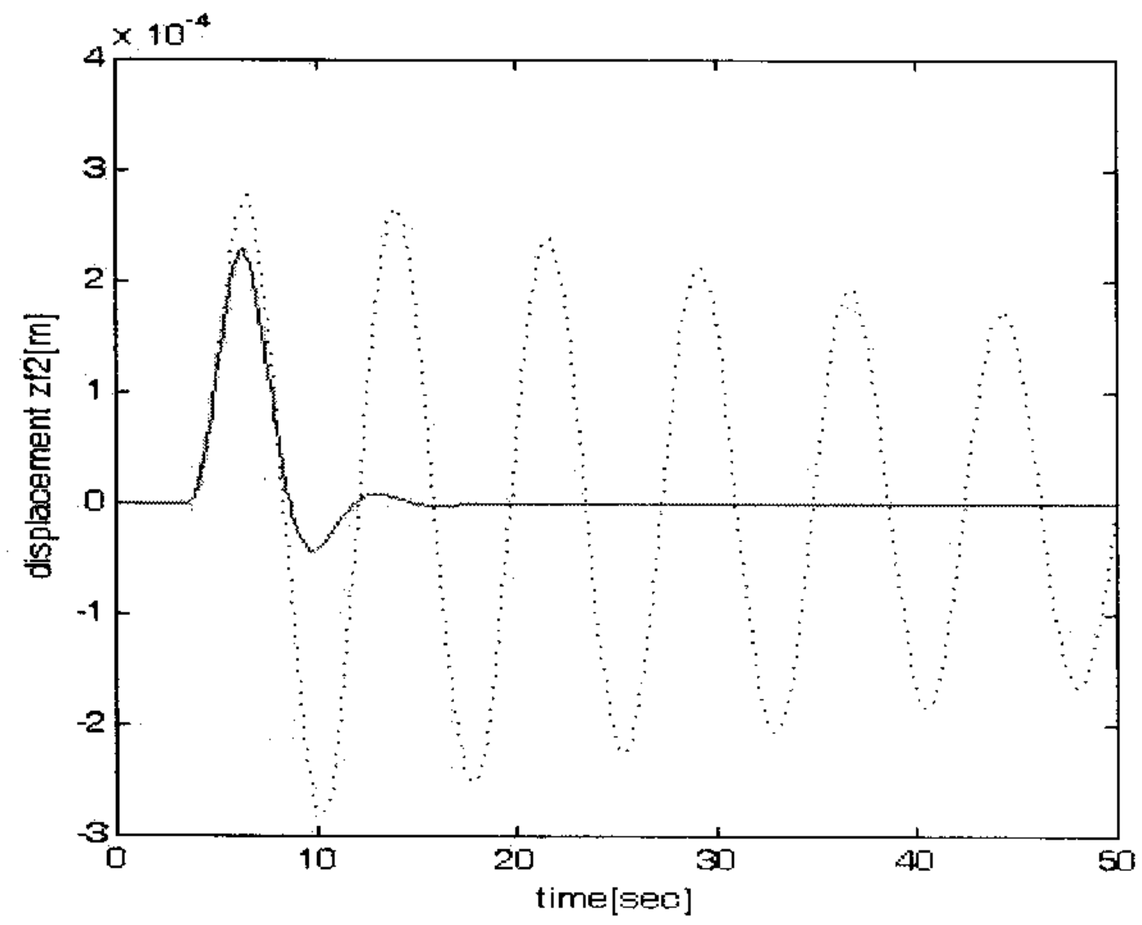


Fig. 5 Road disturbance response z_{f2}.

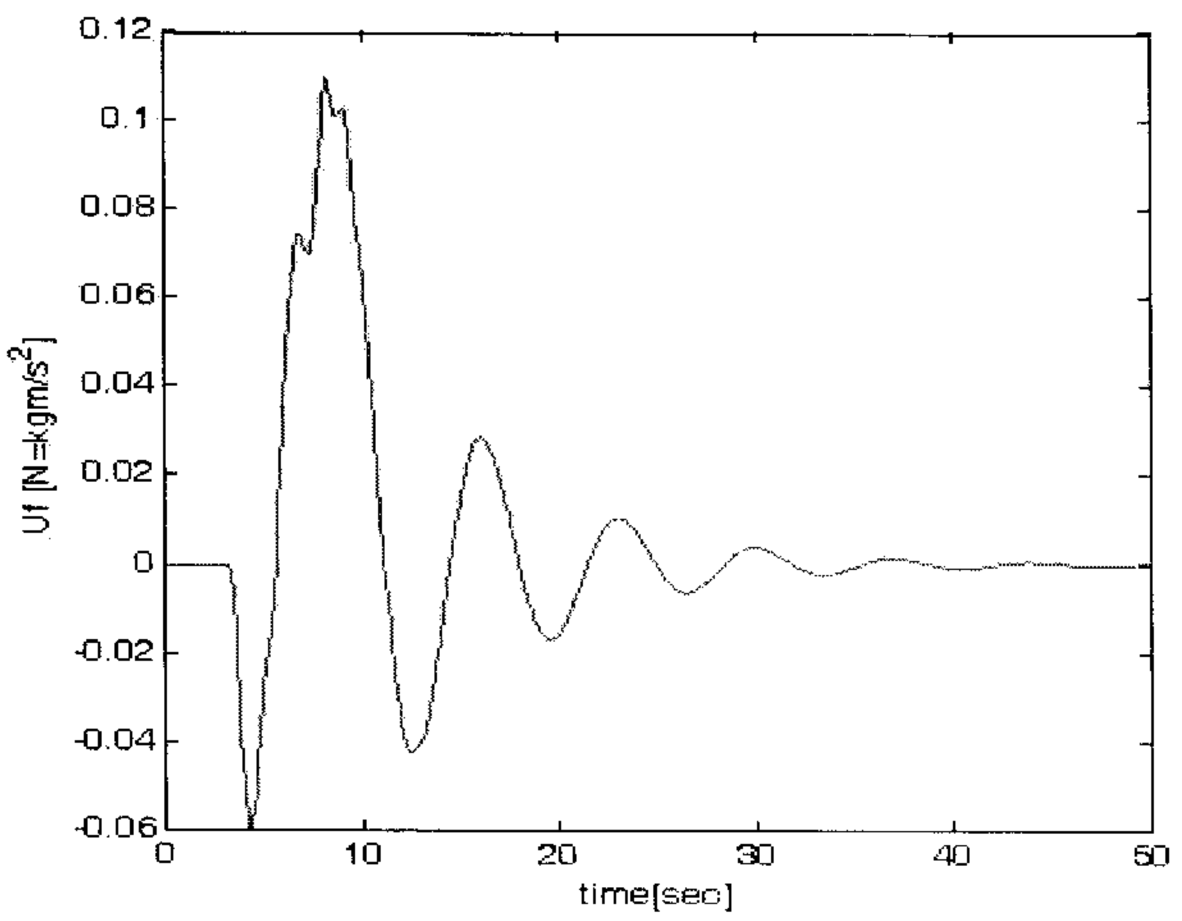


Fig. 6 Front part control input U_f.

Fig. 7, Fig. 8 and Fig. 9 also show the z-direction displacements in uncontrolled, controlled cases, and control input of the rear part, respectively.

Fig. 4 to Fig. 9 show that the active suspension 4-DOF vehicle model with the descriptor LMI robust h^∞ controller exhibit better performances in upper and lower motion displacement vibration control than those of the vehicle with uncontrolled passive spring-damper suspension system.

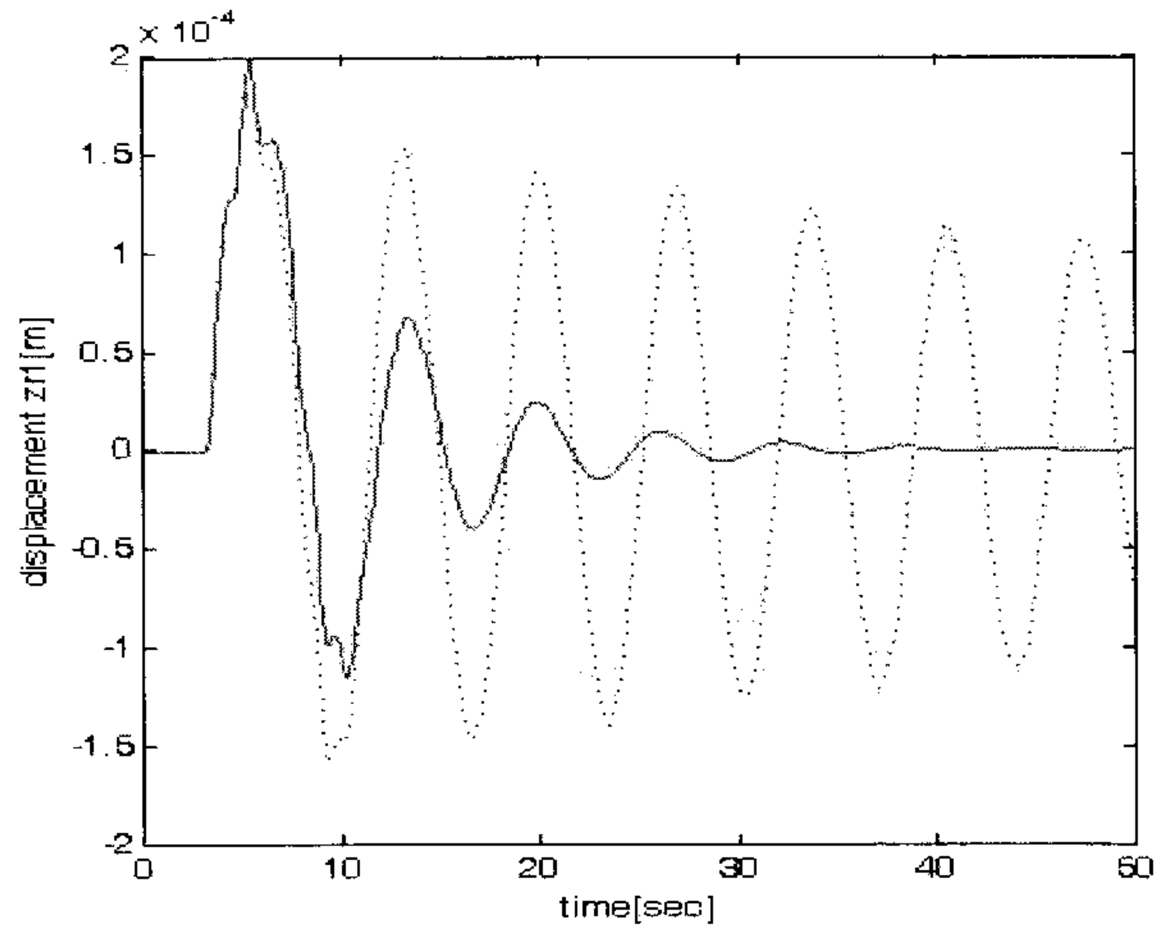


Fig. 7 Road disturbance response z_{r1}.

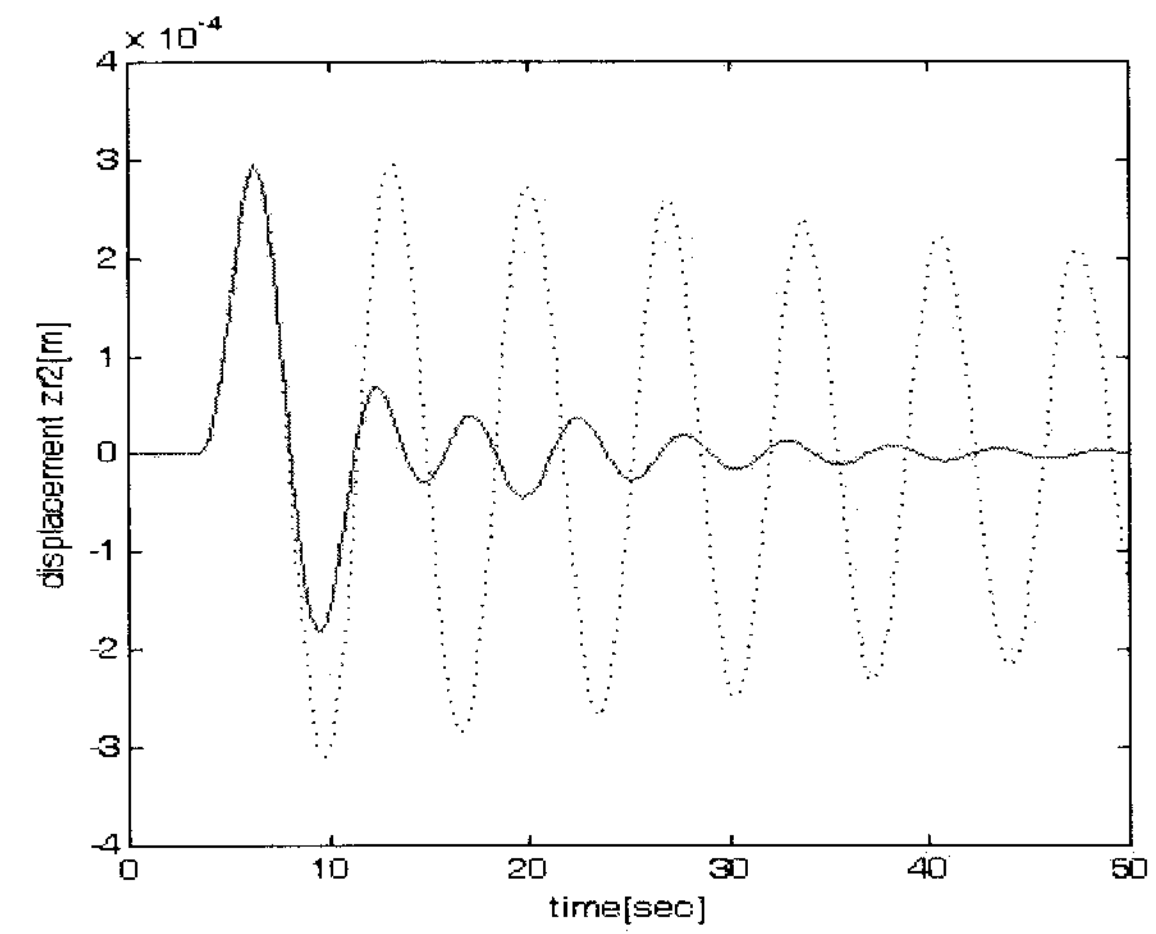


Fig. 8 Road disturbance response z_{r2}.

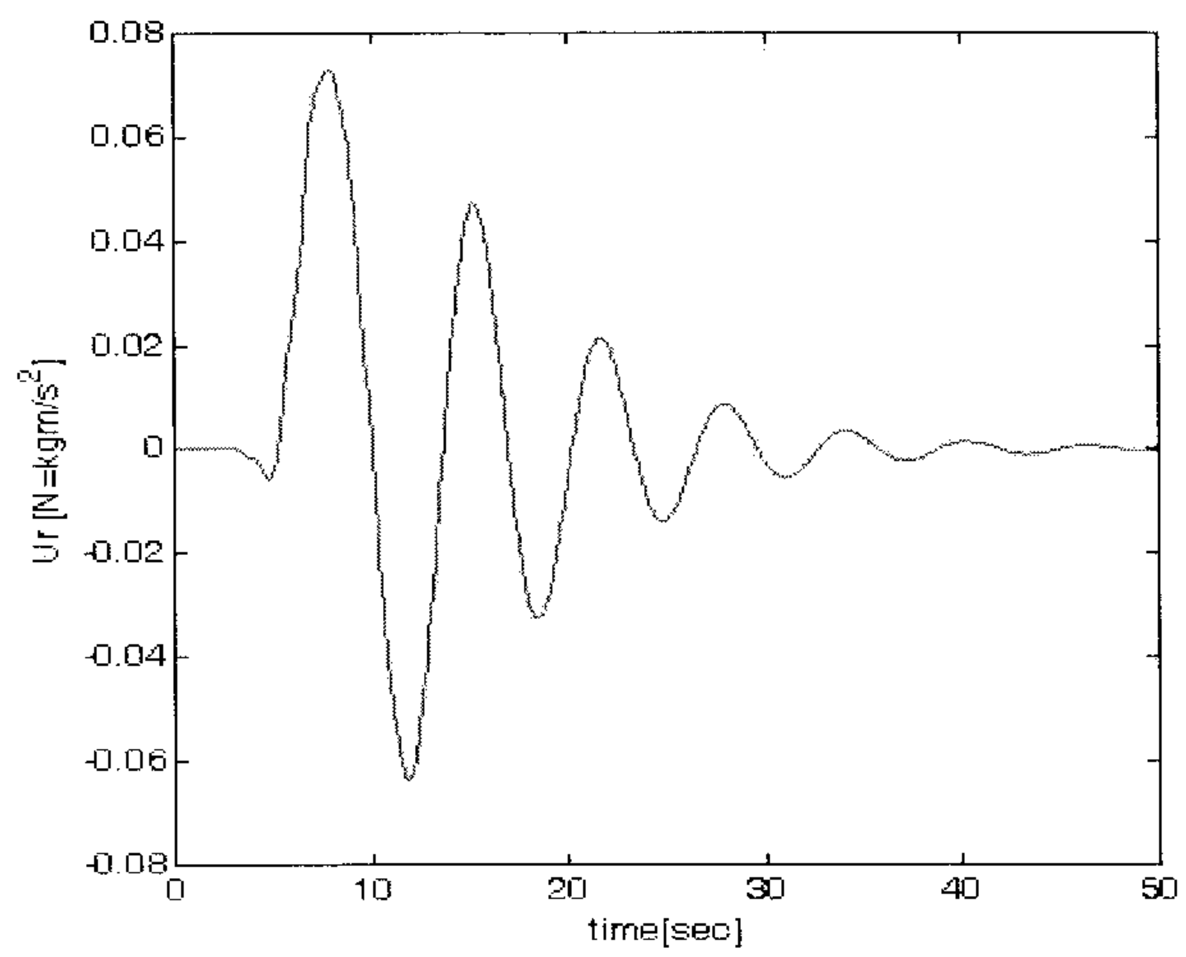


Fig. 9 Rear part control input U_r.

Fig. 10 and Fig. 11 show that the control method in this paper is prominent for improving vehicle performance and driver's ride comfort problems. In Fig. 10, Road disturbance bouncing response z about its mass center becomes smaller after 20 seconds in case of uncontrolled suspension than response of controlled one. However from a point of view which is concerned with ride comfort, steady vibration bouncing response (a solid line in Fig. 10; controlled) to the continuous road disturbances means better performance than irregular bouncing response to the regularly added sinusoidal road disturbance (a dotted line in Fig. 10; uncontrolled). Pitch angle response in Fig. 11 shows the control method in this paper is prominent for improving ride comfort.

IV. CONCLUSIONS

In this paper, I dealt with a design method based upon descriptor LMI robust h^{∞} control solution which is obtained by linear matrix inequalities with descriptor system representation for improving vehicle performance and driver's ride comfort problems. The descriptor linear matrix inequalities robust h^{∞} controller was designed based on a 4-DOF linear vehicle system model which represents the bouncing displacement and pitching angle of a vehicle concerned with front-rear parts bouncing displacements. The active suspension system with considering location of front-rear wheelbase and driving velocity by road disturbance input time delay was analyzed and the robust control system was designed. In order to design descriptor LMI robust controller, the necessary and sufficient conditions for the existence of the linear matrix inequalities to solve robust h^{∞} control problem was investigated.

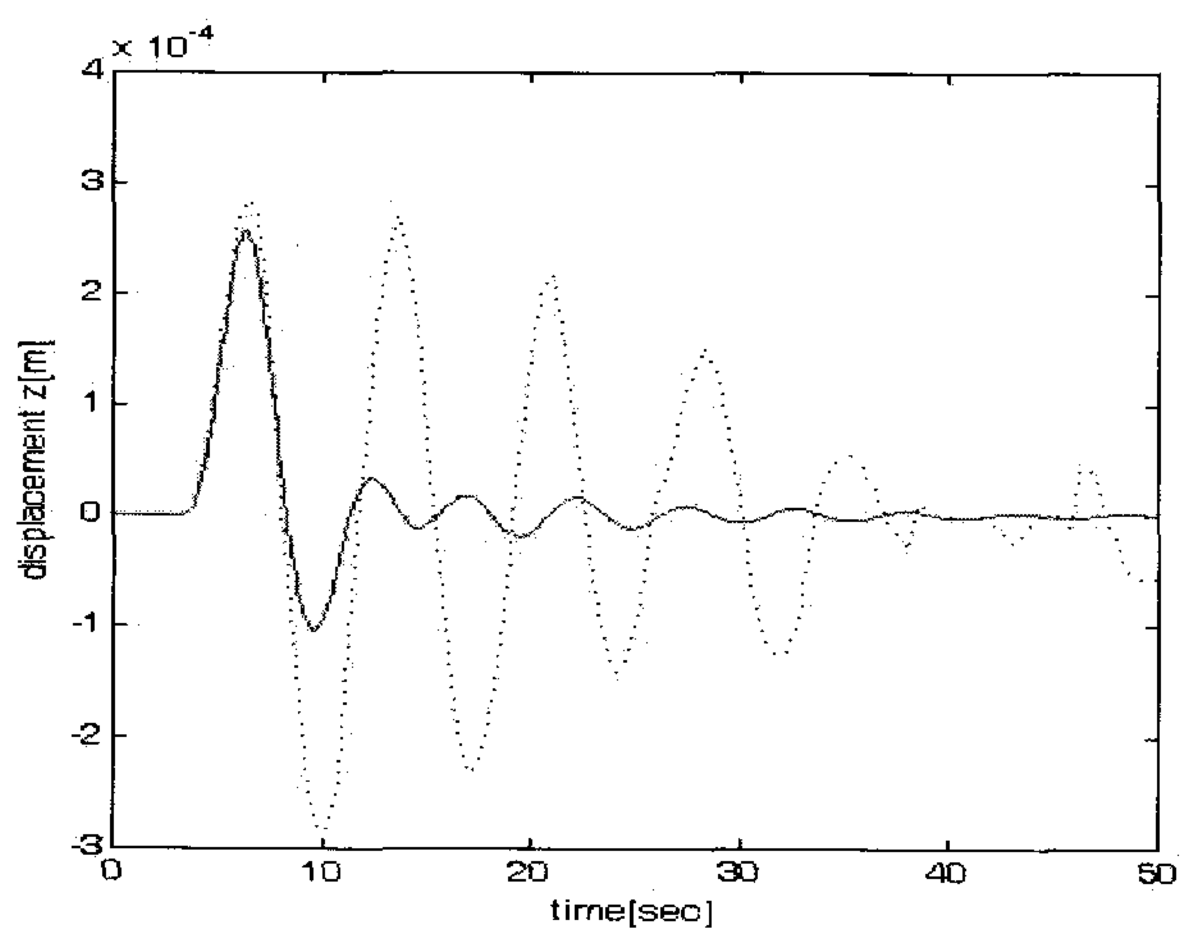


Fig. 10 Road disturbance bounce response z.

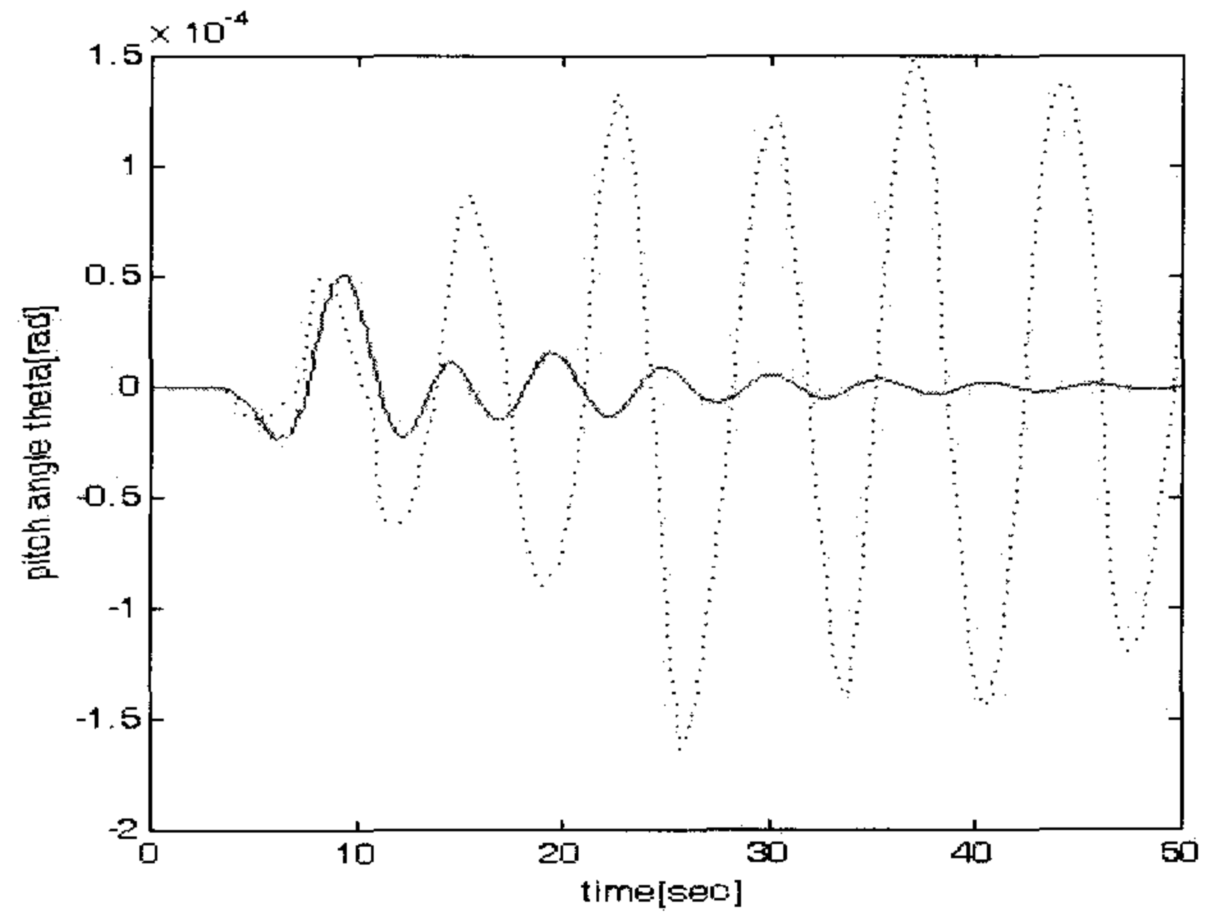


Fig. 11 Road disturbance pitch response θ.

REFERENCES

[1] E. Ono, "A study on the integrated control of automotive dynamics," *Journal of Systems, Control and Information*, vol. 49, No. 6, pp. 205–210, 2005.

[2] J. H. Park, "Combined optimal design with minimum phase system," *Journal of Control, Automation and Systems Engineering*, vol. 10, No. 2, pp. 192–196, 2004.

[3] J. H. Park and S. J. Jang, "A study on active suspension robust control with sensor and actuator location," *Journal of The Korean Institute of Maritime Information & Communication Sciences*, vol. 10, No. 6, pp. 1147–1152, 2006.

[4] J. H. Park and S. J. Jang, "A study on adopting active suspension control in sky hook system," *Journal of The Korean Institute of Maritime Information & Communication Sciences*, vol. 10, No. 5, pp. 950–955, 2006.

[5] J. H. Park and S. H. Kim, "A study on adopting symmetric control system in active suspension equipment," *Journal of The Korean Institute of Maritime Information & Communication Sciences*, vol. 10, No. 4, pp. 769–774, 2006.

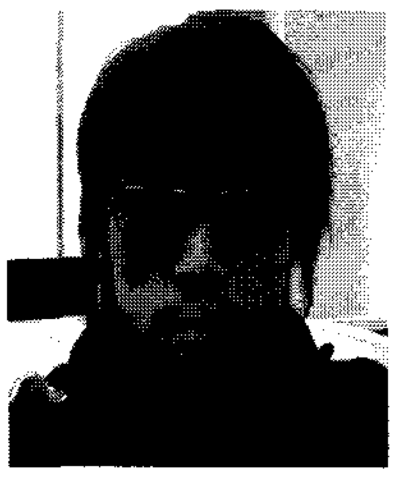
[6] J. H. Park, "A study on active suspension control system in vehicle bouncing and pitching vibration for improving ride comfort," *Journal of The Korea Society of Computer and Information*, vol. 12, No. 2, pp. 325–331, 2007.

[7] J. H. Park, "A study on adopting intelligent control system in active suspension equipment," *Journal of The Korea Society of Computer and Information*, vol. 12, No. 3, pp. 287–293, 2007.

[8] J. H. Park and K. R. Cho, "Optimal structural design for flexible space structure with control system based on LMI," *The Korean Society of Mechanical Engineers/KSME International Journal*, vol. 16, No. 1, pp. 75–82, 2002.

[9] T. Iwasaki, *LMI and control*, Shoko-Do, 1997.

[10] K. M. Grigoriadis and R. E. Skelton, "Low order control design for LMI problems using alternating projection methods," *Proc. Allerton Conf.*, October, 1993.



Jung-Hyen Park
Received Bachelor degree in Mechanical Engineering from Pusan University in 1992, and Master and Doctor of Engineering degrees in Systems Engineering from Kobe University in 1995 and 2000, respectively. Joined the Department of Automotive&Mechanical Engineering of Silla University in 2001, Present an Associate Professor. Research interest in the area of Vehicle System Analysis, System Modeling, System Design, Intelligent Control System Design.